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ROBUST OPTIMIZATION FOR COORDINATION OF INTEGRATED MULTIREFINERY NETWORKS

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1. Introduction

The continued aggressive search for cost savings in the oil industry has forced refineries to modify operations in an effort to improve economic performance (Moro, 2003). A common approach to counter this situation is to seek integration alternatives not only on a single facility but also on an enterprise-wide scale (Chopra and Meindl, 2004). Planning for this kind of operation should be carried out centrally, thus allowing for proper integration among all operating facilities and, consequently, an efficient utilization of available resources (Al-Qahtani and Elkamel, 2008). The benefits from coordinated planning of multiple sites not only appear in economic terms but also in terms of process flexibility (Shah, 1998).

Planning applications are of particular interest due to their high economic incentives and strategic importance. Planning is basically an activity in which production targets are set and market forecasts, resources availability, and inventories are considered. In general, planning is categorized into three different time classifications: strategic (long term), tactical (medium term), and operational (short term).

Considering this type of high level activity, especially in the competitive and volatile refining industry, the impact of uncertainties is inevitable. Sources of uncertainties can be categorized as short-term, mid-term, or long-term according to the time horizon. Short-term uncertainties mainly refer to unforeseen factors in the internal process - for example, operational variations and equipment failure (Subrahmanyam *et al.*, 1994). On the other hand, long-term uncertainties mainly represent external factors that impact the planning process on a longer time horizon (for example, supply, demand, and price fluctuations). Mid-term uncertainties incorporate features from both short-term and long-term uncertainties (Gupta and Maranas, 2003).

The above discussion points out the importance of refinery planning under uncertainty. The uncertainty in optimization models for the oil industry has been noted by several academic studies conducted in the past few years. The main methods employed in these studies are stochastic programming (Escudero *et al*., 1999; Dempster *et al*., 2000; Lababidi *et al*., 2004; Li *et al*., 2004; Al-Othman *et al*., 2008), dynamic programming (Dempster *et al*., 2000; Cheng and Duran, 2003), stochastic robust programming (Pongsakdi *et al*., 2006; Khor et al., 2007; Lakkhanawat and Bagajewicz, 2008), and fuzzy programming (Liu and Sahinidis, 1997; Hsieh and Chiang, 2001). Despite these many contributions for optimization problems under uncertainty, few of them discuss the importance of coordination of integrated multirefinery networks. Therefore, the refinery operational planning problem under uncertainty is still an open issue, which is relevant for both mathematical modeling and actual applications.

In this context, we extend the deterministic model proposed by Al-Qahtani and Elkamel (2008) for the strategic planning of integrated multirefinery networks. The model is formulated as a robust mixed-integer linear model (MILP) to tackle long-term parameters uncertainty in the coefficients of the objective function (raw material costs and final products prices) and the right-hand-side coefficient constraints - RHS (products demand).

The robust optimization methodology focuses on models that ensure solution feasibility given the possible outcomes of uncertain parameters. Under this approach, the decision-maker is willing to accept a suboptimal solution for the nominal values in order to ensure that the solution remains feasible and near optimal when the data changes. On the other hand, this method assumes only limited information about the distributions of the underlying uncertainties, such as known mean value and its range. For example, unlike the approach of stochastic optimization, in the robust optimization technique, there is no necessity of specifying scenarios and their probabilities or expected recourse costs, all of which are often cumbersome to estimate.

Methodologies based on the robust technique have been developed in many works (Beyer and Sendhoff , 2007. The first trial in this direction is reported to Soyster (1973), who proposed a conservative approach that assumes that all random parameters are equal to their worst-case value. Since then, several works have recently extended the Soyster approach (El-Ghaoui and Lebret, 1997; El-Ghaoui *et al.*, 1998; Ben-Tal and Nemirovski, 1998, 1999, 2000; Bertsimas and Sim, 2003, 2004; Bertsimas *et al.*, 2004; Bertsimas and Thiele, 2006). In this paper, the robust optimization methodology proposed by Bertsimas and Sim (2003, 2004) is adopted to account for uncertainties. These authors proposed an approach that attempts to make the trade-off between solution optimality and solution robustness more attractive. An important aspect of this method is that the new robust formulation does not add complexity to the original problem.

