

ABSTRACT NUMBER: 015-0366

**DECISION SUPPORT SYSTEM (DSS) FOR MACHINE SELECTION:  
A COST MINIMIZATION MODEL**

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POMS 21st Annual Conference

Vancouver, Canada

May 7 to May 10, 2010

## **Abstract**

A model-driven Decision Support System (DSS), with mathematical modeling considering machines workload balance and total cost minimization, is used to solve the problem of machine selection for multiple parallel, non-identical electronic assembly machines. The major contribution of this model is on the detailed cost components when estimating total cost. Sensitivity analyses were done, varying amount of products, machines, imbalance factor, and coefficient of variation (CV), and the results show that the higher the CV or the number of machines, total cost of all products assembled increased due to the complexity of balancing machines workload for a large number of products.

Keywords: cost model, machine selection, machine workload balance, mathematical modeling

## **1. Introduction**

Many manufacturing assembly companies use cost estimates of their products without considering a lot of details when calculating unit costs. The most commonly identified reason for the lack of details is the resources needed (i.e., time, knowledgeable personnel, cost, etc.) when estimating costs of each assembled product. As many researchers agree: “Traditional accounting systems allocate overhead as a percentage of direct labor hours and/or machine hours. As the volume of the product increases by 10%, the manufacturing overhead increases by 10%; this is assuming that the direct labor hours and machine hours increase proportionally. While this method of allocation is simple and fast, it does not reflect accurately on the actual product cost” (Cooper et al., 1988; Dhavale, 1990; Miller et al., 1985; Ong, 1995).

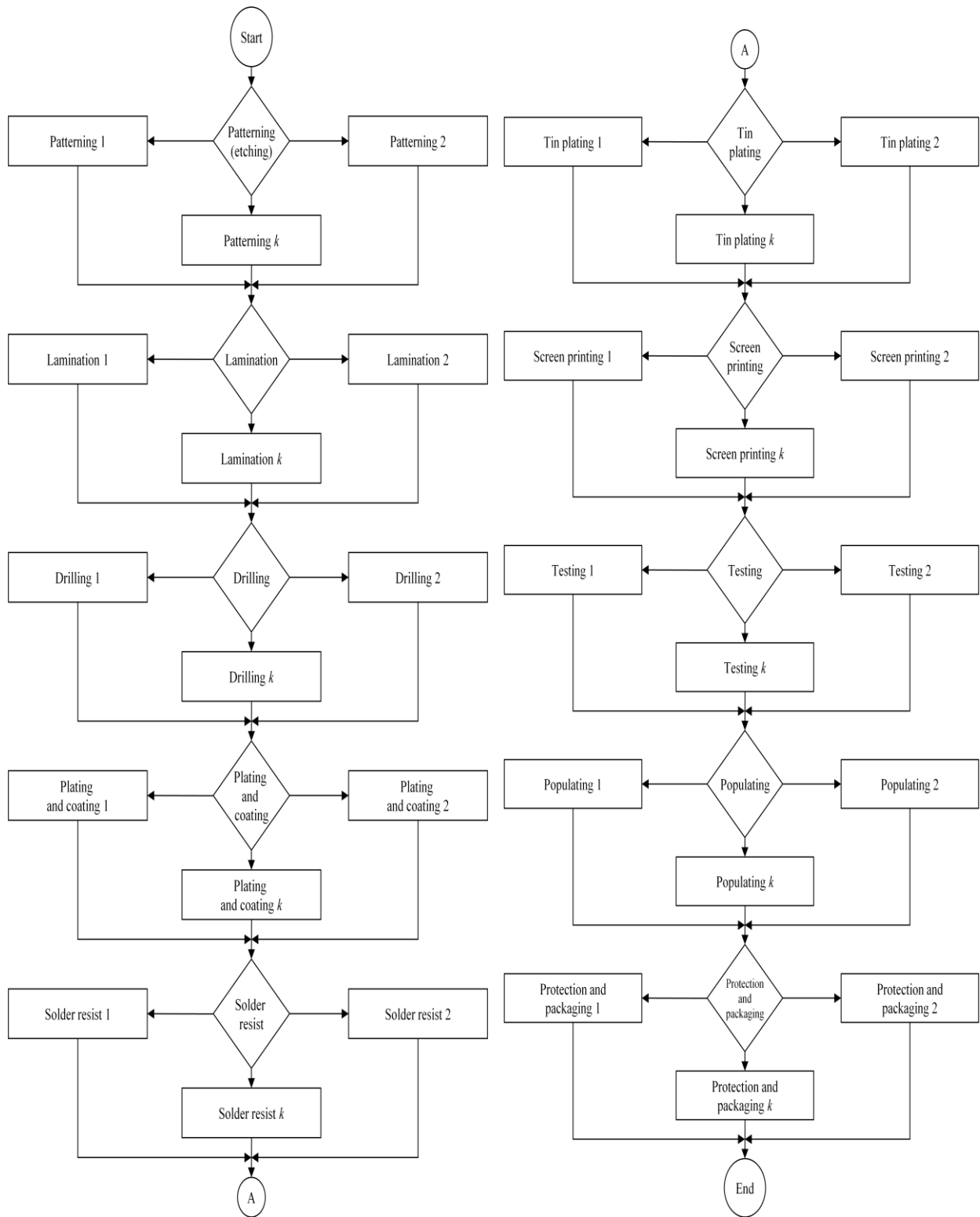
This article presents a tool to make very detailed product cost estimates with the purpose of using them during machine selection to minimize total cost of assemblies. A cost model can help companies in understanding how their earnings are calculated once the product costs are known. The cost model presented in this article considers all the steps required during the assembly of the products and how the cost is generated at each step thus ultimately affecting the company’s net income for a given accounting period. Because the emphasis of this research effort is to provide a short-term decision tool, the components of the products variable and indirect costs are thoroughly studied. The main contribution of this research effort is a detailed cost model that can be used during budget preparation, short-term production planning, and in a continuous basis focusing on cost improvement.

