

UPPER AND LOWER BOUNDS ON INVENTORY CONTROL MODEL WITH A FLUCTUATING CURRENCY EXCHANGE RATE

1. Introduction

In today's hypercompetitive business environment characterized by multiple players and ever-dwindling raw materials and resources, businesses need to refocus on how they manage their supply chain. The recent hikes in fuel prices impacted significantly on the prices they (businesses) can charge their customers, in many instances with falling sales and a lot of inventory lying idle for longer than usual. While such phenomena are common in many economies, there are additional challenges facing the so called emerging economies. Companies conducting business in such nations have to worry about currency devaluations, uncertain political situations and, as of late, piracy issues that are getting out of control. Hence, supply chain management in the emerging economies is, needless to say, more complex and challenging.

In the past few years, the world has witnessed an increased liberalization of trading rules among nations. There is an increased flow of goods across many borders. In fact, it appears evident that there is significant interdependence among different nations on each other for raw materials, though this situation affects emerging economies more than others. Almost every country on the globe imports something from another nation. Hence, the reality of exchange rate fluctuation and its impact on trade flow is something that just cannot be ignored.

In the previous study, Wanorie (2006, 2007) used currency historical data to determine a simple currency forecasting model. It was established that the MA(1) model

would be appropriate. This model is determined by using Box-Jenkins (1976) and autoregressive conditional heteroskedastic (ARCH) modeling approaches. There are indeed many complex currency exchange models, but the purpose of this simpler model is to provide a tool not so sophisticated that an inventory decision maker could use especially in the context of the emerging economies. Using the MA(1) model, inventory decisions can be made as presented in Section 3 below and as presented in the previous study (Wanorie, 2007). Because the decision to include x number of periods of demand ended up being more complex than initially expected, a need arose to develop upper and lower bounds on the model to make it manageable for every day inventory decision making.

2 Literature review

Bahmani-Oskooee and Hegerty (2009) after examining the trade flow between the United States and Mexico show that excessive exchange rate fluctuation does influence trade flows in the short term for most industries. In another study, the authors show how the automobile manufacturers in Europe have suffered significantly because of the currency fluctuations, indicating an adverse effect for BMW of €400 million in 2008 (Kaur, 2009).

Mishra et al. (2009) reiterate the goal of supply chain management as the delivery of the right product to the right price at the right price and right cost. In theory, this idea looks simple. There are so many suppliers all around the world who are looking for customers to sell their products and make profits to reward the shareholders and also to continue to expand their business. Also, there are customers all over who are looking for a reliable supplier to provide what they need when they need it at a price agreeable to both. The key question that needs to be addressed is: How does currency fluctuation affect this “agreeable” price?

Indeed there are many reasons to explain why the supply chain management is challenging and that it needs to be scrutinized with utmost diligence, for it is clear that those that manage their supply chain effectively can cut supply costs by as high as 75 percent (Stevenson, 2009). In this study, the focus is on one variable that is unique to emerging economies though any business conducted in nations that import raw materials from other nations is affected to some extent. Businesses importing goods and raw materials from nations that use a different local currency have to deal with fluctuating currency rates. In this research paper, the goal is to show how an upper bound and a lower bound can be established on inventory model based on a simple currency model, developed by Wanorie (2006) in an earlier study, so as to take into account the uncertainty caused for the fluctuation.

The many volatile situations experienced with regards to exchange rates in many nations such as Thailand and South Korea in the late 1990s cannot be ignored. In Africa, Zimbabwe is another example of nations with a tremendous risk companies face if they ignore the impacts of sudden exchange rate fluctuations (often devaluations) which change the cost of the imported goods. One study developed a model that incorporates various sources of uncertainties in the supply chain and point out the fact that many inventory managers are beginning to recognize that effectively managing risks in their business operations plays an important role in successfully managing their inventories (Handfield, Warsing and Xinmin, 2008). While this model is an important contribution to managing risks, it does not address the specific risk related to exchange rate fluctuations.

The literature on currency exchange rate fluctuation risk appears to offer hedging as the ultimate protection even though this strategy may actually be overstretched (Kaur, 2009). The problem and the major limitation of this strategy is that most emerging economies have

currencies that are weak and that do not have fame in the international arena. Hence, hedging, while it is a viable alternative with strong currencies such as Yen, the US dollar, and the Euro, may not be available for most other currencies in Africa and Asia and many other emerging economies.

3 Inventory model and definition of variables used in the model

For the purpose of developing an upper bound and lower bounds on an inventory model that is used to make ordering decisions at any given time, the following variables are utilized.

$Y_t =$ exchange rate (local currency/USD) at time t ;

$\tilde{Y}_{t+L} =$ the expected exchange rate a lead time later;

$L =$ lead time, which is assumed to be constant.