The remainder of this paper is organized as follows. In section 2, the robust optimization methodology of Bertsimas and Sim (2003, 2004) is explained. Section 3 presents the problem under study. The robust methodology is applied to the refinery network in Section 4. Next, section 5 offers results and discussions in the context of an industrial scale study of a multirefinery network and presents a comparison of the different results obtained from integrated and nonintegrated strategic planning. The paper ends with concluding remarks in section 6.

2. Robust optimization methodology

Consider the following linear optimization problem with a set of n variables:

$$\textbf{Minimize } \sum_{j=1}^{n} c_j x_j$$

$$\textbf{Subject to } \sum_{j=1}^{n} a_{ij} x_j \le b_i, \forall i \qquad \textbf{(1)}$$

$$l_j \le x_j \le u_j, \forall j$$

Without loss of generality, assume that the uncertainties affect only the elements of the matrix $\boldsymbol{A}$ = [a_{ij}]. Actually, if there exists uncertainty regarding the independent coefficients of the constraints (b_i), a new variable x_{n+1} can be introduced into the model, and the model constraint can be rewritten as $\sum_{j=1}^{n} a_{ij} x_j - b_i x_{n+1} \le 0$, $l_j \le x_j \le u_j$, $x_{n+1} = 1$, which represents an inclusion of b_i into the technological matrix $\boldsymbol{A}$ (Bertsimas and Sim, 2003). Moreover, if the objective function is also subject to uncertainties, the model can be rewritten to minimize z and the constraint $z - \sum_{j=1}^{n} c_j x_j \le 0$ is added to the set of constraints $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ (Bertsimas and Sim, 2004).

The coefficients of the technological matrix $\boldsymbol{A}$=[a_{ij}>0] can be also modeled as a symmetric and bounded random variable $\tilde{a}_{ij}$, $j \in J_i^a$ that assumes values in the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$, where $J_i^a = \{ j \mid \hat{a}_{ij} > 0 \}$. For every constraint i a parameter Γ_i^a is introduced (not necessarily integer), which assumes values in the interval $[0, |J_i^a|]$. The main idea behind Bertsimas and Sim's approach is to control the conservatism of the robust solution by introducing a parameter that can be defined by the decision-maker (Γ_i^a). Since in practice it is unlikely that all the uncertain coefficients are equal to their worst case value (such as Soyster's method), Bertsimas and Sim proposed a less conservative approach such that the decision-maker can choose the number of uncertain factors against which he/she wishes to be protected. If $\Gamma_i^a = 0$, the uncertainties in the parameters of constraint i can be ignored (deterministic problem). By contrast, $\Gamma_i^a = |J_i^a|$ represents the most conservative case in which all the uncertainty parameters of the constraint i are considered (Soyster's Model). Accordingly, this parameter limits the number of coefficients that are simultaneously worst-case valued.

The constraint $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ can be violated only if a_{ij} increases; if a_{ij} decreases the constraint will be satisfied for sure. So, consider the worst case for constraint violation (Soyster's case): $\sum_{j=1}^{n} \left(a_{ij} + \hat{a}_{ij}\right) x_j \leq b_i$. If $x_j < 0$, this constraint is never violated. Thus, only the cases with $x_j \geq 0$ need to be controlled. Assuming $\Omega_j = \left|x_j\right|$, the model constraint can be rewritten as: $\sum_{j=1}^{n} \left(a_{ij} x_j + \hat{a}_{ij} \Omega_j\right) \leq b_i$.