## **2. Problem description**

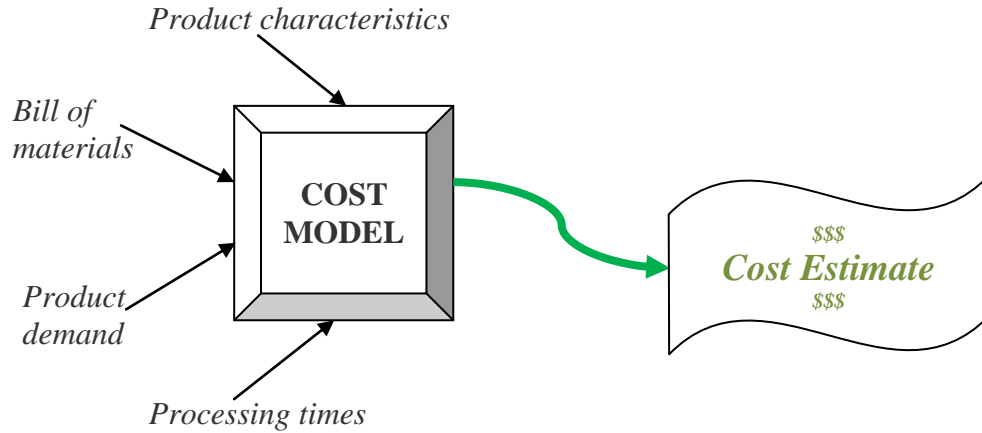
The problem studied during this research is the optimization of the machine selection process when there are multiple parallel, non-identical electronic assembly machines. The emphasis of

the optimization tool (mathematical model) is to use it as part of a decision support system where minimizing total cost of products assembled is the main objective. Due to the repetitiveness of choosing the optimum machine for each product at each assembly process, a cost model was developed following some basic assumptions: (1) an electronic product consists essentially of a printed circuit board (PCB) with electronics components soldered to it following a series of sequential steps, and (2) the cost of the electronic products is calculated assuming typical and generic assembly sequence and processes (refer to Figure 1). Crama et al. (2002) considered a generic assembly process which “consists in placing (inserting, mounting) a number of electronic components of prespecified types at prespecified locations on a bare board”. As pointed out by Ong (1995) on his cost estimate using an activity-based approach, a general process flow that can be used for the cost estimation is considered here even though there will be variations among different manufacturers. The cost model (refer to Figure 2) assumes that in each step of the assembly sequence, resources are consumed, hence cost is incurred. The variables that affect each assembly process are identified and considered in the machine selection problem based on how they influence product cost.

Including a measurement of the expected process variability is done in the mathematical model with the coefficient of variation (CV). As explained by Hopp and Spearman (2001), variability (or “the quality of nonuniformity of a class of entities”) is associated with randomness, but not identical; it is inherent in all manufacturing or assembly processes or systems. The variability of the processing and setup times is considered in this research and is measured based on the processes relative variability by considering the standard deviation and the mean of these randomly generated times;  $CV = \sigma/\mu$ . Based on its CV value process variability is classified as low ( $CV < 0.75$ ), moderate ( $0.75 \leq CV < 1.33$ ), and high ( $CV \geq 1.33$ ).



**Figure 1.** Assembly Processes Flowchart (Méndez, 2001; Clark, 1985)



**Figure 2.** Cost Model Diagram

### 3. Literature review

The review of relevant literature was divided based on the scope of this research, in the following major areas: (1) use and application of Decision Support Systems (DSS) in general; (2) use and application of mathematical modeling on machine assignment problems; (3) cost modeling and time reduction models for electronics products.

A Decision Support System can be defined as an interactive information system that supports business and organizational decision-making activities by compiling useful information from raw data, documents, personal knowledge, and/or business models to identify and solve problems and make decisions. Power and Sharda (2007) explained that “a model-driven DSS includes computerized systems that use accounting and financial models, representational models, and/or optimization models to assist in decision-making”. Model-driven DSS has been used since the late 1960s (Power, 2003) as a management decision system and the first dissertation research using this methodology was presented by Scott Morton in 1967.

Some of the Decision Support Systems reviewed were models where minimizing costs was not necessarily the main objective. Sundararajan et al. (1998) developed a model to determine optimum production scenario based on the tradeoffs between service levels, costs, inventories,

changeovers, and capacity. This model presented an application for a food processing industry where they mentioned minimizing cost as part of the challenge without the specifics been included. Pillai (1990) developed another Decision Support System to identify and select alternatives that provide the highest manufacturing improvements and cost effectiveness. This work was performed for Intel and one of the areas it focused was in providing a baseline for a unit cost analysis. A cost model was developed using Intel's format to calculate the ROI (return on investment) of the alternatives considered.

Mathematical models that present the end result of an operations research model where alternatives, restrictions, and an objective criterion are considered (Taha, 1997), has been used to solve problems in decision making as well. Pentico (2007) presented a survey of assignment problems where two or more sets were to be optimally matched. Within the models for multi-dimensional problems Gilbert and Hofstra (1998) mentioned the axial three-dimensional assignment problem considering jobs, workers, and machines as the three dimensions to match. Another authors, Gavish and Pirkul (1991) developed a mathematical model and heuristics procedures for the generalized assignment problem with multi-resources. These authors included cost on their mathematical models by being part of the objective functions without further details on its calculation. LeBlanc et al. (1999) research differentiated from the others because they presented an extension of the multi-resource generalized assignment problem by splitting batches of products, while considering the effect of setup times and costs independently of the total cost.

Mathematical modeling considering workload balancing of parallel identical machines was presented by Rajakumar et al. (2004) where the researchers were looking to maximize production output optimizing overall performance. Ammons et al. (1997) presented a mathematical model to solve the problem of balancing workload in printed circuit cards

assembly but the focus was to balance component placement times and setup times across the machines.

