Variance of the forecasting error: revisiting top-down and bottom-up approaches under different updating conditions

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Keywords: Sales forecasting approach, Forecast updating, Forecasting errors, Variance, Information exchange.

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Abstract

The literature is still inconclusive as to the most appropriate sales forecasting approach. This paper aims at analyzing the behavior of the variance of the sales forecasting error during the lead-time under Top-Down or Bottom-Up approaches and under different conditions of forecast updating in order to identify the best alternative for achieving lower safety inventory levels. The paper revisits the Top-Down vs. Bottom-Up discussion in light of todays collaborative planning initiatives; corroborates empirical evidences on the choice of the forecasting approach by means of the analytical demonstration of the formulae relating intervening variables; and contributes to practice by providing straightforward equations to accurately determine safety inventory levels under such conditions.

1 Introduction

The decision on the best approach to sales forecasting has been traditionally treated on an empirical basis, leading to a choice whether to adopt a Top-Down or a Bottom-Up perspective. This debate has been overshadowed in the 1990s by collaborative forecasting initiatives and real-time actual sales information exchanges both at the firm level, such as Sales & Operations Planning (Lapide, 2001/2002; 2006), and at the supply chain level, such as Vendor Managed Inventory and CCollaborative Planning, Forecasting & Replenishment (Kelle and Akbulut, 2005; Sari, 2008). More recently, research on hierarchical forecasting methods has been conducted in an attempt to combine both approaches in order to decrease the forecasting error (Hyndman et al., 2007). In fact, the topic is seen as being as relevant as ever (Zotteri and Kalchschmidt, 2006; Lapide, 2006).

In most previous empirical studies, analyzes were carried out using the simple exponential smoothing method on actual sales data (Schwarzkopf et al., 1988; Gordon et al., 1997; Snyder et al., 2004). They led to different conclusions about the adequacy of the alternative approaches for different levels of the correlation coefficient, of the sales variance, and of the share of a given product in total or aggregate sales. Another stream of literature adopted a theoretical approach by deriving an analytical expression for the mean and variance of the lead-time demand and by combining forecasts for hierarchical time series (Hyndman et al., 2007).

This paper analyzes the behavior of the variance of the sales forecasting error during the lead-time under the Top-Down and the Bottom-Up approaches, and under different conditions of forecast updating. The objective is to identify under which circumstances one approach could be more advantageous than the other, as measured by a lower variance and, thus, lower safety inventory levels. We obtain the analytical expressions of the variance of the forecasting error during the lead-time when sales forecasts are frozen or updated, and under a Top-Down and a Bottom-Up approach, linking these approaches to the discussion of demand updating and information exchange in collaborative arrangements.

A close work is Snyder et al. (2002) which provides formulae for calculating the mean and variance of lead-time demand under several forms of exponential smoothing, including the homoscedastic and the heteroscedastic cases. They obtain a lead-time demand variance formulae for heteroscedastic extensions to exponential smoothing, to be used for safety stock determination tailored for changes in trend or changes in season. In the present paper although we do not consider seasonal effects in the data and more complicated smoothing techniques, we do offer a simple decision rule on the choice of the TD or the BU approaches based on data temporal structure (as in ARIMA models) and the values of data first two moments.

The contribution of this paper is anchored on three key elements. The first one is that it revisits the discussion on the choice of a Top-Down or a Bottom-Up approach in light of today's collaborative planning initiatives and unveils the complex relations that involve the variance of the forecasting error under conditions of frozen or unfrozen forecasts during the lead time. The second is that it corroborates the empirical evidences on the choice of the forecasting approach by means of the analytical demonstration of the formulae relating intervening variables. At last, it contributes to practice by providing straightforward equations to accurately determine safety inventory levels under such conditions.

Results indicate that the choice of the best approach to sales forecasting does not depend on the updating of the forecast during the lead-time as well as on the value of the smoothing constant. Whether the forecasts are updated or not, the choice of the best approach is shown to be a function of the main characteristics of the sales data series of a given product when compared to the total aggregate sales. These characteristics are the correlation coefficient, the share of this product in total sales, and the ratio between the standard deviation of demand of this product and that of the remaining products.

However, it should be noted that there is a relevant interaction between the choice of the smoothing constant and the updating of the sales data during the lead-time. This interaction affects the variance of the sales forecasting error during the lead-time. If forecasts are updated during the lead-time, the value of the smoothing constant should be set to one in order to minimize the error variance, that is, the actual demand is the best predictor of the demand of the subsequent period. This corroborates the rationale behind collaborative planning initiatives of using all information available to parties.

The remainder of the paper is structured as follows. Section 2 reviews the discussion on the choice of Bottom-Up or Top-Down approaches, the impact of collaborative planning initiatives on forecasting updates and simple exponential smoothing concepts. Sections 3 and 4 analytically demonstrate the expressions of the variance of the sales forecasting error respectively for the Bottom-Up and Top-Down approaches, each under conditions of frozen and unfrozen forecasting during the lead time. Finally, section 5 concludes the paper by presenting the discussion of the contributions and the managerial implications of the results.

2 Literature Review

2.1 The Top-Down x Bottom-Up Approaches

There is enough consensus in the literature on the conceptualization and operationalization of the Top-Down (TD) and Bottom-Up (BU) sales forecasting approaches, though not on the compared relative advantages.

Under the TD approach, sales forecasting is done first by aggregating all individual items, and then by disaggregating again these aggregate data into individual items, generally based on the historical percentage of each item within the whole group. Conversely, in the BU approach each one of the individual items is forecasted separately and then all the forecasts are summed up in case an aggregate forecast for the group is deemed necessary (Schwarzkopf et al., 1988; Jain, 1995; Lapide, 1998).

Previous literature covers the adequacy of the TD and BU approaches according to different characteristics of the historical data. The majority of these studies consider the correlation coefficient between sales of an individual item and the aggregate sales of the remaining items (Kahn, 1998; Lapide, 1998; Gelly, 1999; Gordon et al., 1997; Schwarzkopf et al., 1988). Others focused on the associations among the share of an item in the aggregate total sales, the ratio between the sales variance of an individual item, and the aggregate sales of the remaining items (Gelly, 1999).

In the TD approach the peaks and valleys inherent to each items sales are cancelled off by the aggregation, and the negative correlation among the individual items reduces the aggregate sales variance (Kahn, 1998). Estimates based on aggregate data are considered to be more precise than those based on individual forecasts also when the individual items present independent sales patterns, that is, a null correlation (Schwarzkopf et al., 1988).

However, Lapide (1998) indicates that the TD approach makes sense only if all individual items sales are growing, decreasing or remaining stable, thus characterizing a positive correlation among the sales of different items. For example, a family of products frequently comprises items that potentially cannibalize each other, as in the case of a family with new and old products. For those items, the sales pattern is very different, given that the sale of some items increases to the detriment of the others (negative correlation), in which case the BU approach would be preferable.

Gordon et al. (1997) studied more than 15,000 aggregate and disaggregate historical data series generating forecasts with Triple Exponential Smoothing. The BU approach yielded more precise forecasts in 75 percent of the series, and higher precision gains were obtained for individual items with strong positive correlation and whenever they represented a large proportion of the aggregate total sales. On the other hand, when the data were negatively correlated, the TD approach resulted in higher accuracy regardless of the item's participation in the aggregate total sales. All these apparently conflicting results call for an analytical quantification of the impact of the variables involved (correlation coefficient, smoothing constant) on the approach selected.

Finally, the TD approach has already been shown to be more appropriate for individual items that present a more predictable sales pattern along time, such as, for example, those with a small sales coefficient of variation. That small coefficient could result from a large participation of the individual item in the aggregate total sales or otherwise from a small ratio between the variance of the individual item sales and the variance of the aggregate sales of the remaining items (Gelly, 1999).

