POMS Abstract Number: 015-0330

Calculating Process Capability Index with Limited Information

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POMS 21st Annual Conference

Vancouver, Canada

May 7 to Mat 10, 2010

1. INTRODUCTION

The question of whether or not a process is capable of meeting specifications has been asked in a wide variety of industries since the early 1980s (Rodriguez, 1992). The theoretical framework for accessing the capabilities of a process began with the development of the C_p index by Juran (1974). Process capability indices continue to be widely used tools for process engineers despite "a growing recognition that these tools are limited and, in particular, that standard capability indices are appropriate only with measurements that are independent and reasonably normally distributed" (Rodriguez, 1992, p176). The popularity of process capability indices, along with the common understanding that in many cases these indices are flawed tools, has led continued research in this area. A recent summary of the state of theory and practice is presented by Wu, Pearn, and Kotz (2009).

Process capability indices are dimensionless measures that relate the output dispersion of a process allowed by its specification tolerance limits to the actual dispersion of the process. These indices allow the comparison across a whole range of processes, industries, and countries (Prasad and Bramorski, 1998). The first process capability index developed was the C_p index. To date the C_p index is still one of the two most popular indices (Rodriguez, 1992). It relates the magnitude of a process's tolerance range to its dispersion as shown below in equation 1.

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

The primary deficiency of the C_p index is that it does not provide an indication of the process' output location with respect to its specification limits. In other words it implicitly assumes that a process is centered between the upper and lower specification limits. The C_{pk} index is the most popular process capability index (Wu, 2008; 2009). Introduced by Kane (1986), the C_{pk} index overcomes the above stated weakness by explicitly considering both "the magnitude of process variation as well as the degree of process centering" (Chen, Pearn, and Lin, 2003, p102). This is shown below in equation 2.

$$C_{pk} = \min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] \tag{2}$$

Many researchers recognize that process capability indices are ultimately measures based on process yield (Boyles, 1991; Chen et al, 2003; Hsu, Pearn, and Wu, 2008; Wu et al, 2009). In this study we present two process capability indices that are based directly on process yield rather than one that is indirectly based on yield through such intermediate measures such as sample mean and sample standard deviation. The first index (C_{pky}) is mathematically equivalent to the C_{pk} index for normally distributed data. This formulation is then modified to make it applicable regardless of distribution type or the number of features/characteristics being measured. The result is a new process capability index (C_{py}) that is simpler than many recently developed indices, yet provides a true yield based measure of a process' capability to produce in-spec products. This new index also overcomes many of the limitations that have plagued previous process capability indices.

2. UNDERPINNING ASSUMPTIONS OF PROCESS CAPABILITY INDICES

Traditionally, process capability indices rely on several assumptions of dubious validity. Misunderstandings, or violations of these assumptions, are likely culprits for the bad publicity process capability indices have received. These assumptions include; process stability, normally distributed outputs, and a single performance characteristic.

2.1. Process Stability

One of the most commonly cited assumptions is that the process being analyzed must be stable (Prasad and Bramorski, 1998; Deleryd, 1999; Hsu et al., 2008). Prasad and Bramorski (1998) and Boyles (1994) both state that to effectively use process capability indices, the process variability for each of the sample subgroups in the sampling regiment must be identically and independently distributed. They further state that the dependency structure between units output from a process, along with outliers, will cause problems with the usage of standard process capability indices.

Many process capability indices, including C_p and C_{pk}, are calculated based on sample statistics collected by statistical process control tools, or other sampling methods. In order for these sample statistics to be meaningful the process must be under a reasonable degree of statistical control (Deleryd, 1999). Deleryd also points out that, "a capability study monitoring an unstable process will only express the capability of the process at that very moment and nothing can be said about the capability of the process in the future" (p320).

2.2. Process Distribution

Another important assumption of process capability indices is that the process being analyzed most be normally distributed (Hsu et al, 2008), or at least symmetric (Gunter, 1989).

Sometimes data transformations are used to generate distributions that are more nearly normal (e.g. Clements, 1989; Franklin and Wasserman, 1992; Shore, 1998; Polansky 1998). Some authors have developed process capability indices that are more appropriate for non-normal data (Hsu et al, 2008). This assumption of normality also implies that the characteristic of interest most be measureable on a continuous scale.

