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Abstract Title: An integrated model for maintenance planning, process quality control and production scheduling

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Abstract

Performance of a manufacturing system depends on the performance at the shopfloor level. Traditionally, shopfloor level operational policies concerning maintenance, quality and production-scheduling have been considered and optimized independently. However, these three aspects of operations planning may also have an interaction effect on each other and hence need to be considered jointly for improving the manufacturing system performance. In this paper, a model for integrating maintenance and process quality is developed to obtain optimal preventive maintenance interval and control chart parameters that minimize expected cost per unit time. Subsequently, the optimal preventive maintenance interval is superimposed onto the production-schedule in order to determine the optimal job-sequence that will minimize penalty-cost incurred due to schedule delay. An illustrative example is included to compare the performance of the proposed integrated model with the performance obtained by using independent models.

Key word: Preventive maintenance, process quality, production scheduling, control chart

Introduction

In the context of the manufacturing, there is a need for a manufacturing system to quickly adapt itself to the demand fluctuations (random requests) and to the internal interruptions (machines breakdowns/process failure). In such an environment, the optimal production and maintenance planning, and scheduling and quality control becomes increasingly challenging.

There is a large gap in the literature in terms to joint consideration of scheduling, maintenance and process/product quality. For instance, in most models, machines are assumed to be always available during the scheduling horizon; while on the other hand, maintenance planning models seldom consider the impact of maintenance on due dates and quality of the process.

This paper has three goals. First, to develop an approach to integrate maintenance planning and process quality control policy. Specifically, the aim is to obtain optimal preventive maintenance interval and control chart parameters that minimize the expected cost per unit time. Subsequently, an approach is proposed in which the optimal preventive maintenance interval obtained is superimposed onto the production-schedule in order to determine the optimal job-sequence that will minimize the penalty-cost associated with schedule delay. Finally, it compares the performance of the proposed integrated model with the methodology that treats these issues independently.

The paper is organized as follows: in Section 2, a brief review of relevant literature is presented. In section 3, the problem statement is discussed in detail and in section 4, a solution methodology is presented. In Section 5, a mathematical model for integrated maintenance planning and process quality control policy is developed and a numerical example is presented for illustration. A superimposition model of preventive maintenance interval obtained in section 3 on production schedule is presented in Section 6 along with an illustrative example to show how the proposed approach works. Comparison of the integrated model with the performance obtained by using independent model is given. Some possible extensions of proposed approach are also given.

2. Literature review

In the ’80s, researches on production scheduling with machines failures or varying machine capacity started appearing in literature. Many of these researches, such as Pinedo and Rammouz (1988), Adiri et al. (1991), Hirayama and Kijima (1992), Federgruen and Mosheiov (1997), Leung and Pinedo (2004), considered a passive approach toward machine unavailability and focused on how to adjust the production schedule to account for the time when machines are unavailable. It is well known that carrying out preventive maintenance on a machine with increasing failure rate can effectively reduce the occurrence of machine failures and increase the machine availability. Graves and Lee (1999), and Lee and Chen (2000) developed approaches that simultaneously schedule jobs with a single preventive maintenance. They assume that each machine is maintained only once during the planning horizon. Their approaches on production and preventive maintenance scheduling consist of two stages:

(1) Determine the interval during which a machine has to be maintained only once to increase its availability.

(2) Within the interval found in (1), schedule the jobs and the single preventive maintenance simultaneously.

Cassady and Kutanoglu (2003) compared the optimal value of total weighted tardiness under integrated production scheduling with preventive maintenance planning with that under separate production scheduling and preventive maintenance planning. They assume that the uptime of a machine follows a Weibull distribution; the machine is minimally repaired when it fails; and the preventive maintenance restores the machine to a state as good as new. Their results indicate that there is an average of 30% reduction in the expected total weighted tardiness when the production scheduling and preventive maintenance planning are integrated. Leng et al. (2006) and Sortrakul and Cassady (2007) further extended the work of Cassady and Kutanoglu (2003) and proposed Chaotic Partial Swarm Optimization (CPSO) heuristic and GA-based heuristics respectively to solve the integrated mathematical model for single machine production scheduling and PM planning as a multi-objective optimization problem.

Similarly, increasing number of practitioners and researchers have recognized that there is a strong relationship between product quality, process quality and equipment maintenance (Ben-Daya and Duffuaa, 1995), and integration of these may be beneficial to organization. But research in this field is still limited. Rahim (1993) determined jointly the optimal design parameters on an x-bar control chart and preventive maintenance (PM) time for a production system with an increasing failure rate. Ben-Daya and Rahim (1999,2000); and Rahim (1994) investigated integration of x-bar chart and PM, when the deterioration process during in-control period follows a general probability distribution with increasing hazard rate. Cassady et al. (2000) studied an x-bar chart in conjunction with an age replacement preventive maintenance policy. Rahim and Ben-Daya (2001) provided an overview of the literature dealing with integrated models for production scheduling, quality control and maintenance policy. Recently, Linderman et al. (2005) developed a generalized analytic model to determine the optimal policy to coordinate Statistical Process Control and Planned Maintenance to minimize the total expected cost. Panagiotidou and Tagaras (2007) have proposed an economic model for the optimization of preventive maintenance interval in a production process with two quality states.