Hyndman et al. (2007), in a recent working paper, make the case for a new approach to hierarchical forecasting, which they consider to provide optimal forecasts, better than those produced by either a TD or a BU approach. These forecasts consist of a combination of both approaches, and the determining criteria are the mean absolute error and the mean absolute percentage error. However, they do not consider the forecast errors during the lead-time. On the other hand, sales forecasting during the replenishment lead-time constitutes a more complex situation (Harrison, 1967; Johnson and Harrison, 1986). Snyder et al. (2004) calculate the means and variances for demand during the lead-time, under a wide variety of exponential smoothing methods, but do not consider the forecasting error.

The variance of the forecasting error is seen as a cornerstone measure for determining safety inventory levels. In this case, the forecasting error variance during the lead-time must be estimated. The variance of the forecasting error and the sales variance are not equal (Silver and Peterson, 1985; Greene, 1997). The best adequacy of the variance of the forecasting error is related to the use of forecasting for sales estimation. Therefore, the safety inventory level must be determined to protect against variations in sales forecasting errors. Usually the variance of the forecasting error tends to be greater than the sales variance, due to the additional sampling error introduced by the forecasting models when using only part of the available historical data (Silver et al., 2002).

According to Silver and Peterson (1985), the exact relationship between the variance of the forecasting error for a single period and the variance of the forecasting error during the lead-time depends upon complicated relations between the sales pattern, the forecasting review procedures and the value of the smoothing constant. One of the reasons for such complexity is that the recurrence procedure in the exponential smoothing introduces a certain degree of dependence between the forecast errors at different periods of time separated by the lead-time (Harrison, 1967).

2.2 Collaborative Planning Initiatives

Using frozen sales forecasts during the lead-time is a very common management practice, since the review of sales forecasts values tends to be worthless when replenishment orders are still being processed (Greene, 1997). Unless it is possible for the decision-makers to change previously placed orders during the replenishment lead-time, there is little value in reviewing sales forecasts during the time interval between the placement and the receipt of an order. This paper departs from previous studies by also considering the case of unfrozen sales forecasts during the lead-time. In this case, sales forecasts are supposed to change during the lead-time, thusbenefiting from the real time information exchange inherent to collaborative initiatives like CPFR.

CPFR, or Collaborative Planning, Forecasting, and Replenishment has been defined as “an initiative among all participants in the supply chain intended to improve the relationship among them through jointly managed planning processes and shared information” (Seifert, 2003, p. 30). It is considered to improve reaction times to consumer demand; yield more precise sales forecasts; enhance communication between buyers and sellers; reduce lost sales due to out-of-stock items; reduce inventory; and reduce costs (Seifert, 2003).

When using collaborative forecasting, buyers and sellers share their demand forecasting processes and information (Aviv, 2001) and can update their sales and order forecasts each time an exception occurs (Caridi et al., 2006). It has been associated to cost benefits in the supply chain (Danese et al., 2004; Aviv, 2001, 2007;

Sari, 2008), especially in environments with highly variable demand uncerntainty (Sari, 2008) and in the presence of supply-side agility (Aviv, 2007).

2.3 Exponential Smoothing

Exponentially smoothing methods date back to Brown (1959, 1963), Winters (1960), among others.

Let $X_1, X_2, \cdots, X_n$ represent a sequence of demands for a certain product at times $1, 2, \cdots, n$. The simplest model, the simple exponential smoothing (SES), (Brown, 1963) assumes that

$$X_t = \mu + \epsilon_t, \quad t = 1, \cdots, n \ , \tag{1}$$

where μ represents the unconditional mean, which can be estimated using historical data up to time n, and where $\epsilon_1, \cdots, \epsilon_n$ are independent and identically distributed (iid) (non-observable) random errors with common distribution F with zero mean and variance σ^2_ϵ. It follows that $E[X_t] = \mu$ and $var(X_t) = \sigma^2_\epsilon$ for all t, and $\rho_\tau(X_i, X_j) = 0$, where $\rho_\tau(X_i, X_j)$ represents the linear correlation coefficient between X_i and X_j, $i \neq j$ and where $\tau =\mid i - j \mid$.

Let W be a discrete random variable assuming values in $\mathcal{I}= \{1, 2, \cdots\}$, and representing the lead time (LT) occurring between the order set for the product (at time n) and its delivery. Assume that the lead time W is independent from the sequence $X_1, \cdots, X_n$, and let Y represent the total demand for the product during the lead time (LTD). This means that the lead time demand Y is a random sum given by

$$Y = \sum_{t=n+1}^{n+W} X_t.$$

The conditional expectation of the LTD given that $W = w$, w in $\mathcal{I}$, and the unconditional expectation of the LTD are respectively given by $E[Y \mid W = w] = wE[X] = w\mu$ and $E[Y] = E[W]E[X] = E[W]\mu$. In addition, the conditional variance of Y given that $W = w$, and the unconditional variance of Y are respectively given by $var[Y \mid W = w] = w \cdot var(X) = w \cdot \sigma^2_\epsilon$, and $var(Y) = var(W)(E[X])^2 + E[W]var(X) = var(W)\mu^2 + E[W]\sigma^2_\epsilon$.

Now assume that the forecasts for the demand of the product are computed based on the SES technique. The smoothed series is given by

$$\bar{X}_t = \alpha X_t + (1-\alpha)\bar{X}_{t-1} \ , \ \bar{X}_0 = X_1 \ , \ t = 1, 2, \cdots, n, \tag{2}$$

where α is the smoothing constant, $0 \leq \alpha \leq 1$, and $\bar{X}_t$ is the exponentially smoothed value at time t.

Let $\widehat{X}_n(h)$ represent the h-steps ahead forecast for the demand of the product made at time n. According to SES, the forecasts for all future values are equal to the last exponentially smoothed value:

$$\widehat{X}_n(1) = \widehat{X}_n(2) = \widehat{X}_n(3) = \cdots = \bar{X}_n. \tag{3}$$

The smoothed value also follows the recurrence equation, that is, from equation (2) we obtain

$$\widehat{X}_n(1) = \alpha X_n + (1-\alpha)\widehat{X}_{n-1}(1).$$

Under model (1) (globally constant mean) and for large n the h-steps ahead forecast estimator is consistent:

$$E[\widehat{X}_n(h)] = \mu, \tag{4}$$

providing unbiased predictions, and, as $n \to \infty$, its asymptotic variance is given by

$$var(\widehat{X}_n(h)) = \frac{\alpha}{2-\alpha}\sigma_\epsilon^2, \tag{5}$$

a result which may be used to construct asymptotic confidence interval for the forecasts.[2]

Now suppose that the lead time W starts at time $n+1$. Let the sum of the forecasts for the demand of the product made during the lead time (the lead time forecast, LTF) be the random variable V. There are two possibilities:

[2] Under the globally constant mean SES model, and assuming that the innovations ϵ follow a Normal$(0, \sigma_\epsilon^2)$ distribution, the $(1-\gamma) * 100\%$ asymptotic confidence interval for the h-steps ahead forecast is given by $(\widehat{X}_n(h) - z_\gamma \sigma_\epsilon \sqrt{\frac{\alpha}{2-\alpha}} \ , \ \widehat{X}_n(h) + z_\gamma \sigma_\epsilon \sqrt{\frac{\alpha}{2-\alpha}})$, where z_γ is the $(1-\gamma)$ quantile of the standard Normal distribution.

(*i*) *The forecasts are updated at times* $n+1, \cdots n+W-1$. According to our notation the lead time forecast is given by

$$V = \widehat{X}_n(1) + \widehat{X}_{n+1}(1) + \cdots + \widehat{X}_{n+W-1}(1),$$

which is equal to

$$V = \sum_{k=1}^{W} \widehat{X}_{n+k-1}(1),$$

which is also equal to $V = \bar{X}_n + \bar{X}_{n+1} + \cdots + \bar{X}_{n+W-1}$.