While reviewing literature on cost modeling for electronics products, Hillier and Brandeau (2001) considered an operation assignment problem for a printed circuit board assembly process with the primary objective of minimizing total manufacturing cost, and a secondary objective of balancing machine workloads. They developed a binary integer program and a heuristic to solve the problem. These scholars considered the manufacturing cost “to be the expected total amount of time required to produce all of the boards during the planning horizon”. The main difference between the problem presented in this article and Hillier and Brandeau work is that they used identical assembly machines which imply that machine assignment at each assembly process was not required, and that machine workload balance was done for all processes together, not at each assembly process. Ong (1995) used Activity-Based Costing (ABC) in the early concept stage of design to estimate manufacturing costs of a printed circuit board assembly. This model provided details of how the product cost was calculated during the design of the product. On a previous research effort, Méndez (2001) developed a very detailed cost model for power electronics assemblies. The cost model was used for the fabrication of the boards and for new electronics’ assemblies and was designed for the manufacturing companies to thoroughly understand their new products’ cost; it did not consider assigning machines and the objective of minimizing costs. On a more recent research effort, Méndez (2009) developed a mathematical model for machine selection minimizing total cost, but it did not consider machine workload balance.

Summarizing the review of relevant literature, models were not found that while optimizing (minimizing) total cost, designing a decision support system, and/or developing a cost model were able to calculate the cost of the products with significant detail. The models where cost was

thoroughly analyzed used Activity-Based Costing (ABC) as the accounting system to allocate their costs (Ong, 1995) or were applicable to assembly processes of new products and not necessarily applicable to existing products with the purpose of minimizing products' costs (Méndez, 2001).

#### **4. Model development**

The cost model calculates direct labor cost, materials cost and overhead or indirect costs of the assembly of electronic products. The sum of these main cost classifications give us the total cost of the product (Horngren et al., 2003; Castillo, 1998). The necessary details in particular with the indirect costs are considered to ensure the estimated cost presented is accurate.

This cost model is based on a variable costing accounting system because the Decision Support System (DSS) developed for this article with the cost model is to be recommended as a short-term decision making tool. Variable costing accounting system dovetails much more closely with other operational analysis that require a separation between fixed and variable costs (Hilton, 1999). With variable costing, the fixed manufacturing overhead costs are not included as a product cost on the manufacturing accounts because they are treated as period costs (costs that are expensed during the time period in which they are incurred) (Hilton, 1999).

The first step of this research effort was the development of a Microsoft Excel simplistic model to calculate and minimize the cost to assemble only one product at a time in each machine (Méndez, 2009). It provided a better understanding of the details when calculating each cost component on an easy-to-use interactive model. As an extension to that model, an integer linear program (ILP) (Méndez, 2009) was also developed with the objective of minimizing the total cost of all products assembled as an optimization tool. This mathematical model considered various machines per assembly process for a multi-product environment.

#### 4.1. Integer linear program with machine workload balance (ILPb)

In an effort to create more realistic scenarios, a factor to measure the required percentage of machine workload balance is now included in the ILP model and this new model is called ILPb. This factor is based on the maximum imbalance percentage allowed by management at each assembly process when assigning products to machines during the machine selection process. The mathematical model used to solve the problem presented in this article is a binary linear program since the decision variables will have binary values (i.e., 0 or 1).

During machine selection in manufacturing environments to assemble products at each assembly process, it is important to maintain machine workload balanced. Machine workload balance can be defined as distributing the available workload among the available machines as equally as possible. Therefore, when machine workload is balanced, waiting times between processes might be minimized and work-in-process inventory (the goods or products between adjacent processes) may be reduced. Due to natural process variability and the variation in processing times, obtaining a perfect machine workload balance is not easily attainable. The machine workload balance is measured by using the required hours of each machine by product at each process to satisfy expected product demand.

Notation presented below in alphabetical order is used throughout the mathematical model for indexes and sets:

$a$  index for product components,  $a \in A$

$b$  index for consumable materials,  $b \in B$

$i$  index for products,  $i \in I$

$j$  index for processes,  $j \in J$

$k$  index for machines,  $k \in K$

$s$  index for support personnel,  $s \in S$

$u$  index for utilities,  $u \in U$

The following notation is used throughout the mathematical model presented for the parameters considered in the formulation:

- Deterministic parameters:

$DLc$  average salary per hour of direct labor employees

$HRS$  worked hours per week

$MH_{jk}$  machine hours available per week for machine  $k$  in process  $j$

$MMc$  total machine maintenance cost per week

$N$  number of products assembled during a given period of time (i.e., week)

$SPc$  average cost of support personnel per year

$WKS$  working weeks per year

$\delta$  machine workload imbalance factor

- Stochastic parameters:

$CM_{ijb}$  units of consumable material  $b$  for product  $i$  in process  $j$ ;  $U(0,15)$

$CMc_b$  cost per unit of consumable material  $b$ ;  $U(0.50,1.00)$

$CP_{ija}$  units of product component  $a$  for product  $i$  in process  $j$ ;  $U(0,50)$

$CPc_a$  cost per unit of product component  $a$ ;  $U(0.10,0.50)$

$DLp_j$  average percent of time direct labor employees worked in process  $j$ ;  $U(30\%,50\%)$

$DLq_j$  quantity of direct labor employees assigned to process  $j$ ;  $U(1,3)$

$Dyr_i$  demand per year of product  $i$ ; uniform distribution with varying parameters values depending on number of machines analyzed,

i.e, U(100000,120000), U(200000,220000), U(300000,320000)

$HT_{ij}$  handling or moving time of product  $i$  from previous process to process  $j$ ;  
N(0.50,0.50)

$MTTF_{jk}$  mean-time-to-failure (in hours) of machine  $k$  in process  $j$ ; U(75,150)

$MTTR_{jk}$  mean-time-to-repair (in hours) of machine  $k$  in process  $j$ ; U(0.50,1.50)

$PT_{ijk}$  process time of product  $i$  in process  $j$  and machine  $k$ ; varies for each level of CV,  
for MCV – N(0.10,0.90)

$SPp_j$  average percent of time of support personnel in process  $j$ ; U(5%,15%)

$SPq_s$  quantity of support personnel  $s$ ; U(1,3)

$SUB_{ijk}$  machine setup time per batch of product  $i$  in process  $j$  and machine  $k$ ; varies for  
each level of CV, for MCV – N(0.50,0.45)

$Ub_i$  units per batch of product  $i$ ; U(500,1500)

$UCc_u$  cost per unit of utility  $u$ ; U(0.05,0.10)

$UCq_{jku}$  units of utility  $u$  consumed by machine  $k$  in process  $j$ ; U(10,300)

