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On Ordering Policies in a Manufacturer-Retailer Model with Advance Demand Information and Production Lead Time

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Abstract

We consider a manufacturer-retailer model with advance demand information. The retailer orders finished products to the manufacturer under base stock policy, with deterministic order lead time, and decides the amount of base stocks to minimize his total average holding and blocking cost. The manufacturer decides the number of products with retailer's order information to minimize his total average holding and additional penalty cost on the excess of production, where there is the deterministic production lead time for this order. Two cases are considered: the production lead time is less than or equal to the due date of demand, and its reverse. In the former case, the retailer has positive safety stocks and the number of product order of manufacturer is also of modified base stock type, whereas in the latter case the retailer always has no stock, and better performance is obtained if advance demand information is available to the manufacturer.

1. Introduction

In manufacturing systems, it is important to manage the stock of finished goods appropriately. Lack of stocks causes the delay for delivery date of the product, which leads to the losses of not only demand but also the customer's confidence. On the other hand, if an excessive stock is maintained in the fear of the delay, the system will occupy the stock space uselessly, and quality degradation and the product obsolescence happen, which drives up costs.

The importance of the inventory control has been recognized in Harris [1], in which an economic order quantity (EOQ:Economic Order Quantity) is designed as an optimum purchasing lot size (lot size) that is determined by a trade-off between fixed order cost and the warehousing expense. To shorten the lead time and reduce the whole production cost, inventory management in a supply chain becomes more important.

Solyai and Surai[2] have treatd the inventory model which has two or more enterprises. In [2], the optimal policy and the optimum inventory level are derived as a single supplier and single retailer problem (ILOP:Integrated Lot-sizing and Order-up-to level Problem) with deterministic demand. Here, it is assumed that the generation of demand occurs at the same time as the receipt of the final products. In many cases, however, the amount of demand is determined before the delivery date, and the amount of the orders and production are based on the demand information. This information is called advance demand information (ADI).

Gallego and Ozer[3] considered a single retailer inventory model which uses ADI. In [3], the retailer's inventory control model with advance demand information in a single-item single-process inventory system on a discrete time is formulated, and it is shown that the base stock policy is optimal when there is no setup cost. Liberopoulos and Tsikis [4] have developed ADI framework for atandem production line with lot sizing, and presented hybrid policies that combine an installation kanban policy and an installation stock policy or an echelon stock policy. Karaesmen, Liberopoulos and Dallery [5] have considered a single-stage M/M/1 make-to-stock production policy. In Hiraiwa and Nakade [6], the production-inventory model with ADI in a single-item single-process inventory system on a discrete time is formulated, and the optimal amount of base stocks and the release lead time to maximize a total expected profit when the demand lead time are given. Here the demand lead time is time from the arrival of demand information to happening of the demand actually, and the release lead time is time from putting out the production order to the occurring of actual demand.

Their models in [3] to [6] are, however, the models intended for only a single firm, not a chain of two or more firms.

In this study, a manufacturer-retailer model with a single item and a single process is considered. Each demand requests one item with ADI. The retailer orders products to the manufacturer, and the manufacturer decides the number of products which he starts for processing each time. A backlog cost of each demand and a holding cost for each product in inventory are imposed on the retailer. To minimize these average costs, the retailer decides the amount of order to the manufacturer. Formulation of retailer's order instruction policy is based on the base point stock policy discussed in [3]. It is assumed that the demand lead time is constant, and there is no ordering and transportation cost. It is also assumed that the order lead time from the retailer to the manufacturer is also constant, and the manufacturer must produce finished products for this order during the order lead time. The manufacturer decides the number of production order at this time based on the order from the retailer. The cost that the manufacturer owes is a holding cost for finished products and a penalty cost for the excessive production. Production lead time is constant, and the backlog for the manufacturer is not permitted because the order lead time is fixed. There is a normal production capacity for the manufacturer, and when it is necessary to start process for products in which the number of products exceeds the capacity, the penalty cost is imposed according to the excessive part. To minimize his average cost, the manufacturer decides the number of products started in process. A Markov decision process is used for deriving manufacturer's production order policy. Moreover, when the demand lead time is longer than the order lead time, a production order policy using advance demand information that has already reached the retailer is considered and it is compared with a production order policy with only information on the orders from the retailer.

The organization of this thesis is as follows. Section 2 explains the model, and the policies for minimizing own average costs on each of the retailer and the manufacturer are analyzed in Section 3. Through numerical experiments, Section 4 discusses retailer's order policy and manufacturer's production order policy. Section 5 concludes this paper.

2. Model

The manufacturer-retailer single-item model in Figure 2.1 is considered on a discrete time. This system consists of a single manufacturer and a single retailer, who have their own inventory respectively.