(*ii*) *The forecasts are all computed at time n according to formula (3) (frozen).* This means that the lead time forecast V is a random sum given by

$$V = \widehat{X}_n(1) + \widehat{X}_n(2) + \cdots + \widehat{X}_n(W),$$

which according to (3) is

$$V = \sum_{k=n+1}^{n+W} \bar{X}_n = W\bar{X}_n.$$

Therefore, all predictions are equal to the last smoothed value, although how many of them will be necessary is still uncertain.

We recall that our objective is to compute the variance of the forecasting error during the lead time under the BU and the TD approaches. A main issue is the assessment of correlations among forecasts at different lags and the demands. For each approach we consider the two possibilities of *updated* and *frozen* forecasts.

In Section 3 we consider the BU approach, and in Section 4 the TD approach.

3 The Bottom-Up approach

The BU approach corresponds to the "traditional" practice of forecasting each product individually. Here X represents the demand of a single item or product.

3.1 The BU approach based on updated forecasts

Under this scenario the LTF is given by $V = \widehat{X}_n(1) + \widehat{X}_{n+1}(1) + \cdots + \widehat{X}_{n+W-1}(1)$, a random sum of correlated forecasts since functions of common variables. Recall that for large n, each of these forecast estimators have the same mean and variance given by formulas (4) and (5). To better understand the behavior of the updated forecasts we first compute the correlation coefficient between any two forecasts during the lead time.

Let $\gamma_F(\tau)$ represent the lag τ covariance between two updated forecasts during the lead time, that is, $\gamma_F(\tau) = cov(\widehat{X}_j(1), \widehat{X}_{j+\tau}(1))$, j in $\{n, n+1, \cdots, n+W-1\}$. We now derive the expressions for $\gamma_F(\tau)$. By definition, $\gamma_F(\tau) = E[\widehat{X}_j(1)\widehat{X}_{j+\tau}(1)] - E[\widehat{X}_j(1)]E[\widehat{X}_{j+\tau}(1)]$, and it remains to find the value of the cross moment $E[\widehat{X}_j(1)\widehat{X}_{j+\tau}(1)]$.

According to SES, for any fixed j, $\widehat{X}_{j+\tau}(1) = \bar{X}_{j+\tau}$, for all $\tau = 0, 1, 2, \cdots$. It can be shown that

$$\widehat{X}_{j+\tau}(1) = \alpha \sum_{k=0}^{\tau-1} (1-\alpha)^k X_{j+\tau-k} + (1-\alpha)^\tau \widehat{X}_j(1). \quad (6)$$

Thus

$$\widehat{X}_j(1)\widehat{X}_{j+\tau}(1) = \left(\widehat{X}_j(1)\right)\left(\alpha \sum_{k=0}^{\tau-1}(1-\alpha)^k X_{j+\tau-k} + (1-\alpha)^\tau \widehat{X}_j(1)\right)$$

$$= (1-\alpha)^\tau (\widehat{X}_j(1))^2 + \left[\widehat{X}_j(1)\alpha \sum_{k=0}^{\tau-1}(1-\alpha)^k X_{j+\tau-k}\right].$$

Now note that $\widehat{X}_j(1)$ is simply $\bar{X}_j$, the smoothed value on time j, based on all past observations $(X_j, X_{j-1}, X_{j-2}, \cdots)$. Therefore, all the r.v.s $X_{j+\tau-k}$, $k = 0, 1, \cdots, \tau-1$ are non-correlated with $\widehat{X}_j(1)$. Moreover, they are, by assumption, non-correlated among themselves. Thus,

$$E[\widehat{X}_j(1)\widehat{X}_{j+\tau}(1)] = (1-\alpha)^\tau E[(\widehat{X}_j(1))^2] + E[\widehat{X}_j(1)]\alpha \sum_{k=0}^{\tau-1}(1-\alpha)^k E[X_{j+\tau-k}]. \quad (7)$$

Now, from (4) and (5) and for large series we have

$$E[\widehat{X}_j(1)\widehat{X}_{j+\tau}(1)] = (1-\alpha)^\tau \left[\frac{\alpha}{2-\alpha}\sigma_\epsilon^2 + \mu^2\right] + \mu^2(1-(1-\alpha)^\tau)$$

$$= \frac{\alpha(1-\alpha)^\tau}{2-\alpha}\sigma_\epsilon^2 + \mu^2 \qquad .$$

Therefore, the covariance between updated forecasts separated by a lag τ, $\gamma_F(\tau)$ is given by

$$\gamma_F(\tau) = \frac{\alpha(1-\alpha)^\tau}{2-\alpha}\sigma_\epsilon^2. \tag{8}$$

It is interesting to note that when $\tau = 0$ we obtain the variance (5).

Thus, using (5) and (8), the autocorrelations among the updated forecasts are given by

$$\rho_F(\tau) = (1-\alpha)^\tau \qquad \text{for} \quad \tau = 1, 2, \cdots, W-1, \tag{9}$$

and it goes exponentially to zero as in a auto-regressive process. Note that the correlation between two forecasts does not depend on their time position, but just on the lag τ, and it becomes smaller as the forecasts separation increases.

Now, consider again V, the sum of forecasts for times $n+1, n+2, \cdots, n+W$, made at times $n, n+1, \cdots, n+W-1$, and suppose that the lead time lasts for w periods, that is, let $W = w$. We now compute the *conditional* mean and variance of the LTF, given that $W = w$.

$$var(V \mid W = w) = var(\widehat{X}_n(1) + \widehat{X}_{n+1}(1) + \cdots + \widehat{X}_{n+W-1}(1) \mid W = w)$$

$$= \frac{w\alpha}{2-\alpha}\sigma_\epsilon^2 + 2(w-1)\gamma_F(1) + 2(w-2)\gamma_F(2) + \cdots + 2(w-(w-1))\gamma_F(w-1)$$

which may be rewritten as

$$var(V \mid W = w) = \left(\frac{w\alpha\sigma_\epsilon^2}{2-\alpha}\right) + \left(\frac{2\alpha\sigma_\epsilon^2}{2-\alpha}\right)\sum_{k=1}^{w-1}(w-k)(\rho_F(1))^k, \tag{10}$$

or, equivalently,

$$var(V \mid W = w) = \left(\frac{\alpha\sigma_\epsilon^2}{2-\alpha}\right)\left[w + 2\sum_{k=1}^{w-1}(w-k)(1-\alpha)^k\right]. \tag{11}$$

The conditional mean is given by

$$E[V \mid W = w] = w\mu. \tag{12}$$

To compute the *unconditional* mean and variance of the LTF under scenario (i), first let $p_W(\cdot)$ denote the discrete probability function of W. The unconditional mean is given by $E[V] = E[W]\mu$. The unconditional variance is given by $var(V) = E[var(V \mid W)] + var(E[V \mid W])$. Using results (11) and (12), we get

$$var(V) = \sum_{w \in \mathcal{I}} \left(\frac{\alpha\sigma_\epsilon^2}{2-\alpha} \right) \left[w + 2\sum_{k=1}^{w-1} (w-k)(1-\alpha)^k \right] p_W(w) + var(W)\mu^2. \tag{13}$$

Let ξ represent the product forecast error during the lead time, that is

$$\xi = Y - V.$$

In order to derive the variance of the lead time forecast error ξ, we first compute the value of $\gamma_{Y,V}$, the covariance between Y and V given that $W = w$. From our previous computations, $\gamma_{Y,V} = E[YV \mid W = w] - w^2\mu^2$.