$WT_{ijk}$  waiting time of product  $i$  in process  $j$  and machine  $k$ ; N(1,1)

- Computed parameters:

$AV_{jk}$  availability of machine  $k$  in process  $j$

$AvgMU_{jk}$  average machine utilization at process  $j$  and machine  $k$

$C_{ijk}$  total cost of product  $i$  in process  $j$  and machine  $k$

$CT_{ijk}$  cycle time of product  $i$  in process  $j$  and machine  $k$

$L_{ijk}$  labor cost of product  $i$  in process  $j$  and machine  $k$

$M_{ijk}$  material cost of product  $i$  in process  $j$  and machine  $k$

$MM_{ijk}$  machine maintenance cost per product  $i$  in process  $j$  and machine  $k$

$MU_{ijk}$	maximum machine utilization for product $i$ at process $j$ and machine $k$
$O_{ijk}$	overhead cost of product $i$ in process $j$ and machine $k$
$SC_{ijk}$	machine setup cost of product $i$ in process $j$ and machine $k$
$SP_{ijk}$	support personnel cost for product $i$ in process $j$ and machine $k$
$SU_{ijk}$	machine setup time of product $i$ in process $j$ and machine $k$
$TT_{ijk}$	total time on the assembly line of product $i$ in process $j$ and machine $k$
$UC_{ijk}$	utilities consumption cost of product $i$ in process $j$ and machine $k$

The decision variables are defined as follows:

$$X_{ijk} = \begin{cases} 1 & \text{if product } i \text{ is assigned to process } j \text{ and machine } k, \\ 0 & \text{otherwise.} \end{cases}$$

The ILPb mathematical model is expressed with the objective function and constraints that follow. The mathematical equations for the computed parameters also follow.

$$\text{Minimize } Z = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} C_{ijk} X_{ijk} \quad \forall i, j, k \quad (1)$$

subject to

$$\sum_{i \in I} X_{ijk} \leq N \quad \forall j, k \quad (2)$$

$$\sum_{k \in K} X_{ijk} = 1 \quad \forall i, j \quad (3)$$

$$\sum_{i \in I} (MU_{ijk} X_{ijk}) \leq MH_{jk} \quad \forall j, k \quad (4)$$

$$\text{Avg}MU_{jk}(1-\delta) \leq \sum_{i \in I} (MU_{ijk} X_{ijk}) \leq \text{Avg}MU_{jk}(1+\delta) \quad \forall j, k \quad (5)$$

$$X_{ijk}=\{0,1\} \quad \forall i,j,k \quad (6)$$

In the mathematical formulation shown above, equation (1) is the objective function to minimize the total cost of all assembled products. The first constraint presented in equation (2) states that each product  $i$  can be assigned to each process  $j$  and machine  $k$  only once. Equation (3) specifies that at each process  $j$  each product  $i$  can be assigned to only one machine  $k$ . Equation (4) is the capacity constraint of the model to make sure that available machine hours for product  $i$  in process  $j$  and machine  $k$  will not be exceeded. Equation (5) is the constraint to balance machine workload up to a given imbalance factor,  $\delta$ . The last constraint, equation (6) defines the decision variables as binary.

When analyzing capacity constraint (refer to equation 4) and machine workload balance constraint (refer to equation 5) additional mathematical expressions are needed to calculate maximum and average machine utilizations; these are shown in equations 4a and 5a respectively.

$$MU_{ijk}=(PT_{ijk}+SU_{ijk})\times\frac{Dyr_i}{WKS} \quad \forall i,j,k \quad (4a)$$

$$AvgMU_{jk}=\sum_{i\in I} MU_{ijk}/K \quad \forall j,k \quad (5a)$$

Additional mathematical equations are required to calculate direct labor cost per product. These equations consider setup times (based on setup per batch of products) and processing times, direct labor employees and direct labor average wages per hour.

$$L_{ijk}=PT_{ijk}\times DLq_j\times Dlp_j\times DLc+SC_{ijk} \quad \forall i,j,k \quad (7)$$

$$SC_{ijk}=SU_{ijk}\times DLq_j\times Dlp_j\times DLc \quad \forall i,j,k \quad (8)$$

$$SU_{ijk}=\frac{SUB_{ijk}}{Ub_i} \quad \forall i,j,k \quad (9)$$

Labor cost per product is calculated using equation (7) where process time per product, quantity of direct labor employees at each process, the percentage of time they worked at each process, the average rate per hour for direct labor employees, and the setup cost per product are considered. With equation (8) the setup cost per product is calculated using setup time per product, quantity of direct labor employees at each process, the percentage of time they worked at each process, and the average rate per hour for direct labor employees. Equation (9) is used to convert the setup time per batch into setup time per product.

To calculate material cost per product a mathematical equation including the cost of the required product components by considering the units used of each product component and the components cost per unit is used (refer to equation 10). It also includes the cost per product of the consumable materials by considering the units required of each consumable material and the consumable materials cost per unit.

$$M_{ijk} = \sum_{a \in A} (CP_{ija} \times CPc_a) + \sum_{b \in B} (CM_{ijb} \times CMc_b) \quad \forall i, j, k \quad (10)$$

To calculate overhead cost per product mathematical equations to calculate support personnel cost per product, utilities consumption cost per product, and machine maintenance cost per product are used.

$$O_{ijk} = SP_{ijk} + UC_{ijk} + MM_{ijk} \quad \forall i, j, k \quad (11)$$

$$SP_{ijk} = \sum_{s \in S} (SPq_s \times SPp_j) \times TT_{ijk} \times \frac{SPc}{WKS \times HRS} \quad \forall i, j, k \quad (12)$$

$$TT_{ijk} = \frac{CT_{ijk}}{AV_{jk}} \quad \forall i, j, k \quad (13)$$

$$CT_{ijk} = SU_{ijk} + HT_{ij} + WT_{ijk} + PT_{ijk} \quad \forall i, j, k \quad (14)$$

$$AV_{jk} = \frac{MTTF_{jk}}{MTTF_{jk} + MTTR_{jk}} \quad \forall j,k \quad (15)$$

$$UC_{ijk} = \sum_{u \in U} (UCq_{jku} \times UCc_u) \times PT_{ijk} \quad \forall i,j,k \quad (16)$$

$$MM_{ijk} = \frac{MMc}{HRS} \times (SU_{ijk} + PT_{ijk}) \quad \forall i,j,k \quad (17)$$