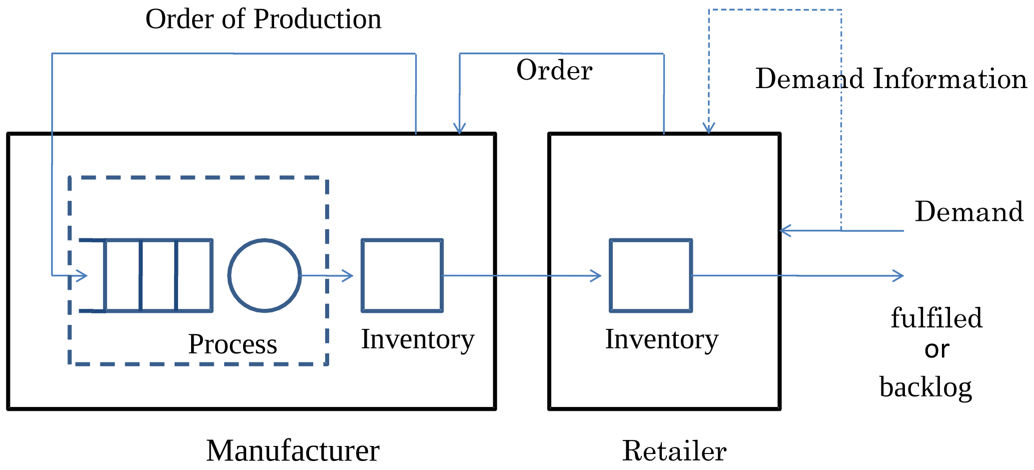


Figure 2.1 The model

Exogenous demand to the retailer occurs at each epoch. The demand information is delivered to the retailer at time t-τ on demand which will generate at time t. This information is called advance demand information (ADI) described in Introduction. The time τ is called demand lead time, and it is a constant positive integer. To satisfy the demand, the retailer orders products to the manufacturer. The products corresponding to the order by the retailer at time t must arrive at t+L, where L is an order lead time. $D_{t,t+\tau}$ is the amount of demand for the consumer to receive at time t+τ. Demand $\{D_{t,t+\tau,}t = 0,1,\ldots\}$ is mutually independent and each demand has an identical probability distribution $P(D = d)(d = 0,1,\ldots)$, where D shows the random variable having the demand distribution. The demand which is not filled at its arrival becomes a backlog, which imposes a cost of the backlog at each unit time on the retailer. The retailer also owes an inventory holding cost to each stock per unit time. When the number of products in its inventory is x, the total of the backlog cost and the inventory holding cost is denoted by $g(x)$, where $g(x)$ is a convex function in x and it is the backlog cost if x is negative and the holding cost if x is positive, and it takes the value of zero at x=0. The ordering and transportation costs are not handled in this study.

The manufacturer decides the amount of products for which production starts at time t, just after the order from the retailer is received by the manufacturer. Since the order by the retailer must be fulfilled, for the order made at time t, the manufacturer has to send the finished products to the retailer as they reach the retailer at time t+L. The transportation time from the manufacture to the retailer is denoted by l (l is an integer and less than L), and thus at the beginning of time t+L-l the manufacturer begins transportation of the same number of finished products as the number of products the retailer orders at time t. Let L' be production lead time. That is, the production process starting at the beginning of time t by the production order made by the manufacturer completes at the beginning at time $t+L'$. The production cost c_p is imposed for each product. The finished product becomes an inventory or is sent to the retailer immediately. The manufacturing system is assumed to have normal maximal production capacity C. If the manufacture decides the production order whose amount x is greater than C, then the penalty cost $p(x)$ is imposed for $x>C$. At the end of each period the inventory cost c_h is incurred for each product in inventory of the manufacturer.

3. Formulation and Analysis

3.1 The retailer's order policy

The retailer model is now formulated. The retailer decides the number of order to the manufacturer under a base stock policy which is based on the amount of inventories at the beginning of time $t+L$. Let s be an amount of base stocks. D_t is actual demand at time t, and it becomes $D_{t-\tau,t}$ from the assumption of the model in Section 2. Let B_t be a backlog at time t, and I_t^2 denote an amount of inventory at the beginning of time t. The retailer decides the amount of the order at the beginning of each period. The amount is denoted by z_t, and the corresponding products must arrive at the retailer at the beginning of time $t+L$. Figure 3.1 shows the timing of the retailer model.

The demand is divided into two types for convenience. Let O_t^L denote the total amount of orders from time t to $t+L$ which have been determined by ADI but not occurred by the beginning of time t, and U_t^L denote the total amount of orders from time t to $t+L$ which will be determined by ADI after the beginning of time t. If $\tau>L$, all demand whose ADI arrived by time $t+L-\tau$ will be satisfied until time $t+L$, and otherwise for ADI which will arrive from time t to $t+L-\tau$ the corresponding demand will arrive at the retailer by time $t+L$. Therefore, we have