While some literature is available for integrating maintenance with scheduling and maintenance with quality, integration of all the three areas i.e. production scheduling, maintenance and quality control has recently started getting attention from the research community (Rahim and Ben-Daya 2001) and hence it presents a good oppurtunity for further research.

Nomenclature

$E[C_{CM}]_{FM_1}$	Expected cost of corrective maintenance (CM) due to failure mode1
$E[C_{PM}]$	Expected cost of preventive maintenance (PM)
$E[TCQ]_{process-failure}$	Expected total cost of quality due to process failure
MT_{CM}	Mean Time to CM
PR	Production rate
C_{lp}	Cost of lost production
LC	Labour Cost
C_{FCPCM}	Fixed cost per CM
P_{FM_1}	Probability of occurrence of failure due to failure mode 1
N_f	Number of failures
MT_{PM}	Mean Time to PM
C_{FCPPM}	Fixed cost per PM
$E[T_I]$	Expected in-control period
ARL_1	Average run length during in-control period
T_0	Expected time spent searching for a false alarm
P_{FM_2}	Probability of occurrence of failure due to failure mode 2
T_{eval}	Evaluation period
$ARL2_{M/c}$	Average run length during an out-of-control period due to machine failure
$ARL2_E$	Average run length during an out-of-control period due to machine failure
τ	Mean elapse time from the last sample before the assignable cause to the occurrence of assignable cause when the maintenance and quality policies are integrated
T_s	Time to sample and chart one item
T_1	Expected time to determine occurrence of assignable cause
T_{reset}	Time to perform the resetting of the process which moves out-of-control due to external reason
α	Type I error probability
$C_{\mathrm{Re}\,j}$	Cost of rejection
$(R_\delta)_{M/C}$	Probability of nonconforming items produced due to machine failure mode II

$\beta_{M/C}$	Type II error probability due to machine failure mode II
$(R_\delta)_E$	Type II error probability due to external reasons
β_E	Probability of nonconforming items produced due to external cause
C_{reset}	Cost of resetting
$E[(C_{CM})_{FM_2}]$	The expected cost of corrective maintenance due to FM_2
$[ECPUT]_{M*Q}$	Expected cost per unit time of integrated maintenance and quality policy

3. Problem Statement

Consider a production system consisting of a single machine producing products of the same type with constant production rate of PR items per hour on a continuous basis (3shifts of 7hrs each, 6 days-a-week). Further, consider a single component operating as a part of machine with time-to-failure following a two parameter Weibull distribution. Let the scale and shape parameters of the distribution be B and η respectively. Suppose the process can best be evaluated by measuring a key quality characteristic of finished products. Let x denotes the measurement of this characteristic for a given product, and assume that x is a normal random variable having mean μ and standard deviation σ. The value of μ is referred to as the process mean, and the value of σ is referred to as the process standard deviation. When the process is in-control (operating properly), the process mean is set at its target value.

The process mean can instantaneously shift, due to equipment/process failure. After a shift has occurred, the new process mean is given by: $\mu = \mu_0+\delta\sigma_0$, where δ is some non-zero real number. After the shift, the process is said to be out-of-control. Usually, the failure which causes this shift is relatively subtle. Therefore, the cause of failure cannot be identified without shutting down the process and performing a close inspection of the equipment.

In this paper, two types of equipment failure mode are considered. If failure mode 1 (FM_1) occurs, then it is immediately detected and the machine has to be stopped. Corrective actions are taken to restore the machine back to the operating conditions. Thus, FM_1 results in an expected corrective maintenance cost ($E[C_{CM}]_{FM_1}$) comprising of cost of down time, and cost of repair/restoration. However, failure mode 2 (FM_2) affects the functionality of the machine and causes the process to shift, resulting in an increase in the rejection level, till it is detected. It is assumed that the occurrence of FM_2 is not immediately detectable but whenever it is detected, the process is stopped immediately and corrective actions are taken to restore the process to the normal conditions. Apart from machine failures due to FM_2, process may also deteriorate and shift due to external causes 'E' like environmental effects, operators' mistake, use of wrong tool, etc. The process is also restored if an external causes 'E' is detected. Since FM_2 and E cannot be directly detected, a control chart is used to monitor the quality characteristic 'x'. Hence the time to detect FM_2 and E depends on the power of the control chart. The parameters of this chart are: (h) the time (in hours) between samples, (n) the sample size, and (k) the number of standard deviations of the sampling distribution between the centre line of the control chart and the control limits. The resulting upper and lower control limits for the $\bar{x}$-chart are given by:

$$UCL = \mu_0 + k\frac{\sigma}{\sqrt{n}}, \qquad LCL = \mu_0 - k\frac{\sigma}{\sqrt{n}}$$

This paper aims at presenting a methodology to simultaneously use the $\bar{x}$-chart and a block-replacement policy to improve the performance of the manufacturing process. The PM action is assumed to be imperfect. Specifically, the objective is to obtain optimal values of h, n, k and t_{PM} (preventive maintenance interval) that minimize the total expected cost associated with poor quality, inspection/sampling, corrective/preventive maintenance and process downtime. Subsequently, the optimal preventive maintenance interval obtained is superimposed onto the production-schedule to determine optimal job-sequence that will minimize penalty-cost incurred due to schedule delay. A numerical example is used to demonstrate the procedure for identifying optimal scheduling and PM interval.