$h =$ holding cost per unit.

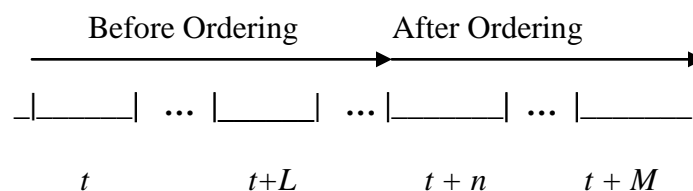
$d =$ duty percentage levied on the arriving goods.

$c =$ the amount paid per unit in foreign currency, US\$.

$\delta =$ discount factor

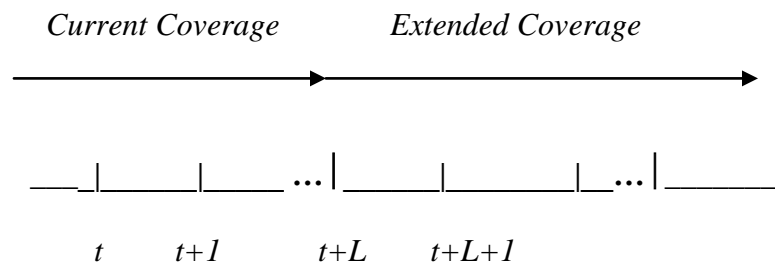
It was shown in the previous study that at any given time t if the purchaser determines a new order has to be placed, the key question to be answered is how many periods of demand to include in that order. In Figures 3.1 and 3.2, the situation as it relates to the decision to be made is presented.

Fig 3.1 General Inventory Ordering Decision at t .



If the current inventory position and prior orders are sufficient to cover the demand only to but not through period $t + L$, a new replenishment order has to be placed so as to avoid stockouts during the period $t + L$, but the manager has to decide whether to order just enough to cover only period $t + L$'s demand or extend the coverage through period $t + M$. Figure 3.2 shows the situation addressed in this particular case.

Fig 3.2 An Inventory Ordering Decision to Extend Coverage.



The inventory manager may order at time t enough to cover demand in periods $t + L + 1$ through $t + M$ or he may wait until period $t + 1$ or later to cover those demands. So, in the case of the situation depicted in the figure above, if $M - L + 1$ periods of demand are to be included, $Q = D_{t+L} + \dots + D_{t+M}$. In each case, whether to extend coverage is dependent on expectations for the exchange rates. If exchange rates for a foreign currency are expected to rise, increasing the acquisition price in terms of the local currency used to purchase inventory, it may be advantageous for the importer to purchase a large quantity at time t so as to extend coverage through many periods. To make this vital decision, the currency model is utilized. $\tilde{y}_t(k) = E_t[y_{t+k}] = y_t e^{k\beta_0 + \beta_1 \epsilon_t}$ is the currency model, where $\tilde{y}_t(k)$ is the estimate of exchange rate for period k made in time t , and β_0 and β_1 are model parameter values determined from historical data for the MA(1) model. As shown in the previous study, the rule to be followed in deciding how many periods of demand to include in a given order is as follows:

- In a general, buy for $L + k$ if

$$\begin{aligned}
cY_1 + cdE[\tilde{Y}_{1+L}]\delta^L + h\delta^L + h\delta^{L+1} + \dots + h\delta^{L+k-1} < \\
& \delta E_1\{\min(c\tilde{Y}_2 + cdE_2[\tilde{Y}_{2+L}] + h\delta^{L+1} + \dots + h\delta^{L+k-2}, \\
& \quad \delta E_2[\min(c\tilde{Y}_3 + cd\delta^L E_3[\tilde{Y}_{3+L}] + h\delta^{L+1} + \dots + h\delta^{L+k-3}, \\
& \quad \quad \delta E_3[\min(c\tilde{Y}_4 + cd\delta^L E_4[\tilde{Y}_{4+L}] + h\delta^{L+1} + \dots + h\delta^{L+k-4}, \\
& \quad \quad \quad \cdot \\
& \quad \quad \quad \cdot \\
& \quad \quad \cdot, \delta cE_{k-1}[\tilde{Y}_k] + \delta^{L+1} cdE_{k-1}[\tilde{Y}_{k+L}])]) \dots)\}. \tag{3-1}
\end{aligned}$$

Let $S_{t,1}$ represent the right side of (2-1), H_1 to represent the holding cost of the left side and P_t the left side of it excluding the holding cost. So

$S_{t,1}$ = the expected discounted cost at time t of meeting demand in period $t + L + 1$ by ordering in period $t + 1$;

$$= \delta cY_{t,t+1} + \delta^{L+1} cdY_{t,t+1+L}.$$

P_t = expected total cost at time t of buying a unit of inventory.