Model (1) has a robust linear counterpart as follows - see Bertsimas and Sim (2003) for proofs. Note that $\Omega_j \leq \left|x_j\right|$ is equivalent to $-\Omega_j \leq x_j \leq \Omega_j$.

$$
\begin{aligned}
&\text{Minimize } \sum_{j=1}^{n} c_j x_j \\
&\text{Subject to } \sum_{j} a_{ij} x_j + \lambda_i^a \Gamma_i^a + \sum_{j \in J^a{}_i} \mu_{ij}^a \leq b_i && \forall i \\
&\lambda_i^a + \mu_{ij}^a \geq \hat{a}_{ij} \Omega_j && \forall i, \forall j \in J_i^a \\
&l_j \leq x_j \leq u_j, && \forall j \in J \\
&-\Omega_j \leq x_j \leq \Omega_j, && \forall j \in J \\
&x_{n+1} = 1 \\
&\Omega_j \geq 0 && \forall j \in J \\
&\lambda_i^a \geq 0 && \forall i \\
&\mu_{ij}^a \geq 0 && \forall i, \forall j \in J^a
\end{aligned}
\qquad \textbf{(2)}
$$

At optimality $\Omega_j = |x_j^*|$ for all *j*, where x_j^* is an optimal solution of problem (2), because the objective function aims to minimize the problem and any value $\Omega_j > |x_j|$ increase the objective function due to $\sum_{j=1}^{n} \hat{a}_{ij} w_j$.

The variables λ_i^a and μ_{ij}^a quantify the system's sensitivity to changes in the uncertain parameters a_{ij}. The sum of the variables λ and μ represents the minimum deviation to account for uncertain parameter deviations.

In addition, the parameter Γ_i^a controls the trade-off between probability of violation and the effect on the objective function of the deterministic problem. If $\Gamma_i^a \in [0, | J_i^a |]$, then the robust solution will be *deterministically* feasible. Even if more than $\lfloor \Gamma_i^a \rfloor$ change, the robust solution will remain feasible with *very high probability*. Assuming a symmetrical distribution of random variables, Bertsimas and Sim (2003, 2004) calculated the probability that the i^{th} constraint will be violated, if more than $\lfloor \Gamma_i^a \rfloor$ coefficients vary. This probability can be approximated by the following expression:

$$\Pr\left(\sum_j \tilde{a}_{ij} x_j^* > b_i \right) \leq 1 - \Phi\left(\frac{\Gamma_i^a - 1}{\sqrt{n_i}} \right) \qquad \textbf{(3)}$$

Where $n = | J_i^a |$, and $\Phi(\theta)$ is the *cumulative distributive function of a standard normal.*

The limiting bound (equation 3) is particularly interesting because it leads us towards a more appropriate choice for Γ_i^a, as shown below in the computational results.

3. Problem Description

This paper proposes a robust mixed-integer linear model in order to generalize the deterministic model developed by Al-Qahtani and Elkamel (2008), which determines an optimal integration strategy across multiple refineries and establishes an overall production and operating plan for each individual site. The refineries are connected in a finite number of ways and the network is supplied by a set of crude oils $cr \in CR$. Each refinery $i \in I$ processes crude oil to produce a variety of marketable petroleum products $cfr \in CFR$. Refineries carry process units $m \in M$ that can operate at different operating modes $p \in P$ and produce several intermediate streams $cir \in CIR$ which can be blended to create distinct commercial offerings. The objective is to minimize the production and investment costs over a given time horizon in order to achieve production flexibility and improve coordination in the network.

The robust model accounts for uncertainties in costs of crude oil, prices of final products, and product demands.

4. Model formulation

The robust formulation for optimal coordination of integrated multirefinery networks is presented below. The definitions of the variables and parameters are given in the nomenclature section at the end of this paper.

$$
\begin{aligned}
Min \quad & \sum_{cr\in CR}\sum_{i\in I} CrCost_{cr}\ sr_{cr,i} \\
& +\sum_{p\in P}\left(OpCost_{p} \sum_{cr\in CR}\sum_{i\in I} z_{cr,p,i}\right) \\
& +\sum_{cir\in CIR}\sum_{i\in I}\sum_{i'\in I} InsCost_{i,i'}\ yt_{cir,i,i'} \qquad \text{where } i\neq i' \\
& +\sum_{i\in I}\sum_{m\in M} InCost_{m}\ ye_{m,i} \\
& -\sum_{cfrex\in PEX}\sum_{i\in I} PR_{cfrex}\ e_{cfrex,i} \\
& -\lambda_{price}\Gamma_{price}+\sum_{cfrex\in PEX}\mu_{cfrex}^{price} \\
& -\lambda_{\cos t}\Gamma_{\cos t}+\sum_{cr\in CR}\mu_{cr}^{\cos t}
\end{aligned}
\qquad \textbf{(4)}
$$