4. Solution Methodology

In order to demonstrate the benefits of combining preventive maintenance and statistical process control for the process, a cost model has been developed that captures the costs associated with the manufacturing process that are affected by quality control policies and maintenance planning. These costs comprise of cost of poor quality, cost of sampling/inspection, cost of preventive maintenance and cost of downtime. In this paper the Duncan's model (Duncan, 1956) of economic design of control chart is modified to capture the cost of process shift incurred due to external reasons 'E' and FM_2.

Due to the stochastic nature of the quality characteristic and machine failures, the actual cost of implementing a specific block-replacement imperfect preventive maintenance policy and $\overline{x}$ chart is a random variable. Therefore, the performance of the manufacturing process is measured by$C(h, n, k, t_{PM})$, the expected cost per unit time. These four variables are therefore referred to as the decision variables. All other parameters $(P, \sigma_0, \mu_0, \delta_E, \delta_{M/C}, T_0, T_{reset}, T_s, MT_{CM}, MT_{PM}, \beta, \eta)$ are assumed to be known and treated as input constants.

5. Cost analysis of the integrated model for Preventive Maintenance and process control

In this section we develop an integrated cost model for joint determination of optimal preventive maintenance interval and the design parameters of control chart.

The total expected cost of the model includes:

1. The expected cost model for minimal corrective maintenance ($E[C_{CM}]_{FM_1}$) due to FM1 and imperfect preventive maintenance ($E[C_{PM}]$) in section 5.1;
2. The expected total cost of quality loss due to process failure ($E[TCQ]_{process-failure}$)

These models are presented in the following sub-sections.

5.1. Expected cost model for corrective maintenance due to FM_1 and preventive maintenance

The expected corrective maintenance cost due to (FM_1) is given as:

$$E[C_{CM}]_{FM_1} = \{MT_{CM} \cdot [PR \cdot C_{lp} + LC] + C_{FCPCM}\} \times P_{FM_1} \times N_f \quad (1)$$

And expected cost per imperfect preventive maintenance action of component will be:

$$E[C_{PM}] = \{MT_{PM} \cdot [PR \cdot C_{lp} + LC] + C_{FCPPM}\} \times \frac{T_{eval}}{t_{PM}} \quad (2)$$

The expected number of failures N_f can be obtained by simulating the machine failures for given η and B. In the present study Blocksim7 (Reliasoft, 2009) is used for simulation.

5.2 Model for Expected total cost of quality loss due to process failure ($E[TCQ]_{processfailure}$)

5.2.1 Process cycle length

The cycle length consists of the in-control time and the out-of-control time and process resetting or machine restoration time. The in-control time is considered first. Assume that the in-control time is a negative exponential distribution with mean $1/\lambda$. The expected in-control time consists of the mean time to failure and the expected amount of time for investigating false alarms (Lorenzen and Vance, 1986),

$$E[T_I] = 1/\lambda + T_0 \times \frac{S}{ARL_1} \qquad (3)$$

The expected number of samples while the process is in-control (S) with a process failure rate of (λ) can be calculated as given in ((Lorenzen and Vance, 1986):

$$S = \sum_{i=0}^{\infty} i \ \mathrm{Pr}[\text{assignable cause occurs between the } i^{\text{th}} \text{ and } (i+1)^{\text{st}} \text{ samples}]$$

$$= \sum_{i=0}^{\infty} i[e^{-\lambda hi} - e^{-\lambda h(i+1)}]$$

$$= e^{-\lambda h} / (1 - e^{-\lambda h})$$

In this paper we consider machine failure in terms of machine operating with degraded functionality and the sudden breakdown which ceases the machine operation. The probability of occurrence of machine failures is captured from past failures data. Similarly process may fail because of machine degradation or due to some external reasons as mentioned above. Let the rate of failure due to machine degradation(FM_2) be λ_2 and due to external reason 'E' be λ_1. Thus the overall process failure rate λ due to (FM_2) and 'E' is $\lambda = \lambda_1 + \lambda_2$.

Where, $\lambda_2 = \frac{N_f \times P_{FM_2}}{T_{eval}}$ and $\lambda_1 = \frac{1}{Mean\ Time\ between\ process\ failure}$

Where, (N_f) is the number of failures during (T_{eval})

The out-of-control time consists of the expected time of the following events:

1. the time between occurrence of an assignable cause and the next sample,
2. the expected time to trigger an out-of-control signal,
3. the expected time to plot and chart a sample,
4. the expected time to validate the assignable cause and
5. the expected time to reset the process if failure is due to external reasons or the expected time to restore the machine if failure is due to FM_2.