In the formulation for overhead cost per product, equation (11) summarizes the variable indirect cost by adding support personnel cost per product, utilities consumption cost per product, and machine maintenance cost per product. To calculate the support personnel cost per product, equation (12) considers the quantity of support personnel with the percentage of time worked at each process, the total time products are in the assembly line and the average weekly support personnel salary. With equation (13) the total time products are in the assembly line by considering the cycle time per product (refer to equation 14) and the availability of the machines (refer to equation 15) is calculated. Availability of the machines is a ratio between the mean-time-to-failure and mean-time-to repair of each machine (Hopp et al., 2001). The utilities consumption cost per product in equation (16) is calculated based on units of utilities consumed and their corresponding costs per unit, and the process time per product. The total machine maintenance expenses are allocated to each product based on the process time per product and the setup time per product. These are used to calculate the machine maintenance cost per product in equation (17).

Equation 18 is the mathematical expression to calculate total cost for all products based on direct labor cost ( $L_{ijk}$ ), material cost ( $M_{ijk}$ ), and overhead cost ( $O_{ijk}$ ). It considers the required weekly demand of each product to be assembled. This is the parameter used with the decision variables in the objective function of the mathematical model presented in this article.

$$C_{ijk} = (L_{ijk} + M_{ijk} + O_{ijk}) \times \frac{Dy r_i}{WKS} \quad \forall i, j, k \quad (18)$$

## 5. Solving methodology

The branch and bound (B&B) algorithm was selected to solve the mathematical model using the mathematical programming language AMPL9 (Fourer et al., 2003) and the solver CPLEX90. There are two methods that are commonly used to solve ILP problems, B&B algorithm being the one that is more successful (Taha, 1997) computationally speaking (the other one is the cutting plane method). The analogy to the problem presented in this article with the B&B algorithm is that with the bounding part, an upper bound is found for the minimization problem, and the branching is done with values zero and one for the decision variables.

Different sets of data are used to solve the mathematical model with the purpose of understanding the behavior of the model as a decision tool under different scenarios. The scenarios studied are based on a moderate process variability using a coefficient of variation (CV) of 0.9, different number of assembly machines at each process (i.e., 4, 7, and 10), different amounts of assembled products (i.e., 10, 100, 1000), and two different levels of the machine workload imbalance factor, 25% and 40%. The scenarios considered are presented in Table 1. The combination of the selected scenarios makes a total of eighteen sets of runs of the mathematical model. These mathematical models are increasing their size in terms of variables and constraints as the number of products and machines increase. Table 2 shows the size of these mathematical models for each selected scenario mentioned in Table 1.

Because this cost model is recommended to be used as a short-term decision making tool and considering that the size of some models is fairly large, a time limit of 3600 seconds (1 hour) was used with the solver. Since the solver checks the remaining time at only certain points while the logic of the model is being followed, it could run over the established time limit. For the

purpose of this article, solutions obtained up to 25% over the time limit (up to 4500 seconds or 1.25 hours) were considered optimum if a feasible solution was found.

**Table 1-** Scenarios to solve the ILPb models with a CV = 0.9

<i>Machines</i>	<i>ILPb; <math>\delta = 0.40</math></i>			<i>ILPb; <math>\delta = 0.25</math></i>		
	<i>Products</i>			<i>Products</i>		
4	10	100	1000	10	100	1000
7	10	100	1000	10	100	1000
10	10	100	1000	10	100	1000

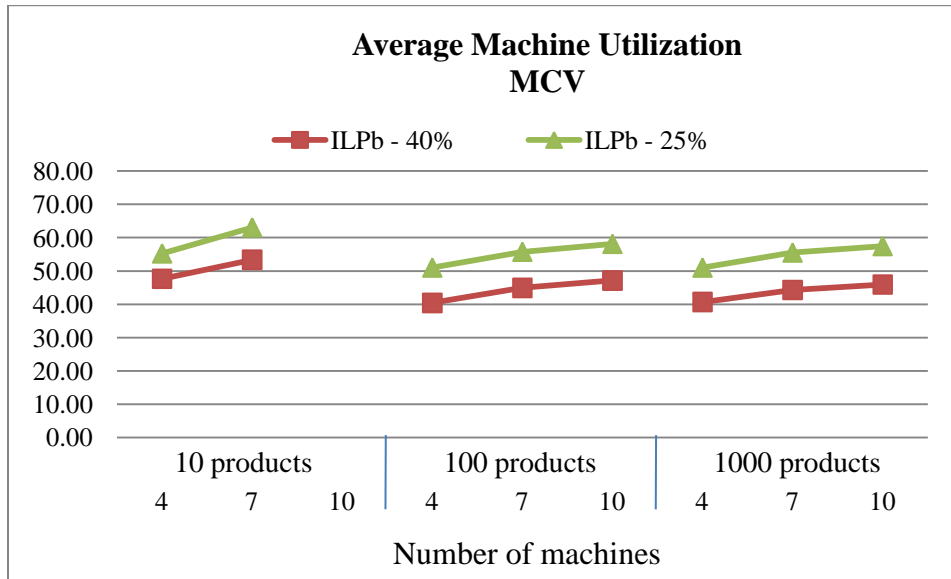
**Table 2 -** Sizes of the ILPb models (CV = 0.9)

<i>Products</i>	<i>Machines per Process</i>	<i>ILPb models</i>	
		<i>Number of Variables</i>	<i>Number of Constraints</i>
10	4	400	240
	7	700	280
	10	1000	340
100	4	4000	1120
	7	7000	1180
	10	10000	1240
1000	4	40000	10120
	7	70000	10180
	10	100000	10240