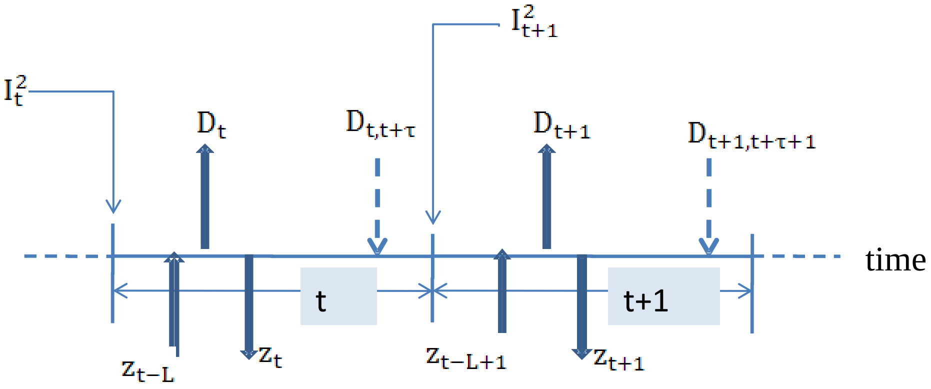


Figure 3.1 Timing in a retailer model

$$O_t^L = \begin{cases} \sum_{i=0}^{\tau-1} D_{t-\tau+i,\ t+i} & \tau \le L, \\ \sum_{i=0}^{L} D_{t-\tau+i,\ t+i} & \tau > L, \end{cases}$$

$$U_t^L = \begin{cases} \sum_{i=0}^{L-\tau} D_{t+i,\ t+\tau+i} & \tau \le L, \\ 0 & \tau > L. \end{cases}$$

Let x_t denote the inventory position (the number of products in real inventory + quantity of products which have been ordered but not arrived – the amount of backlog – the amount of products sent to retailer which have been decided) at time *t*. Then we have

$$x_t \equiv I_t^2 + \sum_{i=t-L}^{t-1} z_i - B_t - O_t^L.$$

Since $I_{t+1}^2 - B_{t+1} = I_t^2 - B_t + z_{t-L} - D_{t-\tau,t}$, x_{t+1} can be shown as follows.

$$x_{t+1} = \begin{cases} x_t + z_t - D_{t,t+\tau} & \tau \le L, \\ x_t + z_t - D_{t+L+1-\tau,t+L+1} & \tau > L. \end{cases}$$

Since $s = x_t + z_t$ under the base stock policy with base stock *s*, it follows that

$$z_{t+1} = \begin{cases} D_{t,\ t+\tau} & \tau \le L\,, \\ D_{t+1+L-\tau,\ t+1+L} & \tau > L. \end{cases}$$

Thus

$$z_t = \begin{cases} D_{t-1,\ t-1+\tau} & \tau \le L, \\ D_{t+L-\tau,\ t+L} & \tau > L. \end{cases} \tag{3.1}$$

The cost imposed on the retailer consists of a backlog cost and a holding cost. The number of products in the inventory or backlog (if it is negative) at the end of time *t*+*L* is given by the following expression:

$$I_t^2 + \sum_{i=t-L}^{t} z_i - B_t - O_t^L - U_t^L = x_t + z_t - U_t^L = s - U_t^L.$$

The expected backlog and inventory cost $G(s)$ imposed at time t is assumed to be based on the real inventory or backlog at time $t+L$, and thus it becomes

$$G(s) \equiv E[g(s - U_t^L)]. \qquad (3.2)$$

Note that $G(s)$ does not depend on t. Under the base stock policy, the optimal base stock s^* for the retailer minimizes $G(s)$. Because the value of U_t^L becomes 0 for $\tau>L$, the optimal base stock s^* is zero when $\tau>L$. If $\tau\leq L$, the optimal base stock s^* is usually positive and depends on the distribution of the total demand for L-τ+1 periods.

By (3.1), the order process from the retailer to the manufacturer does not depend on the number of base stocks stochastically. That is, the number of base stocks does not affect the following analysis on production order and inventory policies of the manufacturer.

3.2 Manufacturer's production order and inventory policy

Next the production order and inventory policy of the manufacturer is considered. A Markov decision process is applied to this optimization. Let I_t^1 denote an amount of finished products in the inventory of the manufacturer at time t, and P_t be an amount of orders whose production starts at time t made by the manufacturer.

The manufacturer must decide the number of production order P_t at time t just after receiving the order z_t from the retailer at time t. Since the backlog is not permitted, if it starts the process of products whose number x exceeding the capacity C, a penalty cost $p(x)$ on the excess x-C is imposed. Production lead time is L', and thus the production beginning at t ends at time $t + L'$. The finished product corresponding order at time t is transported to the retailer and the transportation time is l.

The amount I_t^1 is the number of finished products just before the order z_t and after the beginning of transportation of finished products to the retailer at time t. Figure 3.2 shows the timing of the manufacturer model, where $L' = L - l$.

The amount of the production order is decided based on the inventory position and the quantity of order by the retailer. In Sections 3.2 and 3.3, the order lead time from the retailer to the manufacturer is assumed to be equal to the sum of the production lead time and the transportation time, that is $L' = L - l$. In Section 3.4 the cases that $L' > L - l$ and $L' < L - l$ will be discussed.