This can be expressed using the following mathematical form.

$$\text{Out} - \text{of} - \text{control time} =$$
$$\{h \times (ARL2_{M/c} \frac{\lambda_2}{\lambda} + ARL2_E \frac{\lambda_1}{\lambda}) - \tau + n \times T_s + T_1 + (T_{reset} \times \frac{\lambda_1}{\lambda} + MT_{CM} \times \frac{\lambda_1}{\lambda})\} \quad (4)$$

Thus the expected process cycle length is equal to

$$E[T_{Cycle}] = 1/\lambda + T_0 * \frac{S}{ARL_1} + \{h \times (ARL2_{M/c} \frac{\lambda_2}{\lambda} + ARL2_E \frac{\lambda_1}{\lambda})\} - \tau + nT_s + T_1 + (T_{reset} * \frac{\lambda_1}{\lambda} + MT_{CM} * \frac{\lambda_1}{\lambda}) \quad (5)$$

5.2.2 Process quality cost

The process quality cost consists of three main components: the cost of rejection incurred while operating process in-control (C_I) and out-of-control (C_O), the cost of sampling, and the cost of evaluating the alarms-both false and assignable and cost of resetting or restoring (through corrective maintenance) the process.

Let C_f be the cost of false alarm. This includes the cost of searching and testing for the cause. Then the expected cost for false alarm is given as:

$$E[C_f] = C_f \cdot (S / ARL1) \cdot T_0 \quad (6)$$

Let C_F be the fixed cost per sample and C_V be the variable cost per unit. Thus the expected cost per cycle for sampling is given as:

$$E[C_S] = \frac{(C_F + C_V \cdot n) \times (1/\lambda + T_0 \times \frac{S}{ARL_1} + \{h \times (ARL2_{M/c}\frac{\lambda_2}{\lambda} + ARL2_E\frac{\lambda_1}{\lambda})\} - \tau + nT_s)}{h} \quad (7)$$

The expected cost of rejection when the process is in-control is as follows:

$$E[C_I] = (\alpha \times PR \times C_{\text{Re}\,j}) \times (1/\lambda + T_0 \times \frac{S}{ARL_1}) \quad (8)$$

Similarly, the cost of rejections incurred when the process is in out-of- control state due to machine failure is given as:

$$E[C_O]_{M/C} = \left(PR \times \frac{(R_\delta)_{M/C}}{1-\beta_{M/C}} \times C_{\text{Re}\,j} \right) \times (h \times (ARL2_{M/c}\frac{\lambda_2}{\lambda} + ARL2_E\frac{\lambda_1}{\lambda}) - \tau + n \times T_s) \times (\frac{\lambda_1}{\lambda}) \quad (9)$$

And, the cost of rejection incurred when the process is in out-of- control state due to external reason is given as:

$$E[C_O]_E = \left(PR \times \frac{(R_\delta)_E}{1-\beta_E} \times C_{Frej} \right) \times (h \times (ARL2_{M/c}\frac{\lambda_2}{\lambda} + ARL2_E\frac{\lambda_1}{\lambda}) - \tau + n \times T_s) \times (\frac{\lambda_2}{\lambda}) \quad (10)$$

It is assumed that process is stopped during search and repair. Let C_{reset} be the cost for finding and repairing the assignable cause plus the downtime cost as the process is stopped. Thus expected cost of finding and repairing for a valid alarm for assignable cause due to external failure is given by:

$$E[C_{reset}] = [C_{reset} \times T_{reset}] \times (\lambda_1 / \lambda) \quad (11)$$

The expected cost of finding and corrective action for a valid alarm due to failure mode FM_2 is given by:

$$E[(C_{CM})_{FM_2}] = \{(MT_{CM}) \cdot [PR \cdot C_{lp} + LC] + C_{FCPCM}\} \times (\lambda 2/\lambda) \quad (12)$$

Adding equation 6, 7, 8, 9, 10, 11 and 12 gives the expected cost of process failure per cycle as:

$$E[C_{process}] = E[C_f] + E[C_I] + E[C_O]_E + E[C_O]_{M/C} + E[C_S] + E[C_{reset}] + E[(C_{CM})_{FM_2}] \quad (13)$$

Hence the expected process quality control cost for the evaluation period is given as:

$$E[TCQ]_{process-failure} = E[C_{process}] \times M \quad (14)$$

Where M is calculated as: $\frac{T_{eval}}{E[T_{cycle}]}$

The expected total cost per unit time of preventive maintenance and process quality control chart policy $[ECPUT]_{M*Q}$ is the ratio of the sum of the expected total cost of the process quality control ($E[TCQ]_{process-failure}$), expected total cost of the preventive maintenance ($E[C_{PM}]$) and expected total cost of machine failure ($E[C_{CM}]_{FM_1}$) to the evaluation time. Therefore the expected total cost per unit time for the integrated model is given as:

$$[ECPUT]_{M*Q} = \frac{E[C_{CM}]_{FM_1} + E[C_{PM}] + E[TCQ]_{process-failure}}{T_{eval}} \quad (15)$$

5.2.3. Stand-alone models

To examine the effectiveness of the integrated model, it is compared with the two stand-alone models. The first one considers only maintenance while the second one considers only the process quality control. This section addresses the two stand-alone models.