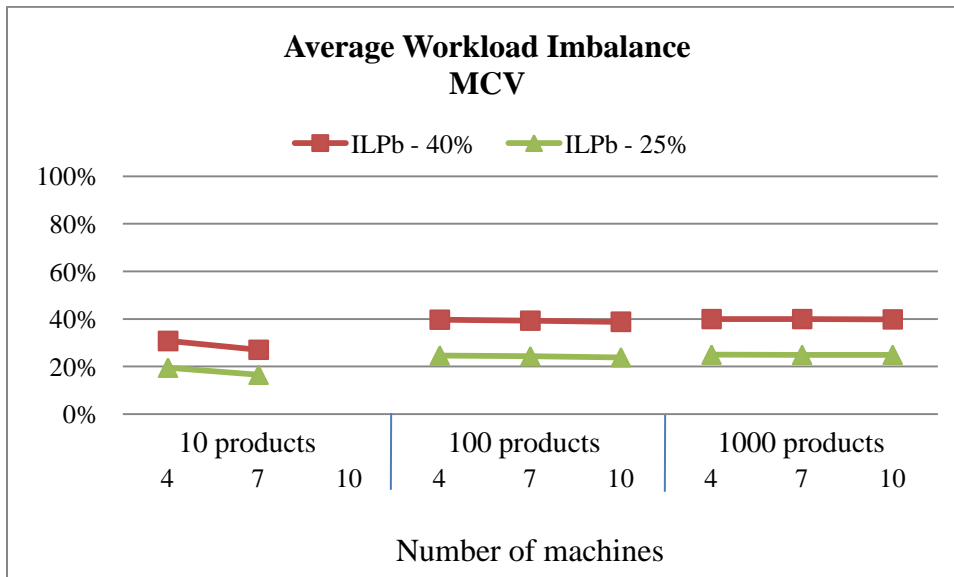
## 6. Analysis of results and sensitivity analyses

The results of the ILPb models are presented for the different scenarios considered with the moderate coefficient of variation (MCV) in terms of average machine utilization, average machine workload imbalance and total cost of assembled products. When the machine workload imbalance factor,  $\delta$ , was 25% and 40%, the results indicate that the higher the number of machines, the higher the machine utilization given the same number of products. As expected, the average workload imbalance is almost constant given that the model has constraints to

balance machine workload. Refer to Figure 3 for the average machine utilization with a MCV and to Figure 4 for the average machine workload imbalance with a MCV.

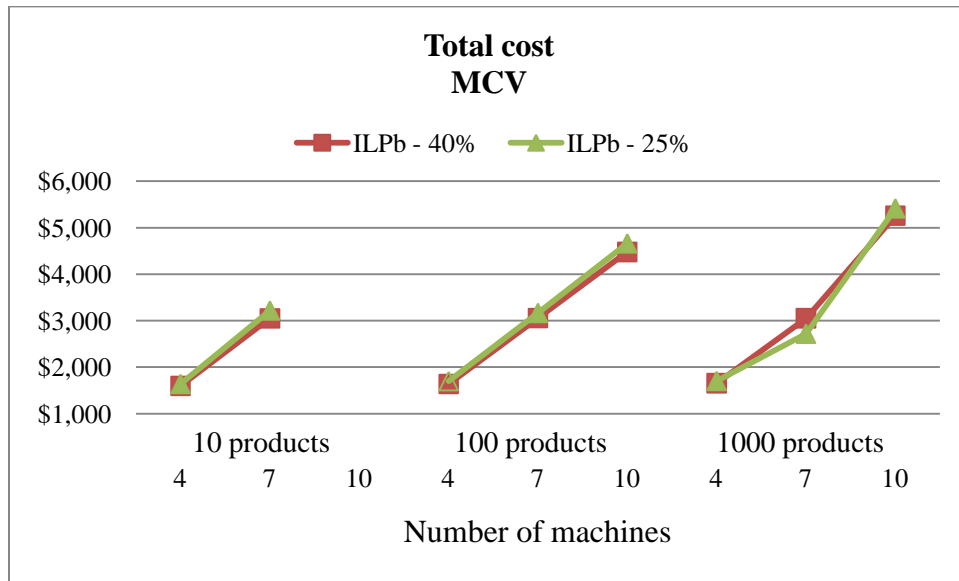


**Figure 3.** Average Machine Utilization – MCV (ILPb;  $\delta = 40\%$  &  $25\%$ )



**Figure 4.** Average Machine Workload Imbalance – MCV (ILPb;  $\delta = 40\%$  &  $25\%$ )

On the ILPb model (with  $\delta = 25\%$  and  $40\%$ ), the total cost, when evaluated at each scenario, has the tendency to increase. This is because the machines chosen at each process for each product will not necessarily be the minimum cost ones to satisfy the machine workload balance constraint. As can be seen in Figure 5, this is not the case for the 10 products scenarios.



**Figure 5.** Total Cost - MCV (ILPb;  $\delta = 40\%$  &  $25\%$ )

To better understand the behavior of the mathematical model studied during the course of this research, sensitivity analyses were done. Because of the importance of controlling the process variability at any manufacturing plant, the sensitivity analyses were done by varying the coefficient of variation (CV). Therefore, additional scenarios were created by considering a low CV (LCV) of 0.4 and a high CV (HCV) of 1.4. As explained by Hopp and Spearman, low variability is when  $CV < 0.75$ , moderate variability is when  $0.75 \leq CV < 1.33$ , and high variability is when  $CV \geq 1.33$  (Hopp et al., 2001). The additional scenarios increase the total runs of the mathematical model to fifty four for the ILPb model. Table 3 summarizes the scenarios for the sensitivity analyses.