$$= cY_t + cdE_t[\tilde{Y}_{t+L}]\delta^L.$$

H_1 = discounted cost at time t of carrying a unit of inventory at the end of period $t + L$;

$$= \delta^L h.$$

and in general let

$S_{t,n}$ = the minimum expected discounted cost at time t of meeting demand in period $t + L + n$ if this period's demand is not met by ordering at time t .

$$= \delta E_t[\min(P_{t+1} + H_{n-1}, S_{t+1,n-1})]$$

H_n = discounted cost at time t of carrying a unit of inventory at the end of periods $t + L$ through $t + L + n - 1$.

$$= \delta^L h + \dots + \delta^{L+n-1} h$$

$$= H_{n-1} + \delta^{L+n-1} h$$

$$H_0 \equiv 0.$$

$P_{t+i} + H_{n-i}$ = expected discounted cost per unit at time $t + i$ of meeting demand in period $t + L + n$ by buying in period $t + i$.

The result is that there is now a more simplified approach using the new notation introduced.

In general, buy for $t + L + n$ if

$$P_t + H_n < \delta E_t[\min(P_{t+1} + H_{n-1}, S_{t+1, n-1})], \quad n > 1. \quad (3-2)$$

4 Upper bound on inventory model

If and when a new order is placed at time t , what is the appropriate level to raise inventory up to? The upper bound represents the maximum number of periods of demand to include in the order. How will the inventory purchaser decide how many periods of demand to include when a new order is placed? In order to show an upper bound for the ordering decision, suppose there is an easy to compute model $U_{t,n} \geq S_{t,n}$. Suppose that we buy according to the following policy. Lemma 4.1 shows that this would be an upper bound on the solution that would have been obtained if (3.2) were used.

Lemma 4.1. Buy for n additional periods if

$$P_t + H_k < U_{t,k} \quad \text{for } k = 1, \dots, n \text{ but}$$

$$P_t + H_{n+1} \geq U_{t,n+1}.$$

Then the amount purchased will be \geq the optimal amount.

Proof for Lemma 4.1:

The optimal policy purchases m periods where

$$P_t + H_k < S_{t,k} \quad \text{for } k = 1, \dots, m$$

$$P_t + H_{m+1} \geq S_{t,m+1};$$

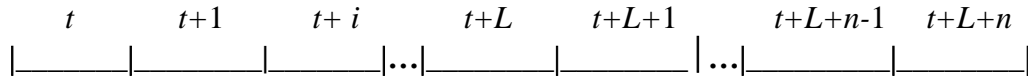
Now $P_t + H_k < S_{t,k} \leq U_{t,k} \quad \text{for } k = 1, \dots, m.$

Therefore, $n \geq m$. This ends the proof.

To get a better picture about the ordering decision, we examine the ordering time line in Figure 4.1. This study proposes the following as an upper bound on the right side of the inventory ordering model of (3.2):

$$U_{t,n} = \delta \min_{1 \leq i \leq n} \left\{ \delta^{i-1} (E_t[P_{t+i}] + H_{n-i}) \right\}$$

Fig 4.1 Ordering Time Line



In Figure 4.1, when one considers whether or not to include the demand for the n^{th} period (period $t + L + n$), there are $n + 1$ different alternatives for placing the order, the last one being one ordered at $t + n$. However, there are $n - i$ periods to carry the inventory if the order is placed at time $t + i$. The expected discounted costs at time t of ordering in periods $t + 1, \dots, t + n$ are:

- For order placed at time $t + 1$, the discounted cost is $\delta \{E_t[P_{t+1}] + H_{n-1}\}$;
- For $t + 2$, $\delta^2 \{E_t[P_{t+2}] + H_{n-2}\}$;
- For $t + n$, $\delta^n \{E_t[P_{t+n}] + H_0\}$.

It is noted that the left-side of (3.2) above involves the cost of ordering decisions at time t time and is not hard or too complex to compute. However, when we apply the bounds, we are interested in examining the costs of ordering in periods other than time t . In fact, the proposed upper bound considers discounted costs associated with orders made at $t + 1, \dots, t + n$. So a reasonable heuristic policy would be to order in period t if the unit cost of ordering in period t and carrying inventory until it is needed is less than the discounted cost of ordering at a later period. Lemma 4.2 below shows that the minimum expected discounted

cost at time t of meeting demand in period $t + L + n$ is less or equal to the proposed upper bound