5.2.3.1. Maintenance models

In this model, we assume that only Planned Maintenance is considered and that there are no inspections. Therefore, the expected cost per corrective maintenance action can be expressed as:

$$E[C_{CM}] = \{MT_{CM} \cdot [PR \cdot C_{lp} + LC] + C_{FCPCM}\} \times N_f \tag{16}$$

The expected cost per preventive maintenance action can be expressed as:

$$E[C_{PM}] = \{MT_{PM} \cdot [PR \cdot C_{lp} + LC] + C_{FCPPM}\} \times \frac{T_{eval}}{t_{PM}} \tag{17}$$

Thus cost per unit time for the planning period is given as:

$$[CPUT]_M = \frac{E[C_{CM}] + E[C_{PM}]}{T_{eval}} \tag{18}$$

Optimal preventive maintenance interval is determined by minimizing the $[CPUT]_M$ (eq. 18).

5.2.3.2. Statistical process control (SPC) model

This model has been investigated a lot in the literature. The expected cycle length and the expected cost of control chart is given as:

$$E[T_{Cycle}]_{SPC} = 1/\lambda_E + T_0 * \frac{S'}{ARL1} + \{h \times (ARL2_E)\} - \tau + nT_s + T_1 + T_{reset} \tag{19}$$

$$E[C]_{SPC} =$$

$$C_f \cdot (\frac{S'}{ARL1}) \cdot T_0 + \frac{(C_F + C_V \cdot n) \times (1/\lambda_E + T_0 \times \frac{S'}{ARL_1} + h \times (ARL2)_E - \tau + nT_s)}{h} +$$

$$(\alpha \times PR \times C_{\text{Re}\,j}) \times (1/\lambda_E + T_0 \times \frac{S'}{ARL1}) + \left(PR \times \frac{(R_\delta)_E}{1 - \beta_E} \times C_{Frej}\right) \times (h \times (ARL2)_E - \tau + n \times T_s) + [C_{reset} \times T_{reset}]$$

(20)

where, $\acute{S}$ is expected number of samples when the process is in-control while using SPC in isolation. Therefore the expected total cost per unit time for the SPC model is given as:

$$E[CPUT]_{SPC} = \frac{E[C]_{SPC}}{E[T_{Cycle}]_{SPC}} \quad (21)$$

Optimal values of control chart variables (n, h, k) are determined by minimizing the $E[CPUT]_{SPC}$ (eq. 21).

5.2.4. Numerical Illustration

Eq. (15) indicates that optimizing the four variables (n^*, h^*, k^*, t_{PM}^*) is not a simple process. In this section, we present a numerical example to illustrate the nature of the solution obtained from the economic design of the proposed integrated model.

Consider a single machine whose failure is assumed to follow a two parameter Weibull distribution with $\eta = 1000$ and $\beta = 2.5$ as the characteristic life and shape parameter respectively. Machine considered here is expected to operate for three shifts of seven hours each for 6 days in a week. Time to carry out preventive maintenance action $(MT_{PM}) = 3$ time units with restoration factor $(RF_{PM}) = 0.6$ (it implies 60% restoration of life and sets the age of the block to 40% of the age of the block at the time of the maintenance action) and time to corrective maintenance $(MT_{CM}) = 12$ time units with restoration factor $(RF_{CM}) = 0$ (repair is minimal, i.e., the age of a repaired machine is the same as its age when it failed).

Case of camshaft from industrial context is presented. The ability to create a perfect camshaft is what most automotive manufacturers want for their car. They want to have a good camshaft that optimizes the performance level they need at a given revolution per minute (rpm) level. However, this requires proper machining of the camshaft as per the specifications. There are a number of processing steps involved in manufacturing of a camshaft, camshaft diameter being an important quality characteristic. The manufacturer has used $\bar{x}$ control chart to monitor the manufacturing process producing that product. Assuming that the process is characterized by an in-control state with process standard deviation of $\sigma = 1$ and a single assignable cause due to external failure is of magnitude $\delta_E = 1.5$ and deviation due to machine failure be $\delta_{M/C} = 1.5$, which occurs randomly and results in a shift of process mean from μ_0 to ($\mu_0 + \delta\sigma$). The initial values of relevant parameters are given in Table1.

Table 1: Relevant Parameters Used in Numerical Example

Parameter value	δ_E	$\delta_{M/}$	T_s	C_F	C_v	C_f	$C_{\text{Re}\,j}$	C_{FCPCM}
	1.5	**0.6**	**20/60**	**100**	**50**	**1200**	**3000**	**10000**
Parameter value	C_{reset}	T_0	T_1	$T_{\text{Re}\,set}$	PR	C_{lp}	LC	C_{FCPPM}
	5000	**1**	**1**	**2**	**10**	**400**	**500**	**1000**

The global optimization tool box of Maple 13 has been used to solve the optimization problem. The optimal values of decision variables that minimize the expected total cost of system per unit time $[ECPUT]_{M*Q}$ are as follows: $n^* = 10.41$, $k^* = 3$, $h^* = 9$, $t^*_{PM} = 232$ and the corresponding expected total cost of system per unit time is $[ECPUT]_{M*Q} = 409$. For the two stand-alone models, the same values of the relevant parameters and policy variables are assigned and corresponding cost per unit time are obtained, which are $[CPUT]_M = 150$ and $E[CPUT]_{SPC} = 273$. This proves that the integrated model has better economic behaviour then the model in isolation.