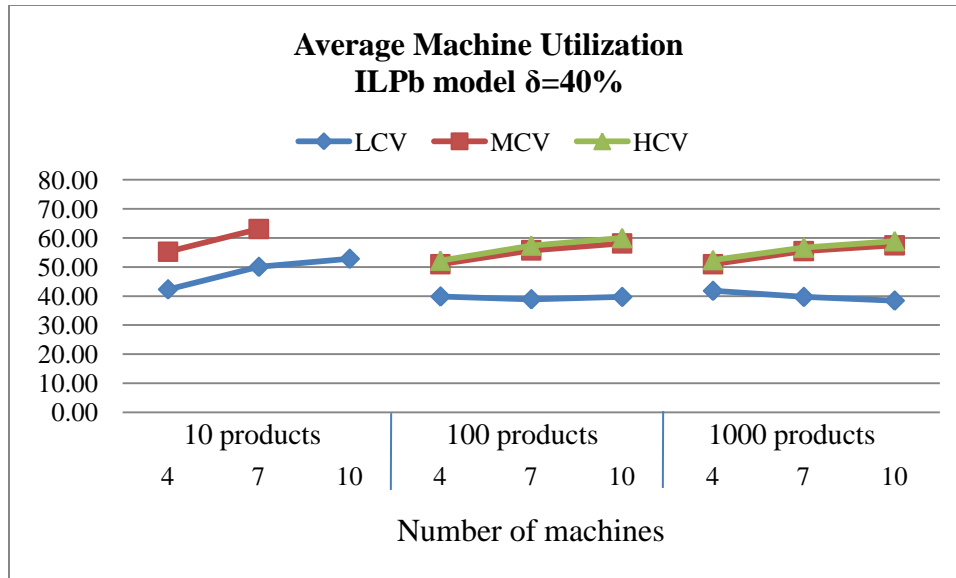
**Table 3** - Scenarios of the ILPb models for the sensitivity analyses

<i>CV</i>	<i>Machines</i>	<i>ILPb; <math>\delta = 0.40</math></i>			<i>ILPb; <math>\delta = 0.25</math></i>		
		<i>Products</i>			<i>Products</i>		
0.4	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000
0.9	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000
1.4	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000

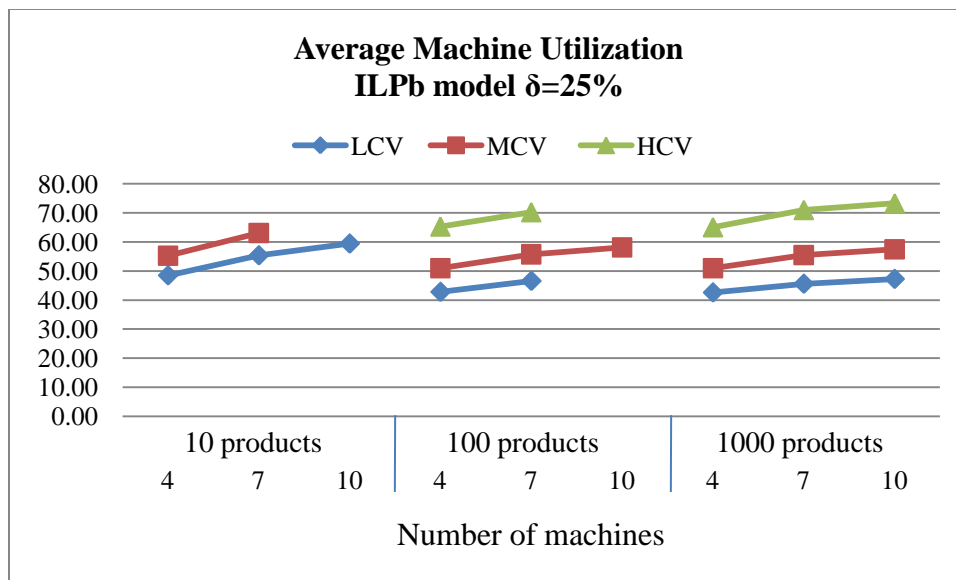
When running the ILPb model with  $\delta = 40\%$  and a LCV, there were two scenarios that ran over the time limit to find a feasible solution; 100 products with seven and ten machines. With a LCV and  $\delta = 25\%$ , there were three scenarios that ran over the time limit and were able to find feasible solutions; 100 products with seven machines, and 1000 products with seven and ten machines. For the scenario with 100 products and ten machines, the solver was not able to find an integer feasible solution even when running during 4528 seconds through over four millions of iterations and almost four hundred thousand B&B nodes analyzed.

The results for the sensitivity analyses for the ILPb model with HCV and  $\delta = 25\%$  and  $40\%$  show that the scenarios with 10 products were not feasible and the solver determined that in less than twenty-two seconds for each scenario. Also, for eleven of the 100 products and 1000 products scenarios, optimum solutions were not found within the time limit, but feasible solutions were obtained instead.

The following graphs (refer to Figures 6 and 7) show the average machine utilization under each level of CV for the ILPb with  $\delta = 40\%$ , and ILPb with  $\delta = 25\%$ , respectively.



**Figure 6.** Average Machine Utilization – ILPb Model;  $\delta = 40\%$



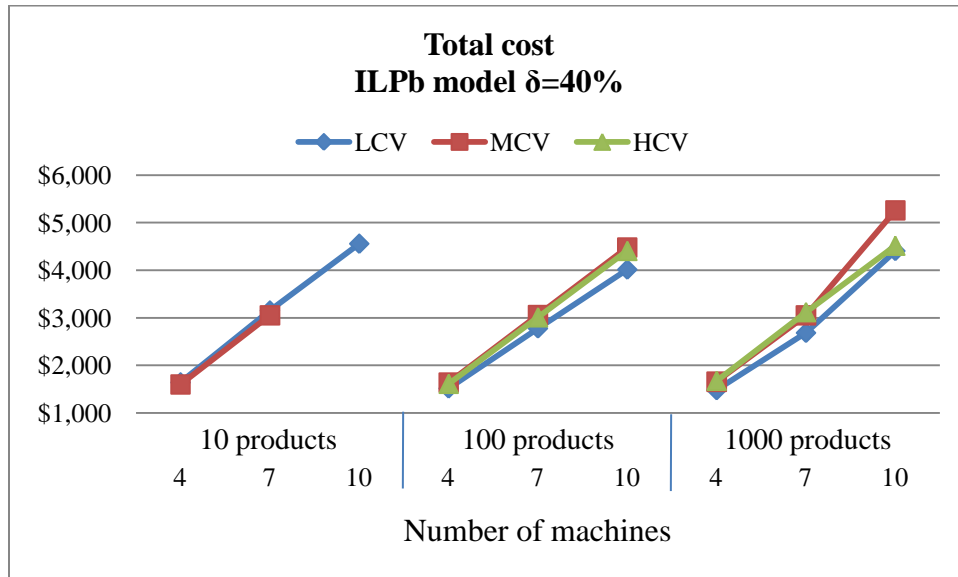
**Figure 7.** Average Machine Utilization – ILPb Model;  $\delta = 25\%$

Under the ILPb model scenarios with  $\delta = 40\%$  (refer to Figure 6), the higher the CV, the higher the average machine utilization for each combination of number of products and machines. When comparing MCV and HCV, there are no significant differences in the average

machine utilization given the same combination of number of products and machines. The missing points on the graphs represent infeasible scenarios.