To compute the conditional cross-moment $E[YV \mid W = w]$ we note that (i) X_{n+j} and $\hat{X}_n(1)$ are uncorrelated for $j = 1, 2, \cdots$; (ii) $E[X_j X_{j+k}] = \mu^2$ for any k; (iii) $E[X_{n+j}^2] = \sigma_\epsilon^2 + \mu^2$ for $j = 1, 2, \cdots$; (iv) $E[\hat{X}_n(h)] = E[X_j] = \mu$; and, finally (v) that $var(\hat{X}_n(h)) = \frac{\alpha}{2-\alpha}\sigma_\epsilon^2$. After straightforward allthough tedious computations we derive the expression of the expectation of the random variable YV, given that $W = w$. It is

$$(\mu^2 + \sigma_\epsilon^2)\alpha \left[\sum_{k=0}^{w-2} (1-\alpha)^k + \sum_{k=0}^{w-3} (1-\alpha)^k + \cdots + \sum_{k=0}^{1} (1-\alpha)^k + 1 \right] +$$

$$\mu^2 \left[w\sum_{k=0}^{w-1} (1-\alpha)^k + (w-1)\alpha \sum_{k=0}^{w-2} (1-\alpha)^k + (w-1)\alpha \sum_{k=0}^{w-3} (1-\alpha)^k + \cdots + (w-1)\alpha \sum_{k=0}^{1} (1-\alpha)^k + (w-1)\alpha \right],$$

which reduces to

$$E[YV \mid W = w] = w^2\mu^2 + \sigma_\epsilon^2 \left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha} \right), \tag{14}$$

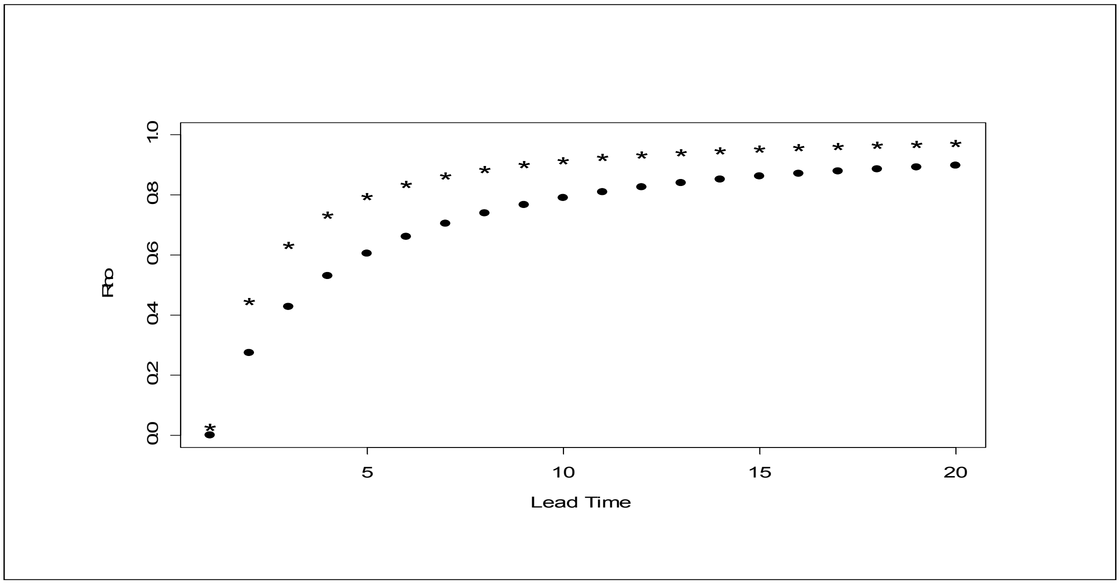

Figure 1: *The correlation between the lead time demand Y and the lead time forecast V for $\alpha = 0.3$ (balls) and $\alpha = 0.7$ (stars), and as a function of the lead time $w \in \{1, 2, 3, ..., 20\}$.*

and therefore

$$\gamma_{Y,V} = \sigma_\epsilon^2 \left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha} \right) \quad \text{given that } W = w, \tag{15}$$

which holds for $w = 1, 2, \cdots$.

The correlation between Y and V, $\rho(Y, V)$, given that $W = w$ is

$$\rho(Y, V) = \frac{\left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha} \right)}{\sqrt{\frac{\alpha w}{2-\alpha} [w + 2 \sum_{k=1}^{w-1} (w-k)(1-\alpha)^k]}}, \tag{16}$$

which does not depend on the mean and variance of the demand process, just on the strength of smoothing constant α. For example, Figure 1 shows for $\alpha = 0.3$ (balls) and for $\alpha = 0.7$ (stars) the value of the correlation coefficient as the lead time varies in $\{1, 2, 3, ..., 20\}$. Independently of the α value, for $w = 1$ it is equal to zero, and for large w it converges to 1.

We are now in position to compute the variance of the lead time updated forecast error ξ given that $W = w$, that is, $var(\xi \mid W = w) = var(Y \mid W = w) + var(V \mid$

$W = w) - 2 * \gamma_{Y,V}$. It is given by

$$var(\xi \mid W = w) = w\sigma_\epsilon^2 + \left(\frac{\alpha\sigma_\epsilon^2}{2-\alpha}\right)\left[w + 2\sum_{k=1}^{w-1}(w-k)(1-\alpha)^k\right] - 2\sigma_\epsilon^2\left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha}\right) \tag{17}$$

which depends solely on the innovations variance σ_ϵ^2, on α, and on the length of the lead time. This conditional variance *increases with* α. See Figure 2.

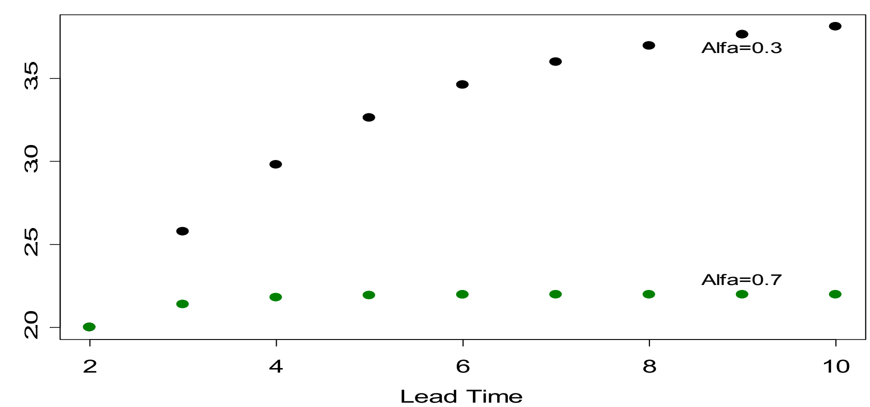

Figure 2: *Variance of the updated forecast errors for* $w \in \{1, 2, 3, ..., 10\}$. *The black balls represent the variance of* ξ *when* $\alpha = 0.3$. *The green balls correspond to* $\alpha = 0.7$.

The variance of the *updated* forecast error during the lead time converges to a finite (small) value as $w \to \infty$. This is in line with the previous result on the correlation coefficient between the observed demand and the forecasts during the lead time (formula (16)), which converges to 1, making the forecast errors and their variances to become smaller as w increases. See Figure 3 where we compare the updated and frozen forecasts errors variance during the lead time, as functions of the lead time.

The *unconditional* variance of the updated lead time forecast error is just the expected value of (17) under the probability distribution of W:

$$E[var(\xi \mid W = w)] = \sum_{w \in \mathcal{I}} var(\xi \mid W = w)\ p_W(w)\ . \tag{18}$$

For example, we computed (18) using Monte Carlo methods assuming W is uniformly distributed over $\{1, 2, \cdots, 10\}$ (note $E[W] = 5.5$) and $\sigma_\epsilon^2 = 1$. For $\alpha = 0.3$ it is equal to 3.0686, and for $\alpha = 0.7$ it is equal to 2.21131. When W is uniformly distributed over $\{1, 2, 3, 4, 5\}$ (one week, $E[W] = 3$) and $\sigma_\epsilon^2 = 1$, for $\alpha = 0.3$ it is equal to 2.4731, and for $\alpha = 0.7$ it is equal to 2.2245.