Lemma 4.2 $S_{t,n} \leq \delta \min_{1 \leq i \leq n} \left\{ \delta^{i-1} (E_t[P_{t+i}] + H_{n-i}) \right\}$ for all t and n . The proof for this

proposition is done by induction. First we show it is true for $n = 1$ and all t . One can show that

$$\begin{aligned}
 U_{t,1} &= \delta \min \{ E_t[P_{t+1}] + H_0 \} \\
 &= \delta E_t[cY_{t+1} + cdE_{t+1}[Y_{t+1+L}]] \delta^L \\
 &= \delta cY_{t,t+1} + \delta^{L+1} cdE_t[E_{t+1}[Y_{t+1+L}]] \\
 &= \delta cY_{t,t+1} + \delta^{L+1} cdY_{t,t+1+L} = S_{t,1}
 \end{aligned}$$

For $n = 1$, we see that in fact $U_{t,n} = S_{t,n}$. Hence, it is true that $U_{t,n} \geq S_{t,n}$. Next, to show it is true for $n + 1$, assume that $U_{t,n} \geq S_{t,n}$ and $U_{t,n+1} \geq S_{t,n+1}$. We know that for any two random variables x and y :

$$E[\min(x, y)] \leq \min(E[x], E[y]), \text{ and } \min(x, y) \leq x.$$

So

$$E[\min(x, y)] \leq E[x], E[y].$$

We can also show that

$$\begin{aligned}
 S_{t,n+1} &= \delta E_t[\min(P_{t+1} + H_n, S_{t+1,n})] \\
 &\leq \delta E_t[\min(P_{t+1} + H_n, U_{t+1,n})] \\
 &\leq \delta \min(E_t[P_{t+1} + H_n], E_t[U_{t+1,n}])
 \end{aligned}$$

So for $n = n + 1$:

$$\begin{aligned}
U_{t,n+1} &= \delta \min_{1 \leq i \leq n+1} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{n+1-i}) \} \\
&= \delta \min \{ E_t[P_{t+1}] + H_n, \min_{2 \leq i \leq n+1} \delta^{i-1} (E_t[P_{t+i}] + H_{n+1-i}) \} \\
&= \delta \min \{ E_t[P_{t+1}] + H_n, \min_{1 \leq j \leq n} \delta^j (E_t[P_{t+1+j}] + H_{n-j}) \} \\
&\quad \text{where } j = i - 1 \\
&= \delta \min \{ E_t[P_{t+1}] + H_n, \delta \min_{1 \leq j \leq n} \delta^{j-1} E_t[E_{t+1}[P_{t+1+j}] + H_{n-j}] \} \\
&\geq \delta \min \{ E_t[P_{t+1}] + H_n, E_t[\delta \min_{1 \leq j \leq n} \delta^{j-1} (E_{t+1}[P_{t+1+j}] + H_{n-j})] \} \\
&= \delta \min \{ E_t[P_{t+1}] + H_n, E_t[U_{t+1,n}] \} \\
&\geq \delta E_t[\min(P_{t+1} + H_n, U_{t+1,n})] \\
&\geq \delta E_t[\min(P_{t+1} + H_n, S_{t+1,n})] \\
&= S_{t,n+1}. \text{ This ends the proof.}
\end{aligned}$$

Next, Lemma 4.3 and Lemma 4.4 show that it is not required to examine an infinite number of periods before deciding how many periods of demand to include in an order if and when one is placed. It can be shown, for example, that if it is not economical to buy at time t for the period $t + L + 1$, it would not be economical to buy for period $t + L + 2$.

Lemma 4.3. $a_k \leq a_{k+1}$.

$$\text{Let } a_k = P_t + H_k - U_{t,k} < 0 \quad \Leftrightarrow \quad P_t + H_k < U_{t,k}.$$

So one needs to show that:

$$P_t + H_k - U_{t,k} \leq P_t + H_{k+1} - U_{t,k+1}, \text{ for all } t, k \geq 1.$$

Proof for Lemma 4.3

Again, the proof for this proposition is completed by induction. First show true for $k = 1$ and all t . One can see that

$$U_{t,k} = \delta \min_{1 \leq i < k} \left\{ \delta^{i-1} E_t[P_{t+i}] + H_{k-i} \right\}$$

For $k = 1$,

$$P_t + H_1 - U_{t,1} \leq P_t + H_2 - U_{t,2}$$

$$\Leftrightarrow U_{t,2} + H_1 - H_2 \leq U_{t,1}$$

$$\Leftrightarrow \delta \min\{E_t[P_{t+1}] + H_1, \delta E_t[P_{t+2}]\} - \delta^{L+1} h \leq \delta E_t[P_{t+1}]$$

$$\Leftrightarrow \delta \min\{E_t[P_{t+1}], \delta E_t[P_{t+2}] - \delta^L h\} \leq \delta E_t[P_{t+1}]$$

Now assuming true for $k = 1$ and all t and show true for $k = k + 1$ and all t .