2.3 Single Characteristic

Most process capability indices are applicable only when considering a single characteristic, while production processes nearly always entail many steps. When there are multiple steps in a process, Bothe (1992) suggests that the process capability index associated with the least capable step be used to determine the capability of the process as a whole. As pointed out by Pearn and Wu (2006), this may represent an upper bound on capability but, in general, "the overall process yield of a multi-process product is lower than any individual process yield" (p640). Pearn and Wu (2006) present a relatively recent review of the literature concerning process capability and multiple characteristics.

3. DEVELOPMENT OF A PROCESS YIELD-BASED CAPABILITY INDEX

In practice, the only information frequently available on a process is the first pass yield rate. Wu et al. (2009, p342) state that, "process yield has been for some time the most common and standard criterion used in the manufacturing industries for judging process performance". They define process yield as shown below where, F(x) is the cumulative distribution function of the measured random characteristic x.

$$Yield(Y) = \int_{LSL}^{USL} dF(x) \qquad (3)$$

Throughout the literature, process capability indices are calculated based upon measured performance characteristics of the process, and process yield is estimated from the resulting index. The main purpose of this study is to develop a measure of process capability that is based directly on yield rather than on the sample mean and standard deviation of the process. In order to do this, F(x) is divided into three sections, the range between the upper and lower specification limits, the range below the lower specification limit, and the range above the upper specification limit. This is easily accomplished by determining yield, and the portion of non-conformance in each tail of the distribution. The C_{pk} index implicitly ignores the non-conformance in the smaller tail of the distribution. Because of this, the index is considered to be "one sided" (Boyle, 1994).

Because the non-conformance in the smaller tail of the distribution is ignored, the C_{pk} index will typically overstate a process' capability.

As stated above, C_{pk} ignores non-conformance in the smaller tail of the process distribution. This is an implicit assumption that based on the belief that due to the shift in the process mean, there is non-conformance only in one tail. This assumption does not always hold true in reality. As pointed out by Rodriguez, this means that "C_{pk} does not uniquely determine the proportion nonconforming" (1992, p177). Depending on the circumstances, this can be a serious concern. When the process yield is very high and/or the process is highly off center relative to the specification limits, it may be valid to assume that non-conformance occurs in only a single tail of the distribution. In other cases this assumption may be highly inappropriate.

When non-conformance occurs in both tails, the result is a misleading capability index value. Misleading, or misunderstood, index values have caused some organizations to abandon the use of indices after discovering that "their misuse is an obstacle to improvement" (Rodriguez, 1992, p176). This has leaded some authors to state that the "widespread, uninformed use of C_{pk} to benchmark all processes is of questionable validity" (Gunter, 1989, p72).

To further this discussion, a definition of the proportion of non-conformance found in the smaller tail of a process' output distribution is proposed as follows;

DEFINITION: N is equal to the portion of non-conformance in the smaller of the two tails, and is mathematically expresses as shown in equation 4.

$$N = Min\left(\int_{-\infty}^{LSL} dF(x), \int_{USL}^{\infty} dF(x)\right). \tag{4}$$

Due to the implicit assumption that C_{pk} dismisses non-conformance in the smaller tail of a process distribution, a process yield based version of the C_{pk} index must include N. This new index is expressed in equation 5.

$$C_{pky} = \frac{\Phi^{-1}(\mathrm{Y} + \mathrm{N})}{3} \tag{5}$$

As noted in equation 5, the C_{pky} index suffers from the same problems as the C_{pk} index when it comes to ignoring non-conformance in one of the tails. Fortunately the structure of the C_{pky} index is such that correcting this problem is quite simple: remove *N* from the formulation. The result is

C_{py}, a process capability index that considers all non-conformances, regardless of which tail it occurs in. This new process capability index is expressed in equation 8.

$$C_{py} = \frac{\Phi^{-1}(\mathrm{Y})}{3} \tag{8}$$

5. DISCUSSION

Throughout the literature it is assumed that once a process is under control and stable that it will remain so. Porteus (1986) notes that the probability of a process becoming unstable will increase each successive unit produced. Furthermore, literature cites that process mean and standard deviation data is usually unknown, or flawed due to sampling error, or violates the assumption of normality (Gunter, 1989; Boyles, 1994; Pearn and Wu, 2006; Hsu et al, 2008; Wu, 2008; 2009). The C_{py} index is not based on any sample statistics. Instead, it is based on the actual yield of the process, which is a population parameter. Therefore, process capability can be calculated using the C_{py} index regardless of whether or not the process is stable.