3.2 The BU approach based on frozen forecasts

Under this scenario the LTF is given by $V = \widehat{X}_n(1) + \widehat{X}_n(2) + \cdots + \widehat{X}_n(W) = W\bar{X}_n$, where all forecasts are computed at time n and are all equal to the last smoothed value. Again, for large n, each of these forecast estimators have the same mean and variance given by formulas (4) and (5). It is clear that the correlation between two forecasts separated by τ lags, $\rho_F(\tau)$, is equal to 1 for all τ which implies that, for large n, the covariance $\gamma_F(\tau)$ between two forecasts separated by a lag τ, is equal to $\frac{\alpha}{2-\alpha}\sigma_\epsilon^2$.

Now suppose that the lead time lasts for w periods, that is, let $W = w$. We first compute the *conditional* mean and variance of the (frozen) LTF, given that $W = w$.

$$var(V \mid W = w) = w^2 \frac{\alpha}{2-\alpha}\sigma_\epsilon^2,$$

and

$$E[V \mid W = w] = w\mu,$$

as $n \to \infty$.

Again, using the above results, it is easy to compute the *unconditional* mean and variance of the (frozen) forecasts during the lead time. The unconditional mean is given by $E[V] = E[W]\mu$, and the unconditional variance is given by

$$var(V) = (\frac{\alpha}{2-\alpha}\sigma_\epsilon^2)E[W^2] + \mu^2 var(W). \tag{19}$$

Let ξ represent the forecast error of the product during the lead time, $\xi = Y - V$. Given that $W = w$:

$$\xi \mid w = \sum_{k=1}^{w}(X_{n+k} - \bar{X}_n) \ .$$

It is clear that the conditional and unconditional expectation of ξ is zero (as obtained by Eppen and Martin (1988). We now derive the variance of ξ given $W = w$. Since $\bar{X}_n$ is non-correlated with the future observations $X_{n+1}, X_{n+2}, \cdots$, we have that $var(\xi \mid W = w) = var(Y \mid W = w) + var(V \mid W = w)$, and thus $var(\xi \mid W = w) = w^2 \frac{\alpha}{2-\alpha} \sigma_\epsilon^2 + w\sigma_\epsilon^2$, or

$$var(\xi \mid W = w) = w\sigma_\epsilon^2 \left(\frac{2 - \alpha + w\alpha}{2 - \alpha} \right), \tag{20}$$

which also coincides with equation (19) of Eppen and Martin (1988). Thus the variance of the frozen forecasts error during the lead time and under the BU approach depends solely on the innovations variance σ_ϵ^2, on α, and on the value of the lead time.

Figure 3 compares the variance of the forecast error during the lead time under the two types of forecast, and for $w = 1, 2, \cdots, 10$. The black balls represent the variance of ξ when the forecasts are frozen, which explode as w increases. The green balls represent this variance when the forecasts are updated. We note that this corresponds to situations where the providers are able to follow their clients needs in real time — probably because they share the demand through the internet, or join to other active partners — and, in this case, the variance grows slowly and tends to stabilize. This could be interpreted as the value of a "logistic partnership" among firms, with impact in the safety stocks.

The *unconditional* variance of the frozen lead time forecast error is just the expected value of (20) since the conditional expectation is zero. Assuming any discrete probability distribution of W with mean μ_W and variance σ_W^2, we obtain

$$var(\xi) = \sigma_\epsilon^2 \left(\mu_W + \frac{\alpha}{2 - \alpha} (\mu_W^2 + \sigma_W^2) \right) . \tag{21}$$

For example, we computed (21) assuming W is uniformly distributed over $\{1, 2, 3, 4, 5\}$ (note $\mu_W = 3$ and $\sigma_W^2 = 2$) and $\sigma_\epsilon^2 = 1$. For $\alpha = 0.3$ it is equal to 4.9412, and for $\alpha = 0.7$ it is equal to 8.9231, approximately two times greater than the updated ones.

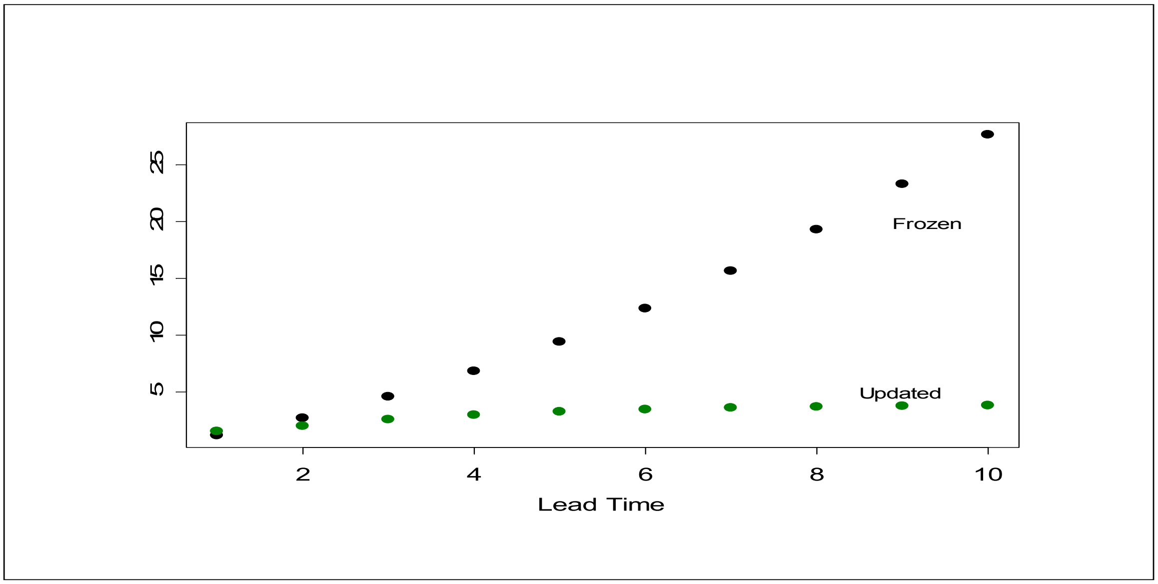

Figure 3: *Conditional variance of the forecast errors for* $w \in \{1, 2, 3, ..., 10\}$*. The black balls represent the variance of* ξ *when the forecasts are frozen. The green balls represent this variance when the forecasts are updated, for* $\alpha = 0.3$ *and* $\sigma_\epsilon^2 = 1$*, fixed.*

4 The Top-Down approach

In this section we derive the variance of the forecast error during the lead time assuming the TD approach. We consider the two possibilities: updated and frozen forecasts.

We assume that the *total aggregate sales* T may be splitted into two parts (or two sub-products), that is

$$T_t = X_{1t} + X_{2t}, \quad t = 1, 2, \cdots, n \ ,$$

where each part X_{it}, $i = 1, 2$, as well as the total T_t follow model (1). We denote the means and variances of X_1, X_2 and T by μ_1, σ_1^2, μ_2, σ_2^2, μ_T and σ_T^2, respectively.