$$P_t + H_{k+1} - U_{t,k+1} \leq P_t + H_{t+2} - U_{t,k+2}$$

$$\Leftrightarrow U_{t,k+2} + H_{k+1} - H_{k+2} \leq U_{t,k+1}$$

$$\Leftrightarrow \delta \min_{1 \leq i \leq k+2} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+2-i}) \} - \delta^{L+k+1} h \leq \delta \min_{1 \leq i \leq k+1} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+1-i}) \}$$

$$\Leftrightarrow \delta \min_{1 \leq i \leq k+2} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+2-i} - \delta^{L+k+1-i} h) \} \leq \delta \min_{1 \leq i \leq k+1} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+1-i}) \}$$

$$\Leftrightarrow \delta \min_{1 \leq i \leq k+2} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+1-i}) \} \leq \delta \min_{1 \leq i \leq k+1} \{ \delta^{i-1} (E_t[P_{t+i}] + H_{k+1-i}) \}$$

Lemma 4.4. $b_k \leq b_{k+1}$

$$\text{Let } b_k = P_t + H_k - S_{t,k} < 0 \quad \Leftrightarrow \quad P_t + H_k < S_{t,k}.$$

So we need to show that:

$$P_t + H_k - S_{t,k} \leq P_t + H_{k+1} - S_{t,k+1} \text{ for all } t, k \geq 1.$$

Proof for Lemma 4.4:

Once again, the proof for this proposition is completed by induction. One needs to first show true for $k = 1$ and all t . We see that

$$S_{t,k} = \delta E_t[\min(P_{t+1} + H_{k-1}, S_{t+1,k-1})]$$

For $k = 1$,

$$\begin{aligned} P_t + H_1 - S_{t,1} &\leq P_t + H_2 - S_{t,2} \\ \Leftrightarrow S_{t,2} + H_1 - H_2 &\leq S_{t,1} \\ &= \delta E_t[\min(P_{t+1} + H_1, S_{t+1,1})] - \delta^{L+1}h \leq \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &= \delta E_t[\min(P_{t+1} + H_1, \delta cY_{t+1,t+2} + \delta^{L+1}cdY_{t+1,t+2+L})] - \delta^{L+1}h \leq \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &= \delta E_t[\min(P_{t+1}, \delta cY_{t+1,t+2} + \delta^{L+1}cdY_{t+1,t+2+L} - \delta^L h)] \leq \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &= \delta E_t[\min(cY_{t+1} + cdE_{t+1}[Y_{t+1+L}]\delta^L, \delta cY_{t+1,t+2} + \delta^{L+1}cdY_{t+1,t+2+L} - \delta^L h)] \\ &\quad \leq \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &\leq \min(\delta cY_{t,t+1} + cd\delta^{L+1}Y_{t,t+1+L}, \delta cY_{t,t+2} + \delta^{L+1}cdY_{t,t+2+L} - \delta^L h) \\ &\quad \leq \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &\leq \min(\delta cY_{t,t+1} + cd\delta^{L+1}Y_{t,t+1+L}, \delta cY_{t,t+2} + \delta^{L+1}cdY_{t,t+2+L} - \delta^L h) \\ &\quad \leq S_{t,1} \end{aligned}$$

This is true for $k = 1$. Next, assume true for $n = k$ and all t and show true for $n = k + 1$ and all t .

$$\begin{aligned} P_t + H_{k+1} - S_{t,k+1} &\leq P_t + H_{k+2} - S_{t,k+2} \\ \Leftrightarrow S_{t,k+2} + H_{k+1} - H_{k+2} &\leq S_{t,k+1} \\ \Leftrightarrow \delta E_t[\min(P_{t+1} + H_{k+1}, S_{t+1,k+1})] - \delta^{L+k+1}h &\leq \delta E_t[\min(P_{t+1} + H_k, S_{t+1,k})] \end{aligned}$$

$$\Leftrightarrow E_t[\min(P_{t+1} + H_k, S_{t+1,k+1} - \delta^{L+k}h)] \leq E_t[\min(P_{t+1} + H_k, S_{t+1,k})]. \quad \text{////}$$

This ends the proof.