6. Batch sequencing model

Consider a single machine that processes three batches having the processing time, setup time, penalty cost, due dates, and other production parameter as given in Table2.

Table 2: Production parameters for the illustrative example

Batch (I)	Processing time in minutes (II) per job	Batch size (III)	Setup time in hrs (IV)	Total processing time (hrs) V=(II*III)/60+IV	Release time (VI)	Due date (hrs) (VII)	Penalty cost/hr/batch (P_i) (in hrs)(VIII)	Carrying cost/job/hrs (IX)
1	6	500	3	53.0	0	100	75	1.71
2	3	500	1	26.0	0	50	50	1.71
3	2	500	2	18.66	0	40	45	1.71

Following assumptions are made to solve the problem:

1. A job cannot be preempted by another job.

2. There are no failures of machine during the schedule.

3. Raw material for all the batches is released at starting of the schedule.

4. The batch processing time is equal to the sum of the processing times of its jobs.

The objective is to obtain the batch sequence that minimizes the Cost Per Unit Time of the schedule$(CPUT)_S$. $(CPUT)_S$ can be calculated as:

$$(CPUT)_S = \frac{Total\ penalty\ cost\ due\ to\ batch\ delay + Total\ raw\ material\ inventory\ carrying\ cost}{Schedule\ completion\ time} \quad (22)$$

Penalty cost is incurred only when a batch is delayed beyond its due date. Penalty cost for a batch can be calculated as:

$$Batch\ penalty\ cost = (Batch\ completion\ time - Batch\ due\ date) \cdot P_i \quad (23)$$

Since it is assumed here that the raw materials for all the batches are released at the starting of the schedule, raw material inventory for a batch is carried until it starts getting processed, i.e. for the duration of the processing and setup time of all the previous batches (if any) and the setup time of the current batch. Hence the inventory carrying cost is calculated based on the whole batch size for this period. Secondly, during the processing of a batch, raw material of the batch depletes at a constant rate and therefore the inventory carrying cost is calculated for this period based on the average inventory (half the batch size).

To obtain the optimal production schedule, a total enumeration method is used. In the present problem for the three batches, a total of 3! batch sequences are possible. These batch sequences are shown in Table 3, where the $(CPUT)_S$ for all the six possible sequences are presented. It is clear from Table 3 that sequence [B2-B3-B1] gives the minimum Cost Per Unit Time of the production schedule and hence the same is selected as the optimal sequence. Since this analysis ignores the probability of machine failure, this solution is termed as 'OS' solution. 'OS' stands for "only scheduling".

Table 3 $(CPUT)_S$ calculation for all possible sequences

Batch sequence	Completion time			Tardiness			Penalty Cost	Inventory Cost	$(CPUT)_S$
	Batch 1	Batch 2	Batch 3	Batch 1	Batch 2	Batch 3			
[B1-B2-B3]	53	79	98	0	29	58	4045	11407	158
[B1-B3-B2]	53	98	72	0	48	32	3808	12248	164
[B2-B1-B3]	79	26	98	0	0	58	2595	9654	125
[B2-B3-B1]	**98**	**26**	**45**	**0**	**0**	**5**	**210**	**8742**	**92**
[B3-B1-B2]	72	98	19	12	68	0	7484	11336	193
[B3-B2-B1]	98	45	19	38	15	0	3558	9583	135

6.1. Superimposing production and maintenance schedule

When the optimal schedule obtained in section 6 is implemented on shop floor it may get interrupted due to scheduled optimal preventive maintenance interval obtained in section 5.2.3. In order to implement both the policies, it is required to superimpose the maintenance interval on the optimal batch sequence. Assuming that the machine can not be stopped for PM until all the jobs in a batch are completed, production manager has four possible alternatives for superimposing the PM schedule. These are represented through Figure 1.

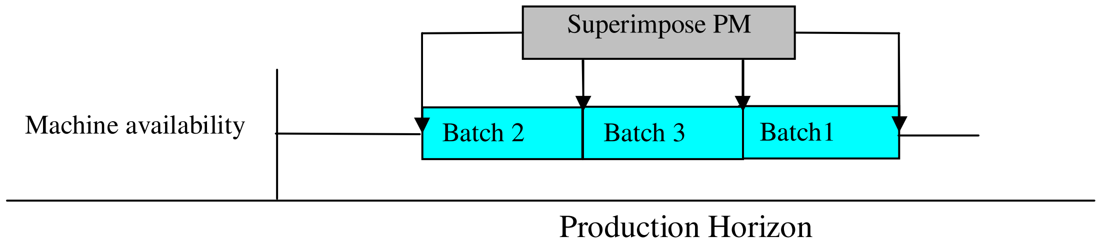

Figure 1: Different alternatives for superimposing PM schedule

The objective of combining these two policies is to determine the optimal production sequence for which the Cost Per Unit Time of joint consideration is minimized. However, the problem of superimposition is complicated by the fact that tardiness values for the jobs are stochastic, since the machine may or may not fail during processing of a job and PM decisions affect the probability of machine failure.