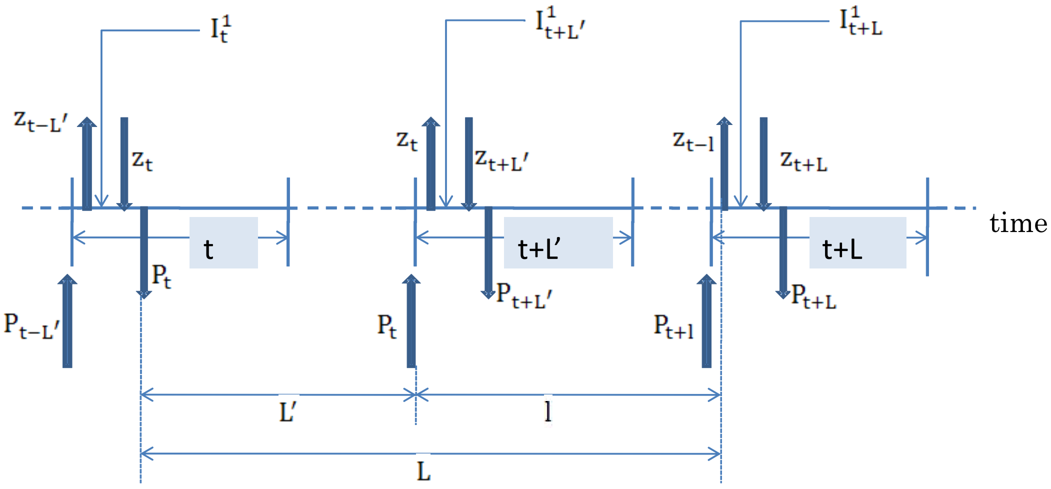


Figure 3.2 Timing in a manufacturer model ($L' = L - l$)

Inventory position x_t at the beginning of time t is given by

$$x_t \equiv I_t^1 + \left(P_{t-L'+1} + \cdots + P_{t-1}\right) - \left(z_{t-L'+1} + \cdots + z_{t-1}\right).$$

It is necessary that $x_t \geq 0$, because all orders must be satisfied. By (3.1) the order quantity of the retailer has the same probability distribution $P(D=d)$ ($d=0,1,\ldots$) as demand and the orders in different times are mutually independent. Let x_{max} and a_{max} be large values on inventory positions and orders enough for deriving optimal production order policy, respectively, and d_{max} be the maximum of the possible demand amount in one period. Since the production order is decided after observing the order, the state in a Markov decision process consists of the current inventory position and the number of just arriving order. Hence the state space S is

$$S = \{(x,d): 0 \leq x \leq x_{max}, 0 \leq d \leq d_{max}\}.$$

Let $A\big((x,d)\big)$ be the decision space in state $(x,d) \in S$. The inventory position x' at the next time is for decision $a \in A\big((x,d)\big)$

$$x' = x - d + a.$$

Note that x' is the number of products in inventory L' times later. When $x \geq d$, the minimal value of the possible order is 0, and when $x < d$, it is d-x, because x' must be non-negative. Thus $A\big((x,d)\big)$ has the following expression.

$$A\big((x,d)\big) = \{a: max(0, d - x) \leq a \leq a_{max}\}.$$

The transition probability is given by

$$P\left((x',d')|(x,d),a\right) = \begin{cases} P(D = d') & x' = x - d + a, 0 \leq d' \leq d_{max}, \\ 0 & \text{otherwise.} \end{cases}$$

The expected cost when action a is taken in state (x,d) is denoted by $c\big((x,d),a\big)$, which is imposed for the manufacturer. This cost is assumed to consist of production cost for each item ordered at the time, the holding cost imposed on the real inventory at the end of L’ time later, and the penalty cost by production exceeding the normal production capacity. The expected cost can be shown by the following expression.

$$c\big((x,d),a\big)=\begin{cases} c_p a + c_h(x-d+a) & 0 \le a \le C, \\ c_p a + c_h(x-d+a) + p(a-C) & a > C. \end{cases}$$

3.3 Production order and inventory policy using advance demand information reaching the retailer

The policy discussed in Section 3.2 is under assumption on the use of information based on the orders from the retailer. When $\tau > L$, z_t is $D_{t+L-\tau,t+L}$. In that case, however, advance demand information $D_{t+1+L-\tau,t+1+L}, D_{t+2+L-\tau,t+2+L}, \dots, D_{t-1,t-1+\tau}$ has reached to the retailer. This is illustrated in Figure 3.3.

When demand information that has reached to the retailer is transmitted to the manufacturer, manufacturer's optimal production policy can be derived by formulating the process as the following Markov decision process.

Let d_i denote the advance demand information which has reached to the retailer τ-L-i+1 period before (i=1,2,…,τ-L). Then the information $d_1, d_2, d_3, \dots, d_{\tau-L}$ is available for the manufacturer if they are informed to the manufacturer from the retailer.