If one assumes that initial inventory and inventory on order are sufficient to cover demand up to but not including period $t + L$, he or she has to place a new order so as to avoid stockouts. Hence, in this special case using the upper bound, it is possible to use Lemma 3.3 to show inventory ordering decisions at time 1 as follows:

- Buy in Period 1 for Period $L + 2$ if

$$P_1 + H_1 < U_{1,1}$$

- Include Period $L + 3$ if

$$P_1 + H_2 < U_{1,2}$$

- Include Period $L + 4$ if

$$P_1 + H_3 < U_{1,3}$$

- In general, buy for $L + k$ if

$$P_1 + H_{k-1} < U_{1,k-1} \quad (4.3)$$

It is assumed that at $t = 1$ the initial inventory (inventory on hand) and inventory on order are sufficient to cover at least periods 1 through L . So suppose that initial inventory plus on-order is sufficient to cover periods 1 through $L + k$ where k is an integer, $k \geq 0$. One would order now for period $L + k + 1$ if $P_1 + H_k < U_{1,k}$. Note that if $k = 0$, an order must be placed for period $L + 1$. One also would order for $L + k + 2$ if $P_1 + H_{k+1} < U_{1,k+1}$. It can be summarized the general inventory ordering decision at $t = 1$ as follows:

- Find the smallest k such that

$$P_1 + H_k \geq U_{1,k}, \text{ where } k \geq 1.$$

$$\bullet \text{ Order } \max(0, \sum_{t=1}^{L+k} D_t - \text{current inventory position}). \quad (4.4)$$

So applying (4.4) and Lemma 4.3, one can state the ordering decision at time 1 as follows:

$k = 1$ if $P_1 + H_1 \geq U_{1,1}$; otherwise k is the unique integer that satisfies

$$P_1 + H_k \geq U_{1,k} \text{ and } P_1 + H_{k-1} < U_{1,k-1}. \quad (4.5)$$

Using (4.5) one can present efficient computations and in fact show a numerical example that shows how inventory decisions are made. Before leaving this subject, it is worthwhile to see how information available at time t can be utilized in this decision process.

Let H_t be the history of errors and past observations known at time t . Therefore, the knowledge of H_t and Y_{t+1} determines H_{t+1} . That is,

$$H_{t+1} = H_t \cup \{t+1, Y_{t+1}, \varepsilon_{t+1}\}.$$

Assume that ε 's are independent and have a zero mean. Define the expected value of a random variable x , conditional on the history at time t , as $E_t[x] = E[x | H_t]$;

In Lemma 4.5 below is presented the exchange rate information known at time t to forecast the rates in future periods so one can apply the upper bound. The problem one could face with (3.1) is that the expectations taken at future periods are not easy to compute. However, (4.5) provides easy to compute upper bounds on the inventory problem using the information about the exchange rates known as of time t .

Lemma 3.5. $E_1[E_j[Y_k]] = E_1[Y_k]$, where $1 < j < k$.

$$= Y_{1,k}.$$

Proof for Lemma 4.5:

$$E_1[Y_k] = \int_{-\infty}^{\infty} Y_k f(Y_k | Y_1, Y_0, \dots, \varepsilon_1, \varepsilon_0, \dots) dy_k ,$$

$$E_j[Y_k] = \int_{-\infty}^{\infty} Y_k f(Y_k | Y_j, Y_{j-1}, \dots, Y_1, Y_0, \dots, \varepsilon_j, \varepsilon_{j-1}, \dots, \varepsilon_1, \varepsilon_0, \dots) dy_k , \text{ and}$$

$$E_1[E_j[Y_k]] = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Y_k f(Y_k | Y_j, \dots, \varepsilon_j, \dots) dy_k f(Y_j, \dots, Y_2, \varepsilon_j, \dots, \varepsilon_2 | Y_1, \dots, \varepsilon_1, \dots) dy_j \dots d\varepsilon_j \dots$$

Using the fact that $f(A|B) = \frac{f(A \cap B)}{f(B)}$, we can show that

$$f(Y_k | Y_j, \dots, \varepsilon_j, \dots) = \frac{f(Y_k, Y_j, \dots, \varepsilon_j, \dots)}{f(Y_j, \dots, \varepsilon_j, \dots)}$$

and

$$f(Y_j, \dots, Y_2, \varepsilon_j, \dots, \varepsilon_2 | Y_1, \dots, \varepsilon_1, \dots) = \frac{f(Y_j, \dots, \varepsilon_j, \dots)}{f(Y_1, \dots, \varepsilon_1)}$$

We see that the product of (4.27) and (4.28) yields

$f(Y_k, Y_j, \dots, \varepsilon_j, \dots | Y_1, \dots, \varepsilon_1, \dots)$. Therefore,

$$\begin{aligned} E_1[E_j[Y_k]] &= \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} Y_k f(Y_k, Y_j, \dots, \varepsilon_j, \dots | Y_1, \dots, \varepsilon_1, \dots) dy_j \dots d\varepsilon_j \dots \\ &= f(Y_k | Y_1, \dots, \varepsilon_1, \dots) \\ &= E_1[Y_k]. \text{ This ends the proof.} \end{aligned}$$

5 Lower Bounds on inventory model

If and when an order is placed, what is the minimum number of periods of demand it should cover? This section is devoted to answering this question. Since the inventory model

formulated in the previous study does not provide us with a closed-form optimal solution, we have to establish an easy-to-compute minimum bound on the optimal solution.