It follows that $var(T) = var(X_1) + var(X_2) + 2\rho_{12}\sqrt{var(X_1)var(X_2)}$, where ρ_{12} denotes the correlation coefficient of the underlying unconditional bivariate distribution of (X_1, X_2). That is

$$\sigma_T^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2. \tag{22}$$

Let $K = \frac{\sigma_1}{\sigma_2}$. Then

$$\sigma_T^2 = \sigma_1^2 \left(1 + \frac{1}{K^2} + \frac{2\rho_{12}}{K} \right). \tag{23}$$

Let p_1 represent the proportion of product X_1 in the total aggregate sales T, which is usually estimated using historical data. Considering the series of size n of total sales, as before we define W as the random variable representing the lead time duration starting at $n+1$, with mean and variance denoted by μ_W and σ_W^2.

The total sales demand during the lead time is represented by

$$Y_T = \sum_{t=n+1}^{n+W} T_t \ .$$

During the lead time the total demand have expected value given by $E[Y_T] = \mu_W \mu_T$, and variance given by $var(Y_T) = \sigma_W^2 \mu_T^2 + \mu_W \sigma_T^2$.

Let Y_1 and Y_2 represent the lead time demand for the parts X_1 and X_2 composing T. Note that

$$Y_1 = p_1 Y_T \quad \Rightarrow \quad var(Y_1) = p_1^2 var(Y_T).$$

Since T as well as the components follow model (1), the proportion p_1 also applies to

$$\bar{X}_{1n} = p_1 \bar{T}_n,$$

where $\bar{T}_n = \alpha T_n + (1-\alpha) T_{n-1}$. Thus

$$var(\bar{X}_{1n}) = p_1^2 var(\bar{T}_n). \tag{24}$$

Treating T as a single item in the BU approach, it holds that $var(\bar{T}_n) = \frac{\alpha}{2-\alpha}\sigma_T^2$. Substituting in (24) we get

$$var(\bar{X}_{1n}) = p_1^2 \frac{\alpha}{2-\alpha} \sigma_T^2.$$

Using (22) we obtain

$$var(\bar{X}_{1n}) = p_1^2 \frac{\alpha}{2-\alpha} \left(\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2 . \right)$$

Let V_1 represent the lead time forecast of sub-product X_1. During the lead time the proportion p_1 still holds. This means that

$$V_1 = p_1 V_T \quad \Rightarrow \quad var(V_1) = p_1^2 var(V_T).$$

4.1 The TD approach based on updated forecasts

According to SES, the one-step ahead forecast, at any time n is given by

$$\widehat{T_n}(1) = \bar{T}_n$$

where $\bar{T}_n = \alpha T_n + (1-\alpha)T_{n-1}$. The forecast of the total aggregate sales during the lead time, V_T, is given by

$$V_T = \widehat{T_n}(1) + \widehat{T_{n+1}}(1) + \cdots + \widehat{T_{n+W-1}}(1).$$

During the lead time the total forecast have expected value given by $E[V_T] = \mu_W \mu_T$. The correlation between any two forecasts for T separated by lag τ is given by expression (9). Treating T as a single variable (one item), all results from Section 3.1 apply for T, Y_T, V_T, replacing X, Y, V.

The forecast error during the lead time is $\xi_T = Y_T - V_T$. Its variance is $var(\xi_T) = var(Y_T - V_T) = var(Y_T) + var(V_T) - 2cov(Y_T, V_T)$. Note that both the conditional covariance $cov(Y_T, V_T) \mid W = w$, given in (15) and the expression (17) hold for T with σ_T^2 replacing σ_ϵ^2.

Under the TD approach the $var(\xi_T)$ may be written as

$$\begin{aligned} var(\xi_T \mid w) &= var(Y_T \mid w) + var(V_T \mid w) - 2cov(Y_T, V_T \mid w) \\ &= \frac{var(Y_1 \mid w)}{p_1^2} + \frac{var(V_1 \mid w)}{p_1^2} - 2cov(Y_T, V_T \mid w). \end{aligned}$$

Therefore $var(\xi_T \mid w)$ under the TD approach is

$$\frac{var(\xi_1 \mid w) + 2cov(Y_1, V_1 \mid w)}{p_1^2} - 2cov(Y_T, V_T \mid w) \tag{25}$$

By equating (25) and (17) we get

$$\frac{var(\xi_1 \mid w) + 2cov(Y_1, V_1 \mid w)}{p_1^2} = w\sigma_T^2 + \left(\frac{\alpha\sigma_T^2}{2-\alpha}\right)\left[w + 2\sum_{k=1}^{w-1}(w-k)(1-\alpha)^k\right].$$

Finally

$$var(\xi_1 \mid w) = p_1^2 w\sigma_T^2 + p_1^2\left(\frac{\alpha\sigma_T^2}{2-\alpha}\right)\left[w + 2\sum_{k=1}^{w-1}(w-k)(1-\alpha)^k\right] - 2\sigma_1^2\left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha}\right)$$

since $cov(Y_1, V_1 \mid w) = \sigma_1^2 \left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha} \right)$.

Substituting (23) in the above expression we get

$$var(\xi_1 \mid w) = \sigma_1^2 \left\{ p_1^2 w \left(1 + \frac{1}{K^2} + \frac{2\rho_{12}}{K}\right) + p_1^2 \frac{\alpha}{2-\alpha} \left(1 + \frac{1}{K^2} + \frac{2\rho_{12}}{K}\right) \left[w + 2\sum_{k=1}^{w-1} (w-k)(1-\alpha)^k \right] - 2\left(\frac{\alpha w - 1 + (1-\alpha)^w}{\alpha} \right) \right\} \tag{26}$$

The unconditional variance is the expected value of (26).

It would be interesting to compare the error variance during the lead time for the TD approach and for the BU approach, as functions of K. The theoretical value of K associated with indifference between the two approaches (denoted by $K_{critical}$) is found by equating (18) and (26). We obtain

$$K = \frac{\rho_{12} \pm \sqrt{\rho_{12}^2 + 1/p_1^2 - 1}}{1/p_1^2 - 1}.$$

Since $0 < p_1 < 1$, we have $1/p_1^2 > 1$, which implies $\rho_{12}^2 + 1/p_1^2 - 1 > 0$. Now, $K > 0$ and therefore the negative square root must satisfy $\sqrt{\rho_{12}^2 + 1/p_1^2 - 1} < \rho_{12}$. This implies $p_1^2 > 1$, which is not possible. Therefore there is just one solution to equation above and it is given by

$$K_{critical} = \frac{\rho_{12} + \sqrt{\rho_{12}^2 + 1/p_1^2 - 1}}{1/p_1^2 - 1}. \tag{27}$$

Note K depends on ρ_{12} and p_1^2, but is not affected by α. For example, for $p_1^2 = 0.5$ and $\rho_{12} = -0.5$, 0.0, and +0.5 the indifference K is respectively equal to 0.43426, 0.57735, and 0.76759.

4.2 The TD approach based on frozen forecasts

According to SES, the frozen forecasts are given by

$$\widehat{T_n}(1) = \widehat{T_n}(2) = \cdots = \bar{T}_n$$

where $\bar{T}_n$ is as previously defined.

The forecast of the total aggregate sales during the lead time, V_T, is given by

$$V_T = W\bar{T}_n.$$

Treating T as a single variable (one item), all results from Section 3.2 apply for T, Y_T, V_T, replacing X, Y, V. During the lead time total forecast have expected value given by $E[V_T] = \mu_W \mu_T$, and variance given by

$$var(V_T) = \frac{\alpha}{2-\alpha}\sigma_T^2 E[W^2] + \mu_T^2 \sigma_W^2.$$

Our final objective is the variance of the forecasting error during the lead time.