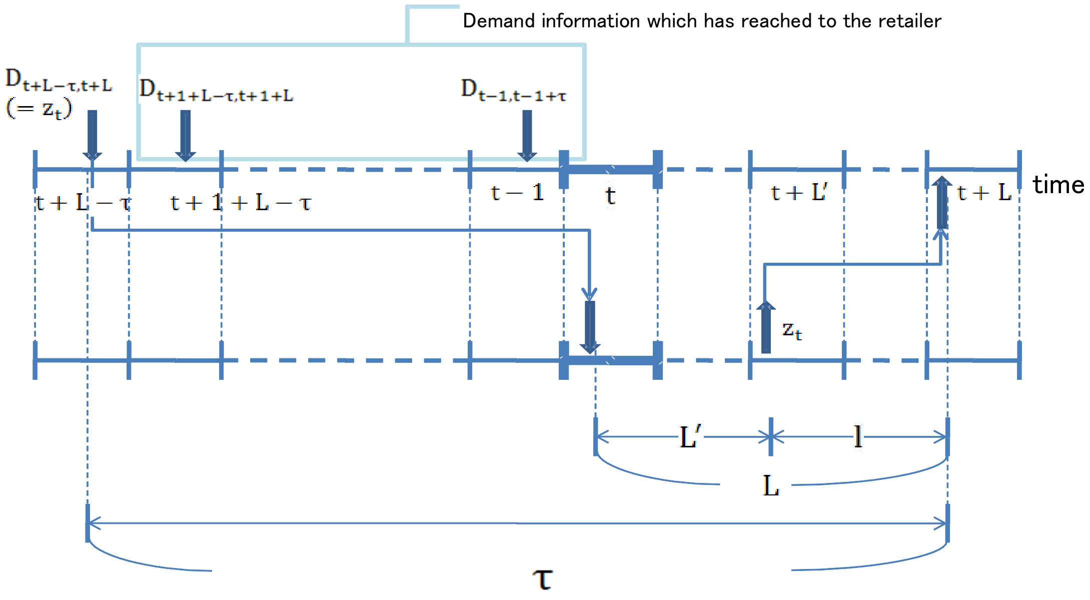


Figure 3.3 Demand Information ($\tau > L$)

The state space is

$$S = \{(x, d_1, d_2, d_3, \dots, d_{\tau-L}): 0 \le x \le x_{max}, 0 \le d_1 \le d_{max}, 0 \le d_2 \le d_{max}, \dots, \\ 0 \le d_{\tau-L} \le d_{max}\}.$$

In the same way as in section 3.2, an action space in state $(x, d_1, d_2, d_3, \dots, d_{\tau-L})$ is given by

$$A\big((x, d_1, d_2, d_3, \dots, d_{\tau-L})\big) = \{a: max(0, d_1 - x) \le a \le a_{max}\}.$$

The transition probability is given by

$$P\left((x', d_1', d_2', d_3', \dots, d_{\tau-L}') | (x, d_1, d_2, d_3, \dots, d_{\tau-L}), a\right)$$

$$= \begin{cases} P(D = d_{\tau-L}') & \begin{pmatrix} 0 \le d_1 \le d_{max}\ 0 \le d_2 \le d_{max}\ 0 \le d_{\tau-L} \le d_{max} \\ d_1' = d_2, d_2' = d_3, \dots, d_{\tau-L-2}' = d_{\tau-L-1}, \\ 0 \le d_{\tau-L}' \le d_{max}, x' = x - d_1 + a \end{pmatrix}, \\ \\ 0 & \text{otherwise.} \end{cases}$$

An expected cost can be obtained similarly in Section 3.2.

$$c\big((x, d_1, d_2, d_3, \dots, d_{\tau-L}), a\big)$$

$$= \begin{cases} c_p a + c_h(x - d_1 + a) & 0 \le a \le C, \\ \\ c_p a + c_h(x - d_1 + a) + p(a - C) & a > C. \end{cases}$$

3.4 Generalization

In Sections 3.2 and 3.3, manufacturer's optimal production and inventory policy is discussed when $L' = L - l$. In this section the production order and inventory policies for the cases $L' < L - l$ and $L' > L - l$ are discussed.

(a) $L' < L - l$

When $\tau \le L$, the inventory position at time τ is assumed to be

$$x_t \equiv I_t^1 + (P_{t-L'+1} + \dots + P_{t-1}) - \left(z_{t-(L-l)+1} + \dots + z_{t-(L-l-L')-1}\right). \quad (3.3)$$

Since $z_{t-(L-l-L')}, \dots, z_{t-1}$ are known as well as z_t at time t, the state space can be stated as

$$S = \{(x, z_{t-(L-l-L')}, \dots, z_{t-1}, z_t) \\ : 0 \le x \le x_{\max}, 0 \le z_{t-(L-l-L')} \le d_{max}, \dots, 0 \le z_{t-1} \le d_{max}, 0 \le z_t \le d_{max}\}.$$

In the similar way as in section 3.3, the system can be formulated into a Markov decision process.