Let $L_{t,n}$ represent a lower bound such that

$$L_{t,n} = \delta E_t[\min_{1 \leq i \leq n} \delta^{i-1} (P_{t+i} + H_{n-i})].$$

In Lemma 5.1 and 5.2, it will be shown that the lower bound is a non-decreasing function and that this proposed lower bound is less than or equal to $S_{t,n}$.

Lemma 4.1. $\delta E_t[\min_{1 \leq i \leq n} \delta^{i-1} (P_{t+i} + H_{n-i})] \leq S_{t,n}$, for all t and n .

One can prove by induction. First show true for $n = 1$ and all t . It can be shown that

$$\begin{aligned} L_{t,1} &= \delta E_t[P_{t+1} + H_0] \\ &= \delta E_t[cY_{t+1} + cdE_{t+1}[Y_{t+1+L}]]\delta^L \\ &= \delta cY_{t,t+1} + \delta^{L+1}cdE_t[E_{t+1}[Y_{t+1+L}]] \\ &= \delta cY_{t,t+1} + \delta^{L+1}cdY_{t,t+1+L} \\ &= S_{t,1} \end{aligned}$$

For $n = 1$, we see that in fact $L_{t,1} = S_{t,1}$. Hence, it is true that $L_{t,1} \leq S_{t,1}$ for all t .

Now assume that $L_{t,n} \leq S_{t,n}$ for all t . Next, show that $L_{t,n+1} \leq S_{t,n+1}$.

$$\begin{aligned} L_{t,n+1} &= \delta E_t[\min_{1 \leq i \leq n+1} \delta^{i-1} (P_{t+i} + H_{n+1-i})] \\ &= \delta E_t[\min(P_{t+1} + H_n), \min_{2 \leq i \leq n+1} \delta^{i-1} (P_{t+i} + H_{n+1-i})] \\ &= \delta E_t[E_{t+1}[\min(P_{t+1} + H_n, \min_{2 \leq i \leq n+1} \delta^{i-1} (P_{t+i} + H_{n+1-i}))]] \end{aligned}$$

$$\begin{aligned}
&\leq \delta E_t[\min(P_{t+1} + H_n, E_{t+1}[\min_{2 \leq i \leq n+1} \delta^{i-1}(P_{t+i} + H_{n+1-i})])] \\
&= \delta E_t[\min(P_{t+1} + H_n, \delta E_{t+1}[\min_{2 \leq i \leq n+1} \delta^{i-2}(P_{t+i} + H_{n+1-i})])] \\
&= \delta E_t[\min(P_{t+1} + H_n, \delta E_{t+1}[\min_{1 \leq i \leq n} \delta^{i-1}(P_{t+1+i} + H_{n-i})])] \\
&= \delta E_t[\min(P_{t+1} + H_n, L_{t+1,n})] \\
&\leq \delta E_t[\min(P_{t+1} + H_n, S_{t+1,n})]. \text{ This ends the proof.}
\end{aligned}$$

Lemma 4.2. $P_t + H_n - L_{t,n} \leq P_t + H_{n+1} - L_{t,n+1}$ for $n, t \geq 1$.

Proof by induction. First show true for $n = 1$ and all t :

$$P_t + H_n - L_{t,n} \leq P_t + H_{n+1} - L_{t,n+1} \text{ can be true only if}$$

$$L_{t,n+1} + H_n - H_{n+1} \leq L_{t,n}.$$

So for $n = 1$

$$P_t + H_1 - L_{t,1} \leq P_t + H_2 - L_{t,2}$$

$$\Leftrightarrow L_{t,2} + H_1 - H_2 \leq L_{t,1}$$

$$\Leftrightarrow \delta E_t[\min(p_{t+1} + H_1, \delta P_{t+2})] - \delta^{L+1} h \leq \delta E_t[P_{t+1}]$$

$$\Leftrightarrow \delta E_t[\min(P_{t+1}, \delta P_{t+2} - \delta^L h)] \leq \delta E_t[P_{t+1}]$$

This is true for $n = 1$. Next show true for $n = n + 1$ and all t .