Analyzing the ILPb model with  $\delta = 25\%$  (refer to Figure 7), the higher the CV, the higher the average machine utilization for each combination of number of products and number of machines. With this model, due to the imbalance factor being more restrictive, there is a noticeable difference on the average machine utilization between MCV and HCV resulting in higher numbers for the HCV. The missing points on the graphs also represent infeasible scenarios.

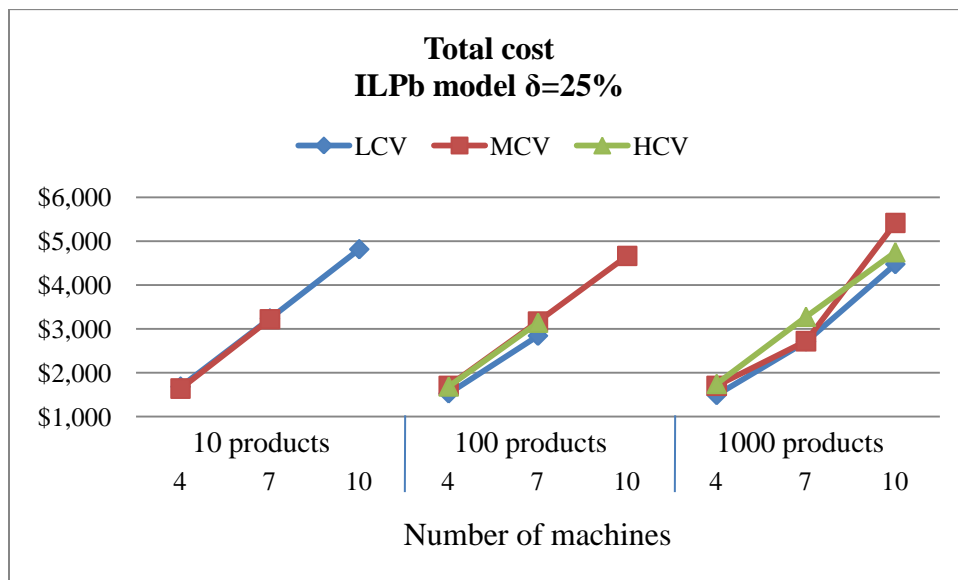
The last comparisons from the sensitivity analyses show the impact on the total cost of all products assembled. Figures 8 and 9 present the total cost for each level of CV for the ILPb with  $\delta = 40\%$ , and ILPb with  $\delta = 25\%$ , respectively.



**Figure 8.** Total Cost – ILPb Model;  $\delta = 40\%$

The graphs for total cost show that the higher the level of the CV and the higher the number of machines within each number of products, the higher the total cost of assembling all products. It can be seen that there is no significant variation on the total cost for the same number of

machines when assembling different amounts of products. This means that when there are four machines, if the products assembled are 10, 100, or 1000, the total cost do not vary significantly because the expected demand per product is not the same to be able to assemble all products given the available capacity. Same reasoning applies when there are seven or ten machines. There is one noticeable exception when there are 1000 products assembled with ten machines per assembly process because the total cost is significantly higher under the MCV scenarios for both imbalance factors ( $\delta = 40\%$  &  $25\%$ ).



**Figure 9.** Total Cost – ILPb Model;  $\delta = 25\%$

## 7. Conclusions

While looking for the best way to select machines when there are multiple related assembly machines available, minimizing the total cost of assembling all products is an appropriate method that can be used as a short-term decision tool. The ILPb does not necessarily assign the minimum cost available machine to be able to balance the machines workload within each assembly process up to the selected imbalance factor. Therefore, the ILPb model requires more machine hours in average than the ILP model (presented on a researcher’s previous work,

Méndez 2009) to assembly the same amount of product demand. This is how the average machine utilization increased when running the ILPb model while the average machine workload balance decreased. As a result, usually the total cost of assembling all products is higher with the ILPb model than with the ILP model given other variables are kept constant.

Analyzing the total cost of all products assembled, since there is not a significant difference, the ILPb model is recommended over the ILP model due to its ability to perform a more complete analysis while solving a machine selection problem. The average difference in total cost fluctuates between 5 and 10% when comparing the ILPb versus the ILP model depending on the level of the CV. Between the two levels of the imbalance factors studied ( $\delta = 40\%$  and  $25\%$ ), the average difference in total cost goes from 2 to 5%. For this reason, the mathematical model considering  $\delta = 25\%$  is highly recommended because it provides a tighter machine workload balance when the machine selection problem is solved.

Even for the more restricted scenarios, the ILPb model when running with AMPL9 and CPLEX90 was able to find feasible solutions. There were scenarios that needed over four millions iterations to be solved and over three hundred B&B nodes to be analyzed within a time range of five to seven hours. As expected, the higher the level of the CV, the higher the average total cost of all products assembled, and the higher the average machine utilization will be when using the ILPb model with any of the imbalance factors identified. The scenarios varying the CV were presented for the purpose of relating to more realistic scenarios in an electronics assemblies environment.

## **8. Recommendations for future research**

There are some options that have been considered by the researchers to expand the use of the integer linear programs developed during the course of this research. These options have two

main purposes: to improve the mathematical models presented in this research considering the same domain, electronics products assemblies; and to extend the applicability of these integer linear programs to other manufacturing scenarios. Following are some recommendations:

- Since there was a significant number of scenarios where an optimum solution was not found during the established time limit (approximately 40%), developing heuristics procedures to solve the ILPb models with emphasis in finding near optimum solutions must be the next step to continue this research effort.
- An additional research focus can be to develop this kind of mathematical model considering a full or absorption costing accounting system for long-term decision making (instead of the variable costing accounting system used in this research). Under a full costing accounting system, the fixed manufacturing overhead costs would have to be included in the calculation of the overhead costs.

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