When $\tau > L$, the amounts $z_{t+1}, z_{t+2}, \dots, z_{t-1+\tau-L}$ of orders in the future are known at time *t*. Thus they are added to the state and in the same way as in section 3.3 the system can be formulated into a Markov decision process.

(b) $L^{'} > L - l$

The case that $\tau \leq L$ is first considered. In this case, the inventory position at time *t* is defined as

$$x_t \equiv I_t^1 + \left(P_{t-L^{'}+1} + \cdots + P_{t-L^{'}+(L-l)-1} + P_{t-L^{'}+(L-l)} + \cdots + P_{t-1}\right) \\ - \left(z_{t-(L-l)+1} + \cdots + z_{t-1}\right).$$

Note that $z_{t+1}, \cdots, z_{t+L^{'}-(L-l)-1}$ have not been decided at beginning of period *t*, whereas real value of inventory at time $t + L^{'}$ is $x_t + P_t - \left(z_t + \cdots + z_{t+L^{'}-(L-l)-1}\right)$. Since backlog is not permitted, $x_t + P_t - z_t$ must be no less than the maximum of possible values of total amounts of orders from time $t+1$ to $t + L^{'} - (L - l) - 1$, that is $(L^{'} + l - L - 1)d_{max}$. Therefore, $P_t \geq (L^{'} + l - L - 1)d_{max} - x_t + z_t$. Under this condition Markov decision process can be applied.

If $L < \tau < L^{'} + l$ and advance demand information is available, $D_{t+L-\tau,t+L}, \cdots, D_{t-1,t-1+\tau}$ are informed to the manufacturer, and $z_t, \cdots, z_{t+\tau-L-1}$ are known. Therefore, in the similar way as above it must be satisfied that $P_t \geq (L^{'} + l - \tau)d_{max} - x_t + z_t + \cdots + z_{t+\tau-L-1}$.

When $\tau = L^{'} + l$, the inventory position similar as (3.3) can be defined, because at time *t*, $z_t, \cdots, z_{t+L^{'}-(L-l)-1}$ are known as $D_{t+L-\tau,t+L}, \cdots, D_{t-1,t-1+\tau}$, respectively. When $\tau > L^{'} + l$ the same formulation as in section 3.3 is available because the orders $(z_t, \cdots, z_{t-1+\tau-L})$ have been known at time *t*.

4. Numerical Experiments

In this section we give numerical examples and discuss optimal production policies of the manufacturer. In the numerical examples set

$$g(x) = \begin{cases} -px & x < 0, \\ qx & x \geq 0, \end{cases}$$

where (p,q) are positive integers. Then (3.2) becomes

$$G(s) = qE[(s - U_t^L)^+] + pE[(U_t^L - s)^+].$$

Define $\alpha^+ = max(\alpha, 0)$, then since $max(\alpha, 0) - max(-\alpha, 0) = \alpha$ it holds that

$$G(s) = (q + p)E[(s - U_t^L)^+] - ps + q\,E[U_t^L]\,.$$

For $\tau \leq L$ the base stock s is defined as follows. Let $F_{U_t^L}(s) = P(U_t^L \leq s)$, and then it holds that $G_t^{'}(s) = (q + p)F_{U_t^L}(s) - p$ and $G_t^{''}(s) > 0$. Thus $G_t(s)$ is convex, and $G_t^{'}(s) = 0$ implies that

$$F_{U_t^L}(s) = \frac{p}{q+p}.$$

Therefore, for m satisfying the following equation, the optimal base stock is m if $G_t(m) < G_t(m+1)$, and $m+1$ otherwise.

$$F_{U_t^L}(m) \leq \frac{p}{q+p} < F_{U_t^L}(m+1).$$

For the manufacturer, the optimal policy discussed in section 3 is derived by policy iteration method. It is assumed that $L^{'} = L - l,$ and $p(x) = p^{'}x^2 + q^{'}x$. Computation is done by C language program on a personal computer with Intel Core2 Duo 2.20 GHz CPU and 2.46GB RAM.

(a) Experiment 1

Parameters in the first experiment are given as τ=2, L=3, p=5, q=2, $c_p = 5$ and $c_h = 2$. Demand has a binomial distribution with parameters (n, p)=(10, 0.5). Since $\tau \leq L$ the retailer's base stock becomes 1 from the above discussion, and optimal expected average cost is 172.182256.

Optimal production order and inventory policy of the manufacturer is discussed. Parameters are given as $x_{max} = 20, a_{max} = 20, p^{'} = 50, q^{'} = 50, C = 6$. This optimal policy is called policy 2, and it is compared with the following policy called policy 1: Policy 1 takes action $a_0\big((x,d)\big)$ in state (x,d) where

$$a_0\big((x,d)\big) = \begin{cases} d - x & x < d, \\ 0 & x \geq d. \end{cases}$$

Policy 1 maintains the inventory position as zero. Table 4.1 shows actions in policy 1 and Table 4.2 shows actions in policy 2. Figures 4.1 and 4.2 show average costs in these policies when penalty costs are linear and quadratic functions on excess of production over capacity, respectively.