$$P_t + H_{n+1} - L_{t,n+1} \leq P_t + H_{n+2} - L_{t,n+2}$$

$$\Leftrightarrow L_{t,n+2} + H_{n+1} - H_{n+2} \leq L_{t,n+1}$$

$$\Leftrightarrow \delta E_t[\min_{1 \leq i \leq n+2} \delta^{i-1}(P_{t+i} + H_{n+2-i})] - \delta^{L+k+1} h \leq \delta E_t[\min_{1 \leq i \leq n+1} \delta^{i-1}(P_{t+i} + H_{n+1-i})]$$

$$\Leftrightarrow \delta E_t[\min_{1 \leq i \leq n+2} \{\delta^{i-1}(P_{t+i} + H_{n+2-i}) - \delta^{L+k} h\}] \leq \delta E_t[\min_{1 \leq i \leq n+1} \delta^{i-1}(P_{t+i} + H_{n+1-i})]$$

$$\begin{aligned} \Leftrightarrow \delta E_t [\min \{ \min_{1 \leq i \leq n+1} (\delta^{i-1} (P_{t+i} + H_{n+1-i}) + \delta^{L+n+1-i}) - \delta^{L+n} h), \delta^{n+1} P_{t+n+2} - \delta^{L+n} h \}] \\ \leq \delta E_t [\min_{1 \leq i \leq n+1} \delta^{i-1} (P_{t+i} + H_{n+1-i})]. \end{aligned}$$

This ends the proof.

6 Findings and conclusion

In this research, an attempt was made to present an upper bound and a lower bound on an inventory control model that can be used to make inventory decisions whenever a new order is to be placed. When the decision maker is faced with the question of how many periods of demand to include in any given order, the upper bound provides the maximum quantity to include. In a similar way, the lower bound provides a limit on the minimum number of periods of demand to be included in the order.

The study assumes that a currency model can be developed using historical data for exchange rates for any nation. Once the importer of goods establishes that such goods need to be imported from another nation that uses a different currency from that of the importer, he or she uses the inventory model developed by the author in the previous that utilizes the exchange rates to make decisions as to how many periods of demand to include in the specific order. Exchange rates do fluctuate often, in some nations more than others, and it is difficult to come up with a reliable exchange rate mode. This study does recognize that limitation. However, because inventory decisions are made every day in nations whose exchange rates do fluctuate, such decisions should not be and cannot be made in a vacuum. One has to study the pattern of the exchange rates and attempt to make the best possible decision under the circumstances. Hence, this study presents an approach that can be used to facilitate decision making in this environment.

It is also noted in this study that nations with popular currencies in the developed world can use hedging to mitigate exchange rate fluctuations. However, even in such nations, hedging alone is not solving their problems related to losses due to exchange rate fluctuations. While such an inventory model and the bounds developed can be used in the developed world, the main focus is on the emerging economies that suffer the most as the result of currency fluctuations and also because they cannot readily use hedging as a risk management tool. Therefore, the author believes that the bounds formulated in this study would make it easier for inventory control in situations where currency exchange rates fluctuate.

The following is an example which gives an idea as to the use of the bounds on inventory decisions. Suppose that lead time, L , is 10 days or 2 weeks. For the other parameter values, let's consider $c = \$2/\text{unit}$, $d = 20\%$ of the purchase price, $h = 1.75$ local currency/unit/week, and interest rate = 12.5% per annum, which is equivalent to 0.002404 per week. The discount rate therefore is $\delta = 0.997601918$. In this case, the exchange rate forecasts can be generated using Monte Carlo simulation with a given sample size for a number of working days. It is assumed there are five working days per week (Monday – Friday) on which exchange rate data is available. The actual number of projected periods used in simulating the upper model is 106 weeks, two of which are extra periods added to account for the lead time. The data used is taken from the historical daily exchange rates for Zambian Kwacha (ZMK)/US\$ from September 7, 2000. The ZMK historical data was used to develop this particular model. Therefore, y_1 is the exchange rate for ZMK/US\$ on this date. Note that periods are in days for the exchange rate but review periods (weeks) in the inventory model. For example, $Y_{1,2}$ is the expected exchange rate

forecast for period 2 made in period 1. To forecast for period 2, therefore, we would need to forecast for 10 days or 2 weeks. In this case, $y_1 = Y_1$. That means $Y_{1,2} = y_{1,6}$.

Based on this data, the purchaser can decide how many periods of demand to include in an order depending on whether it is cheaper to buy each review period or carry inventory for future use. LHS(1) represents the total discounted cost in period 1 of including the demand for period $L + 2$. When the required computations are completed, it can be shown that it is economical to include in the order placed in this period not only the demand for period $L + 1$ but also for demands in periods $L + 2$ through $L + k + 1$, where $k = 96$. This implies that with $L = 2$ weeks, we would order now for weeks 3 to 99. It can be observed that when $k = 97$, the LHS is greater than the RHS, indicating that it is no longer optimal to include demand for another period. At every inventory review point, one must reexamine ordering decisions to see how many periods of demand would be included. Therefore, the benefit of the model can be judged by using a naïve policy of ordering each week or some other approach. In addition, instead of relying on just the upper or lower bound, one can take the average of the two to make the decision.

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