The CPUT for joint consideration of production and maintenance schedule can be expressed as:

$$(CPUT)_{S+(M*Q)} = \frac{\text{Total penalty cost due to batch and maintenance delay} + \text{Total raw material inventory carrying cost}}{\text{Schedule completion time}} \quad (23)$$

The suffix $S+(M*Q)$ indicates that the preventive maintenance schedule obtained through the integrated model of maintenance and process quality control is superimposed on the optimal production schedule obtained independently. The total penalty cost due to batch and maintenance delay can be calculated as follows. (Details can be seen in Pandey et al., (2010)):

The probability that the machine fails while i^{th} job is getting processed can be determined using the Weibull probability distribution as follows,

$$\varphi_{[i]} = F\left(p_{[i]} + \bar{a}_{[i-1]} \middle| \bar{a}_{[i-1]}\right) = 1 - \exp\left[-\left(\frac{p_{[i]} + \bar{a}_{[i-1]}}{\eta}\right)^{\beta} + \left(\frac{\bar{a}_{[i-1]}}{\eta}\right)^{\beta}\right] \quad i = 1,2,\ldots,n \quad (24)$$

The completion time for a job is a discrete random variable that depends on: (i) the age of the machine prior to decision making, (ii) the processing time for jobs; (iii) the time to complete PM and the PM decisions; and (iv) the repair time and the probability of machine failure during batches. Let $C_{[i]}$ denote the completion time for the first job. Then:

$$C_{[i]} = (MTTR)_{PM} \cdot \sum_{i=1}^{i} y_{[1]} + \sum_{i=1}^{i} p_{[1]} + M_{[i]} \qquad i = 1,2,...,n \tag{25}$$

Let $\Theta_{[i]}$ denote the tardiness of the i^{th} job, $i = 1,2,...,n$. Note that $\Theta_{[i]}$ has $i+1$ possible values,

$$\theta_{[1,q]} = \max\left(0, C_{[i,q]} - d_{[i]}\right) \qquad q = 0,1,...,i \tag{26}$$

Thus the total penalty cost incurred due batch tardiness is given as

$$(TPC)_{batch\ tardiness} = \sum_{k=1}^{m}\sum_{i=1}^{n} P_{[i]} E\left(\Theta_{[k]}\right)$$

Where $d_{[i]}$ and $P_{[i]}$ are the due date and penalty cost for the ith job respectively. Table 4 shows the calculations of $(CPUT)_{S+(M*Q)}$ for all the four possible locations of PM.

Table 4 $(CPUT)_{S+(M*Q)}$ at different locations of PM schedule superimposed in optimal production schedule

Batch Sequence	Location of PM	$(CPUT)_{S+(M*Q)}$
[B2-B3-B1]	PM is performed before first batch (in this case it is batch 2 i.e. B2.	217
[B2-B3-B1]	**PM is performed before second batch (in this case it is batch 3 i.e. B3).**	**103**
[B2-B3-B1]	PM is performed before third batch	110
[B2-B3-B1]	PM is performed after third batch (No PM)	639

It is clear from Table 4 that the optimal solution for the superimposed problem is B2-B3-B1 with PM action performed at the starting of second batch in sequence i.e. B3 in this example (marked bold in Table 4).

7. Conclusions and Future scope

This paper proposes a model for integrating PM and process quality control. The model allows joint optimization of quality control charts and preventive maintenance interval

(n, h, k, t_{PM}) to minimize the expected total cost per unit time. To examine the effectiveness of the integrated model, two stand-alone models are also developed. Numerical example taken from a research realistic manufacturing situation indicates that the proposed integrated model performs better than the isolated models. Further, the optimal PM interval obtained from the integrated model of maintenance and process quality control is superimposed on to the optimal production schedule obtained independently. Depending on the nature of the manufacturing system, the average saving may be different but still can be very substantial. Therefore it is believed that integration of maintenance, process quality, and production scheduling is a worthwhile area of study. The proposed model can lead to a number of potential extensions as follows

- The work presented in this paper is limited to a single machine, however it will be interesting to apply the proposed methodology to different shopfloor environments, like flow-shop, open-shop, job-shop, etc., which contain multiple machines and different flow patterns and sequence dependent/independent setup times.
- This study assumes three batches of jobs. However, this can be extended to more number of batches, which will increase the complexity of the problem but take it closer to reality. To solve such problems, different meta-heuristics like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA), TABU Search etc. can be used and their performance may be compared.
- Taguchi quality loss function approach could be employed to quantify loss due to process shift.
- An integrated model of maintenance and production scheduling could also be developed since in this paper only a superimposition model is developed.