Table 4.1　Actions in policy 1（Average cost: 44.824219）

	d										
x	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	0	0	1	2	3	4	5	6	7	8	9
2	0	0	0	1	2	3	4	5	6	7	8
3	0	0	0	0	1	2	3	4	5	6	7
4	0	0	0	0	0	1	2	3	4	5	6
5	0	0	0	0	0	0	1	2	3	4	5
6	0	0	0	0	0	0	0	1	2	3	4
7	0	0	0	0	0	0	0	0	1	2	3
8	0	0	0	0	0	0	0	0	0	1	2
9	0	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0

Table 4.2　Actions in policy 2（Average Cost: 32.45471）

	d										
x	0	1	2	3	4	5	6	7	8	9	10
0	3	4	5	6	6	6	6	7	8	9	10
1	2	3	4	5	6	6	6	6	7	8	9
2	1	2	3	4	5	6	6	6	6	7	8
3	0	1	2	3	4	5	6	6	6	6	7
4	0	0	1	2	3	4	5	6	6	6	6
5	0	0	0	1	2	3	4	5	6	6	6
6	0	0	0	0	1	2	3	4	5	6	6
7	0	0	0	0	0	1	2	3	4	5	6
8	0	0	0	0	0	0	1	2	3	4	5
9	0	0	0	0	0	0	0	1	2	3	4
10	0	0	0	0	0	0	0	0	1	2	3
11	0	0	0	0	0	0	0	0	0	1	2
12	0	0	0	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0

Figure 4.1 Average costs in policies 1 and 2 ($p' = 0$: linear cost)

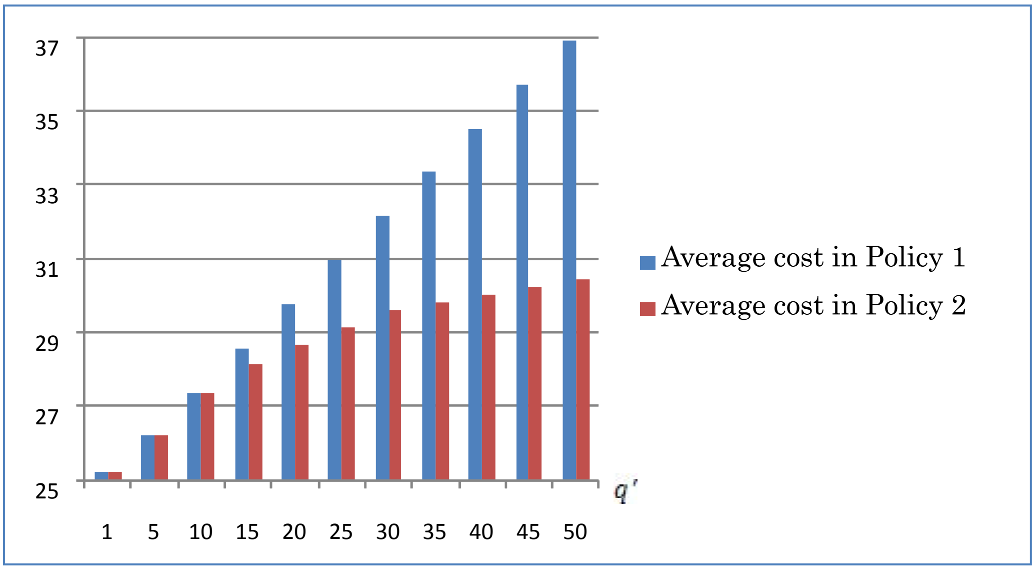
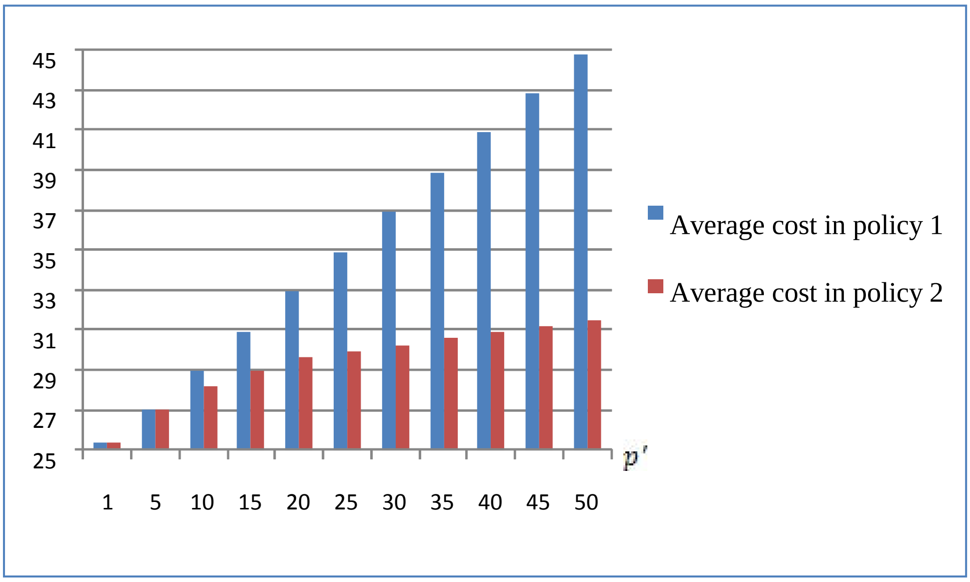