The forecast error during the lead time is $\xi_T = Y_T - V_T$. Its variance is $var(\xi_T) = var(Y_T - V_T) = var(Y_T) + var(V_T) - 2cov(Y_T, V_T)$. Since $cov(Y_T, V_T) = 0$ we obtain

$$var(\xi_T) = \frac{var(Y_1)}{p_1^2} + \frac{var(V_1)}{p_1^2}.$$

Noting that $var(Y_1) + var(V_1) = var(\xi_1)$, where ξ_1 represents the forecasting error during the lead time *due* to subproduct X_1, and noting that $var(\xi_T)$ is given by formula (21),

$$var(\xi_1) = p_1^2 \sigma_T^2 \left(\mu_W + \frac{\alpha}{2-\alpha}(\mu_W^2 + \sigma_W^2) \right).$$

Setting $K = \frac{\sigma_1}{\sigma_2}$ and using (22)

$$var(\xi_1) = p_1^2 \sigma_1^2 \left(1 + \frac{1}{K^2} + \frac{2\rho_{12}}{K} \right) \left(\mu_W + \frac{\alpha}{2-\alpha}(\mu_W^2 + \sigma_W^2) \right). \quad (28)$$

We carried out some simulations experiments which validated (21) and (28). Again, it would be interesting to find out for which values of K the BU (or the TD) approach results in a smaller variance. The theoretical value of K associated with indifference between the two approaches is found by equating (21) and (28). We again obtain

$$K = \frac{\rho_{12} + \sqrt{\rho_{12}^2 + 1/p_1^2 - 1}}{1/p_1^2 - 1}. \quad (29)$$

Figure 4 shows formulas (21) and (28) for fixed $p_1 = 0.5$, $\sigma_\epsilon^2 = \sigma_1^2 = 1$. We assumed that (X_1, X_2) follows a bivariate distribution with correlation coefficient ρ_{12} equal to -0.5 (first row), 0.0 (second row), and 0.5 (third row), and assumed W is uniformly distributed on the integers $\{1, 2, 3, 4, 5\}$, being therefore $\mu_W = 3$ and

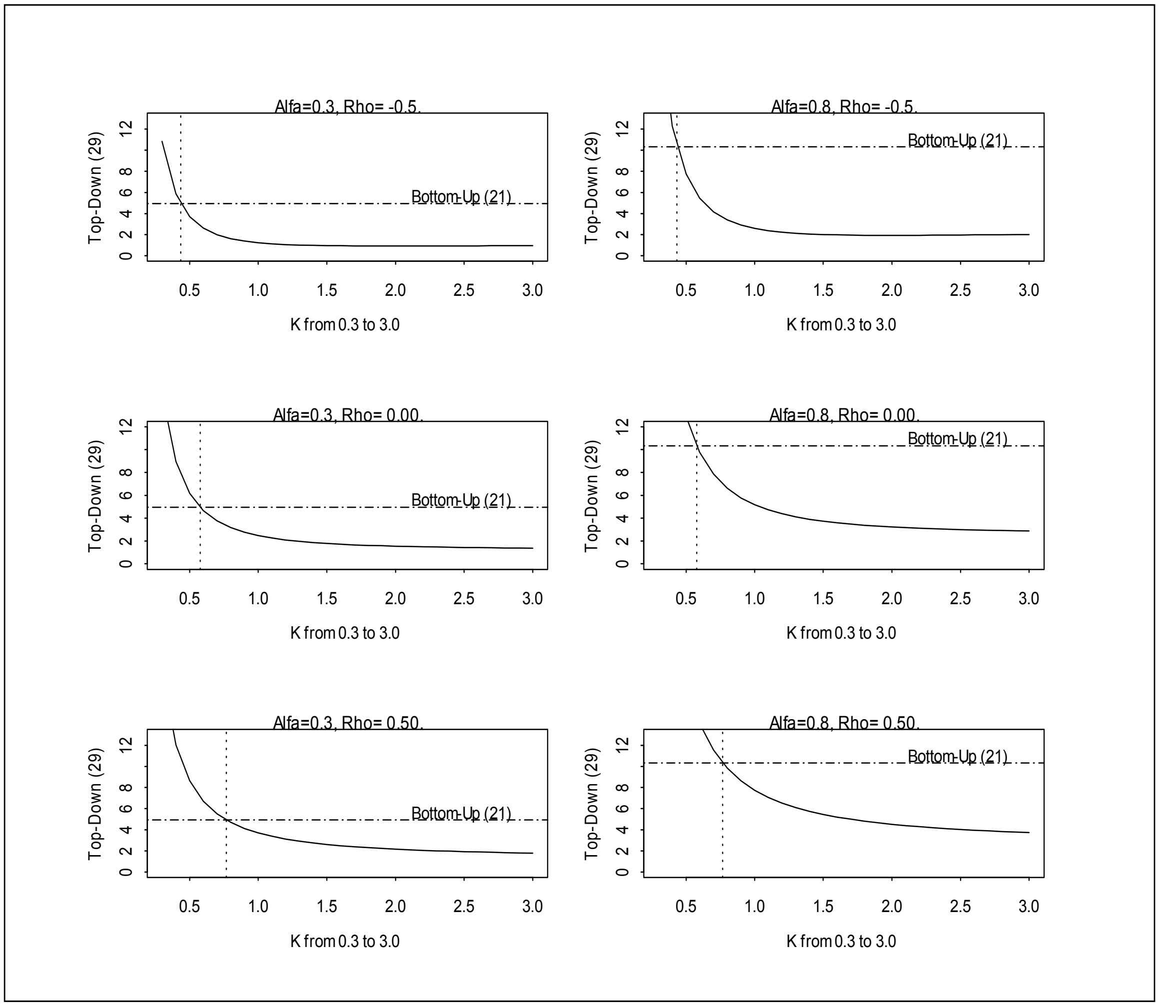

Figure 4: *The value of (21) and the behavior of (28) as K varies from* 0.3 *to* 3.0*, and for α equal to* 0.3 *and* 0.8*. We assume frozen forecasts and $\mu_W = 3$, $var(W) = 2$, and $p_1 = 0.5$.*

$var(W) = 2$. We fix $\alpha = 0.3$ (left column) and $\alpha = 0.8$ (right column) in Figure 4, and examine the behavior of the variance of forecast error during the lead time under the TD and BU approaches and for frozen forecasts, while K ranges from 0.3 to 3.0. We graph in the vertical lines the indifference value K. The horizontal lines are the variances under the BU approach.

5 Further Discussions

This paper analyzed the behavior of the variance of the sales forecasting error during the lead-time under the Top-Down and Bottom-Up approaches and under different conditions of forecasting updating, that is, whether the forecasts are frozen or unfrozen during the lead-time. A main issue in achieving this objective was the assessment of correlations among forecasts at different lags and the demands for each one of the four possible forecasting scenarios, summarized in Table 1.

One should recall that while the Bottom-Up approach corresponds to the traditional practice of forecasting each item individually, the Top-Down approach involves forecasting by first aggregating the sales of all individual items and then disaggregating them based on the historical proportion of the item in the aggregated sales. The operational aspects of the Top-Down and Bottom-Up approaches are very similar indeed, except for the incorporation of the historical proportion in the first approach.

With respect to forecast updating, unfrozen forecasts mean that these are updated at all periods during the lead-time, just like the collaborative planning initiatives presented before. Frozen forecasts correspond to the traditional practice of computing all of them at some time n, just before the beginning of the lead-time. Frequently, these forecasts are considered to be equal during the lead-time.

Results derived in sections 3 and 4 showed that the choice of the most adequate sales forecasting approach is influenced neither by the forecast updating practice during the lead-time nor by the value of the smoothing constant. In other words, whether or not a company is engaged in collaborative planning initiatives, determining the best forecasting approach still deserves a prominent position in the agenda of managers, inasmuch as it constitutes a key decision for reducing the variance of forecasting errors. In fact, the choice of the most adequate sales forecasting approach relies solely on the main characteristics of the sales data series (individual or aggregated) of a given product, namely on its mean, variance, and temporal dynamics, and do not depend on the series probability distribution.