Deleryd points out that "a capability study monitoring an unstable process will only express the capability of the process at that very moment" (1999, p320). If it is unknown whether or not a process is stable, the C_{py} index can still be used to determine that process's capability at that point in time. If management is interested in knowing the process's stability, run chart the C_{py} can be produced and tracked over time. Note that this method of determining stability is based on yield, not process centering and dispersion. If management is concerned only with the stability of yield, this distinction is irrelevant. On the other hand, if management is concerned with the stability of process centering and dispersion, this method of assessing stability does not make traditional statistical process control completely redundant.

Several researchers have noted that when standard capability indices are used to deal with non-normal distributions, that the indices will frequently misrepresent the actual product quality. The condition of process stability on the usage of standard capability indices is a problem because many processes are neither normally distributed, nor symmetric (Pyzdek, 1992; Hsu et al, 2008). Literature also finds that the actual mean and standard deviation of many processes are generally unknown, as well as, their distribution patterns (Kushler and Hurley, 1992; Pearn and Wu, 2006).

To overcome the problems of non-normality, process capability indices are usually based on sample statistics and rely on the central limit theorem to justify the assumption of normality. However, this leads to additional problems. Any time the central limit theorem is used to assure normality, sampling error becomes an issue (Pearn and Wu, 2006; Wu, 2008). Another populate method of dealing with non-normality is data transformation. The use of data transformation techniques lead to its own set of issues including the loss of information, difficulty interpreting transformed data, and difficulty reversing transformations back to a natural scale. The C_{py} index does not rely on the central limit theorem or data transformations. The C_{py} index is based exclusively on yield; therefore, all that is important is what portion of output is within specifications and what portion is not within specifications. This means that normality or non-normality is totally irrelevant when using the C_{py} index.

Another assumption that is often made is that various samples are identically and independently distributed (Prasad and Bramorski, 1998). Stoumbos (2002) states that special cause variations in a process distorts the interpretation of capability indices and renders any predictions dubious. Kane (1986) pointed out that traditional process capability indices are sensitive to departures from normality. Again, the C_{py} index is based exclusively on yield and as such is not sensitive to departures from normality. Although issues such as non-normality or serial correlation may be interesting, they do not necessarily impact process capability and need not be considered when using the C_{py} index.

An implied assumption directly associated with stability and normality is that the performance characteristic being measured is continuous. Deleryd (1999) pointed out that traditional process capability indices have difficulty dealing with attribute characteristics. Attribute characteristics are discrete in nature rather than continuous. The discrete nature of attribute characteristics makes most process capability indices unsuitable for use. Because the C_{py} index is based exclusively on yield, it is equally applicable when dealing with a single characteristic, multiple characteristics, or attribute characteristics. Yield can easily be calculated for any complete multi-step process or process step that has success and failure conditions; thus, the C_{py} index can be used in any of these cases.

6. CONCLUSIONS

In this study we develop a new process capability index which overcomes many of the inherent problems that have plagued existing process capability indices. This new index also answers the call for a yield-based metric. The C_{py} index uniquely associates process capability to process yield by considering both tails of the process distribution. It does not make any assumptions regarding the process distribution such as stability, normality, symmetry, or continuity of the measurement scale. It is applicable when analyzing a process with a single characteristic or multiple characteristics. It is even applicable if one or more characteristics are attributes. Perhaps the most important benefit of this new index is that it is much easier to use than many existing indices. This is due in part to the simple nature of equation 8, but is mainly due to the fact that the index is based exclusively on yield, a measure that is very likely already captured even in organizations that are not implementing more advanced quality tools such as statistical process control.

"The primary goal of quality improvement is to reduce process variability" (Prasad and Bramorski, 1998, p426). The traditional capability indices are good tools for measuring process location and variability, when the process is statistically stable and conforms to the other assumptions noted previously. Lacking these essential conditions, a new index is needed. The C_{py} index developed in this paper fills this gap.

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