Figure 4.2 Average costs in policies 1 and 2 ($q' = 0$: quadratic cost)



Policy 1 always decides the action making the inventory position 0. In policy 2, when $d - x \leq 6$ the action takes the value satisfying $a = d - x + 3$ as the production order does not exceed the production capacity. As a result policy 1 makes the manufacturer more imposed on the penalty cost than policy 2. In numerical experiments with the other parameters on the penalty cost, policy 2 takes more base stocks when penalty cost is high.

When the penalty cost is small, the difference between average costs in policies 1 and 2 is small, and when the cost is high or quadratic the difference increases. In policy 2, actions make the imposed penalty cost smaller, whereas policy 1 implies more penalty cost, not by considering the capacity. In policy 2, when the penalty cost increases, the value on increase of average cost becomes small. If penalty cost is high then the value of $x - d + a$, after the action a is taken, becomes high, and in this case there is little chance on excessive production.

b) Experiment 2

Next the optimal policy derived by Markov decision process, in which advance demand information is available in the manufacturer as discussed in Section 3.3, is considered. This policy is called policy 3. We set τ=4, L=2, p=5, q=2, $c_p = 5$ and $c_h = 2$. Demand has a binomial distribution with parameters (n, p)=(7, 0.5). It is assumed that $x_{max} = 14, a_{max} = 14, p' = 50, q' = 50$ and $C = 4$. Since $\tau > L$, the base stock of the retailer is zero, and as a result the average cost on the retailer is zero. Since $\tau - L - 1 = 1$, in policy 3 the state space is $S = \{(x, d_1, d_2); 0 \leq x \leq x_{max,}\ 0 \leq d_1, d_2 \leq 7\}$. Table 4.3 shows the actions in policy 2, and in Table 4.4 actions for x=0 in policy 3 are shown.

Table 4.3 Actions in policy 2 (Average cost: 26.27062)

	d							
x	0	1	2	3	4	5	6	7
0	4	4	4	4	4	5	6	7
1	3	4	4	4	4	4	5	6
2	2	3	4	4	4	4	4	5
3	1	2	3	4	4	4	4	4
4	0	1	2	3	4	4	4	4
5	0	0	1	2	3	4	4	4
6	0	0	0	1	2	3	4	4
7	0	0	0	0	1	2	3	4
8	0	0	0	0	0	1	2	3
9	0	0	0	0	0	0	1	2
10	0	0	0	0	0	0	0	1
11	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0

Table 4.4 Actions in policy 3 (for $x = 0$)
(Average cost: 23.02859)

d_2	d_1 0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	0	1	2	3	4	5	6	7
2	1	2	3	4	4	5	6	7
3	2	3	4	4	4	5	6	7
4	3	4	4	4	4	5	6	7
5	4	4	4	4	5	5	6	7
6	4	4	4	5	5	5	6	7
7	4	4	5	5	5	6	6	7

For policy 2, if state (d,x) satisfies $d - x > 4$ then action is $a = d - x,$ and otherwise the action takes the nearest value to $4 - x + d$ which satisfying the conditions on no backlog and production capacity. On the other hand, policy 3 takes smaller numbers of orders when the advance demand information d_2 gives the low value and greater values of orders when d_2 takes the high value. This is because policy 3 takes more orders when the next demand is high and the production capacity has a margin, and less orders when the next demand is low. As a result, policy 3 leads to the less average cost for manufacturer than policy 2.

5. Concluding Remarks

In this paper, a manufacturer-retailer model with advance demand information is dealt with. The retailer orders finished products under base stock policy to the manufacturer with deterministic order lead time to minimize his total average holding and blocking cost. The number of base stocks is a positive value when $\tau \leq L$ and zero when $\tau > L$.

The manufacturer decides the number of products with retailer's order information to minimize his total average holding and additional overtime cost, where there is the deterministic production lead time for this order. From numerical results the optimal policy is the modified base stock policy taking the production capacity into consideration. When $\tau > L$ and the advance demand information is available to the manufacturer, his average cost is much less by policy considering this information.

The optimal number of base stocks of the retailer does not affect on the optimal policy of the manufacturer. Though it is not proved that the proposed policies are supply-chain optimal, they will have the near-optimal property in supply chain.

In this paper the transportation cost is not considered for the simplicity of the model. If it is included, the model is more complicated and near-optimal or optimal policy will be a kind of (s,S) policy. This model will be also extended to the cases of multiple products, stochastic demand lead time and so on. They are left for future research.

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