In particular, the smaller the values of ρ_{12} and p_1 (see Eqs. 27 and 30), the greater the chances that the Top-Down approach will minimize the variance of the forecasting error and consequently safety inventory levels. The behavior of the correlation coefficient explains part of the result: an individual item negatively correlated with the aggregate sales of the remaining items presents lower variance of the forecasting error under the Top-Down approach due to the compensation of part of its variance with the aggregate variance of the remaining items.

Table 1: *A summary of the forecasting scenarios analyzed in the paper.*

Main Characteristics		Forecasting Approach: Top-Down	Forecasting Approach: Bottom-up
Forecast updating during the lead-time	Frozen	An aggregated forecast is done first, disaggregation for each item is based on historical shares Traditional practice (no information exchange during the lead-time)	Each item is forecasted individually Traditional practice (no information exchange during the lead-time)
	Unfrozen	An aggregated forecast is done first, disaggregation for each item is based on historical shares Collaborative initiatives (information is exchanged and forecasts are updated during the lead-time)	Each item is forecasted individually Collaborative initiatives (information is exchanged and forecasts are updated during the lead-time)

Even for higher values of ρ_{12} and p_1, the Top-Down approach may present better results when compared to those of the Bottom-Up approach if the value of K is sufficiently high, that is, if σ_1 is sufficiently higher than σ_2. This result is completely explained by the fact that a less than proportional portion of the total variance is added to the variance of the first product. One can note this fact by simply inspecting Eqs. (26) and (29): p_1 is square-rooted, often implying a value much smaller than 1.

Based on Eqs. (27) and (30), the indifference lines ($K_{critical}$ values) between the Top-Down and the Bottom-Up approaches for different values of p_1 and ρ_{12} are presented in Figure 5. Essentially, if the actual value of K is greater than $K_{critical}$, the Top-Down approach should be chosen; otherwise, it should be the Bottom-Up approach. More precisely, given a pair (p_1, ρ_{12}), if the actual value of K is greater than the respective value of $K_{critical}$ linked to that pair, the Top-Down approach should be chosen instead of the Bottom-Up approach.

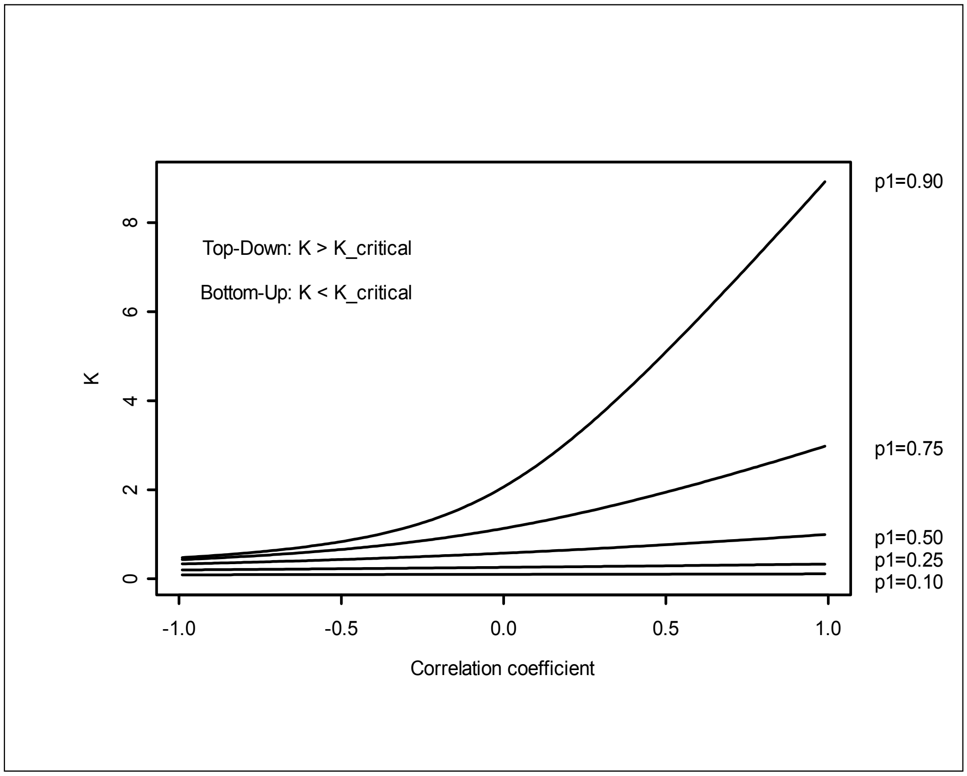

Figure 5: *Indifference lines between Top-Down and Bottom-Up approaches.*

However, it should be noted that, once the best forecasting approach is chosen, a relevant interaction between the value of the smoothing constant and the forecast updating during the lead-time emerges. This interaction also impacts the variance of the sales forecasting error during the lead-time and should be taken into consideration.

For instance, considering that forecasts are updated during the lead-time, one should set the smoothing constant closer or even equal to 1 in order to minimize this variance (cf. Eq. 17 and Figure 2). More precisely, under collaborative planning initiatives and within an information exchange environment, the actual demand is in fact the best predictor of the demand for the subsequent period. Figure 3 depicted the value of information exchange during the lead-time, illustrating its relevant impact in terms of variance reduction when compared to the traditional practice of frozen forecasts. Now, when forecasts are frozen, one should set the smoothing constant closer or even equal to zero, a result that is largely known for time series with no trend and seasonality (Silver and Peterson, 1985). This effect occurs because the variance of the forecasting error during the lead-time, presented in Eqs. (17) and (20), behave differently as a function of the smoothing constant whether the forecasts are updated or not.

Practitioners may also benefit from these results and discussion since the flexibility often sought for the sales forecasting and the safety stock dimensioning is warranted. Primarily, the results presented enable one to determine the approach that leads to the lower variance of the forecasting error during the lead-time with a relatively small computational effort. Secondly, the presented results may be used to segment the sales forecasting analysis with the purpose of determining safety inventory levels, as is shown in Table 2.

Table 2: *Sales forecasting segmentation.*

Main Characteristics		Forecasting Approach $K_{critical} = \frac{\rho_{12} \pm \sqrt{\rho_{12}^2 + 1/p_1^2 - 1}}{1/p_1^2 - 1}$	
		Top-Down	Bottom-up
Forecast updating during the lead-time	Frozen (Set smoothing constant close to zero)	C items Low p_1 High K Actual K tends to be greater than $K_{critical}$	A items High p_1 Low K Actual K tends to be smaller than $K_{critical}$ †
	Unfrozen (Set smoothing constant closer to one)		

†When the correlation coefficient tends to -1, $K_{critical}$ tends to be smaller than the actual value of K, therefore favouring the Top-Down approach.

For example, C items typically present lower participation in total sales (Croxton and Zinn, 2005). Given that p_1 is small, the actual value of K is very likely to be greater than the value of $K_{critical}$, thus making the Top-Down approach preferable for different levels of the coefficient of correlation. This is indicated in Figure 5 by the practically straight lines for different p_1 values up to 0.25. On the other hand, since A items typically present greater participation in total sales and lower a coefficient of variation, they must be individually forecasted (Bottom-Up approach) mainly if they are positively correlated with the aggregate sales of the remaining items. When the correlation is negative, the Top-Down approach may be more adequate if the actual value of K is sufficiently high when compared to the value of $K_{critical}$.

Future research should investigate the behaviour of the variance of the forecasting error under these two approaches when p_1 is not assumed to be constant over time (stochastic). Some pertinent issues are therefore raised. For example, would this assumption favor a given forecasting approach? More precisely, under what conditions is it a good approximation to assume p_1 as a constant? What are the biases incurred in that approximation? How do they relate to the forecasting updating practices?

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