References

Adiri, I., Frostig, E. and Rinnooy Kan, A.H.G. (1991) 'Scheduling on a single machine with a single breakdown to minimize stochastically the number of tardy jobs', *Naval Research Logistics*, Vol. 38, pp. 261–271.

Ben-Daya, M. (1999) 'Integrated production maintenance and quality model for imperfect processes', *IIE Transactions*, Vol. 31, pp.491-501.

Ben-Daya, M. and Duffuaa, S.O. (1995) 'Maintenance and quality: the missing link', *Journal of Quality in Maintenance Engineering*, Vol. 1, No.1, pp.20–26.

Ben-Daya, M. and Rahim, M.A. (2000) 'Effect of maintenance on the economic design of $\overline{X}-$control chart', *European Journal of Operational Research*, Vol. 120, pp.131-143.

Cassady, C.R., Bowden, R.O., Liew, L. and Pohl, E.A. (2000) 'Combining preventive maintenance and statistical process control: a preliminary investigation', *IIE Transactions*, Vol.32, pp.471-478.

Cassady, C.R. and Kutanoglu, E. (2003) 'Minimizing job tardiness using integrated preventive maintenance planning and production scheduling', *IIE Transactions*, Vol. 35, pp.503-513.

Duncan, A.J. (1956) 'The economic design of X - charts used to maintain current control of a process', *Journal of the American Statistical Association*, Vol. 51 No.274, pp.228-42.

Federgruen, A. and Mosheiov, G. (1997) 'Single machine scheduling problems with general breakdowns, earliness and tardiness costs,, *Operations Research*, Vol. 45, pp. 66–71.

Grave, H.G. and Lee, C-Y. (1999) 'Scheduling maintenance and semi-resumable jobs on a single machine', *Naval Research Logistics*, Vol. 46, pp.845-863.

Hirayma, T. and Kijima M. (1992) 'Single machine scheduling problem when the machine capacity varies stochastically', *Operations Research*, Vol. 40, pp. 376–383.

Lee, C.Y. and Chen, Z.L. (2000) 'Scheduling jobs and maintenance activities on parallel machines', *Naval Research Logistics*, Vol. 47, pp.145-165.

Leng, K., Ren, P. and Gao, L. (2006) 'A novel approach to integrated preventive maintenance planning and production scheduling for a single machine using the chaotic particle swarm optimization algorithm'. *Proc. 6th World Congress on Intelligent Control and Automation*. *June 21-23*, *2006*. Dalian, China, pp. 7816-7820.

Leung J.Y.T. and Pinedo, M. (2004) 'A note on scheduling parallel machines subject to breakdown and repair', *Naval Research Logistics*, Vol. 51, pp. 60–71.

Li, Z. and Ierapetritou, M. (2008) 'Process scheduling under uncertainty: review and challenges', *Computers and Chemical Engineering*, Vol. 32, pp.715–727.

Linderman, K., McKone-Sweet, K.E. and Anderson, J.C. (2005) 'An integrated systems approach to process control and maintenance', *European Journal of Operational Research*, Vol. 164, pp.324-340.

Lorenzen, T.J. and Vance, L.C. (1986) "The economic design of control charts:a unified approach,' *Technometrics*, Vol. 28, No.1, pp. 3-10.

Panagiotidou, S. and Tagaras, G. (2007) 'Optimal preventive maintenance for equipment with two quality states and general failure time distributions', *European Journal of Operational Research,* Vol. 180, No. 1, pp.329-353.

Pandey, D., Kulkarni, M.S., and Vrat, P. (2010) 'Joint consideration of production scheduling, maintenance and quality policies: a review and conceptual framework', I*nternational Journal of Advanced Operations Management*, Accepted.

Pinedo, M. and Rammouz, E. (1988) 'A note on stochastic scheduling on a single machine subject to breakdown and repair', *Probab Eng Informat Sci*, Vol.2, pp. 41–49.

Rahim, M.A. (1993) 'Economic design of $\overline{\mathrm{X}}$-control charts assuming weibull in-control times', *Journal of Quality Technology*, Vol. 25, pp.296–305.

Rahim, M.A. (1994) 'Joint determination of production quantity, inspection schedule, and control charts design', *IIE Transactions*, Vol. 26, pp.2-11.

Rahim, M.A. and Ben-Daya, M. (2001) 'Integrated production, quality and maintenance models: an overview', *In:* Rahim, M.A. and Ben-Daya, M. (ed.), *Integrated Models in Production Planning, Inventory, Quality, and Maintenance*, (pp.3-18), Massachusetts, New York: Kluwer Academic Publishers.

Reliasoft (2009) www.reliasoft.com, [accessed, 12 February 2010].

Sortrakul, N. and Cassady, C.R. (2007) 'Genetic algorithm for total weighted expected tardiness integrated preventive maintenance planning and production scheduling for a single machine' *Journal of Quality in Maintenance Engineering*, Vol. 13, No. 1, pp. 49-61.