Subject to

$$\sum_{p\in P} z_{cr,p,i} = sr_{cr,i} \qquad \forall\ cr\in CR,\ i\in I \qquad \textbf{(5)}$$

$$
\begin{aligned}
&\sum_{p\in P}\alpha_{cr,cir,i,p}\ z_{cr,p,i}+\sum_{i'\in I}\sum_{p\in P}\xi_{cir,i',p,i}\ xi_{cir,i',p,i} = \\
&\sum_{i'\in I}\sum_{p\in P}\xi_{cir,i,p,i'}\ xi_{cir,i,p,i'}+\sum_{cfr\in CFCB} w_{cr,cir,cfr,i}+\sum_{rf\in FUEL} w_{cr,cir,rf,i}
\end{aligned}
\qquad \forall\ \begin{aligned}&cr\in CR, cir\in CIR,\\ &i'\ \&\ i\in I, \text{where } i\neq i'\end{aligned} \qquad \textbf{(6)}
$$

$$xf_{cfr,i} = \sum_{cr\in CR}\sum_{cir\in CFCB} w_{cr,cir,cfr,i}-\sum_{cr\in CR}\sum_{rf\in FUEL} w_{cr,cfr,rf,i} \qquad \forall\ cfr\in CFR,\ i\in I \qquad \textbf{(7)}$$

$$xv_{cfr,i} = \sum_{cr\in CR}\sum_{cb\in CFCB}\frac{w_{cr,cb,cfr,i}}{SG_{cr,cb}} \qquad \forall\ cfr\in CFR,\ i\in I \qquad \textbf{(8)}$$

$$\sum_{cir\in FUEL} CV_{rf,cir,i}\ w_{cr,cir,rf,i}+\sum_{c=\text{"}HFO\text{"}\in FUEL} w_{cr,c,rf,i}+\sum_{p\in P}\beta_{cr,rf,i,p}\ z_{cr,p,i}=0 \qquad \forall\ \begin{aligned}&cr\in CR,\ i\in I,\\ &rf\in FUEL\end{aligned} \qquad \textbf{(9)}$$

$$
\begin{aligned}
&\sum_{cr\in CR}\sum_{cb\in CFCB}\left(ATT_{cr,cb,q\in Qv}\ \frac{w_{cr,cb,cfr,i}}{SG_{cr,cb}}+ATT_{cr,cb,q\in Qw}\ w_{cr,cb,cfr,i}\right) \\
&\qquad \geq Q^{L}_{cfr,q\in Qv}\ xv_{cfr,i}+Q^{L}_{cfr,q\in Qw}\ xf_{cfr,i}
\end{aligned}
\qquad \forall\ \begin{aligned}&cfr\in CFR,\\ &q=\{qw,qv\}\in Q,\\ &i\in I\end{aligned} \qquad \textbf{(10)}
$$

$$
\begin{aligned}
&\sum_{cr\in CR}\sum_{cb\in CFCB}\left(ATT_{cr,cir,q\in Qv}\ \frac{w_{cr,cb,cfr,i}}{SG_{cr,cb}}+ATT_{cr,cb,q\in Qw}\ w_{cr,cb,cfr,i}\right) \\
&\qquad \leq Q^{U}_{cfr,q\in Qv}\ xv_{cfr,i}+Q^{U}_{cfr,q\in Qw}\ xf_{cfr,i}
\end{aligned}
\qquad \forall\ \begin{aligned}&cfr\in CFR,\\ &q=\{qw,qv\}\in Q,\\ &i\in I\end{aligned} \qquad \textbf{(11)}
$$

$$MinC_{m,i} \leq \sum_{p \in P} (\gamma_{m,p} \sum_{cr \in CR} z_{cr,p,i}) \leq MaxC_{m,i} + AddC_{m,i} \ \ ye_{m,i} \qquad \forall\ m \in M, i \in I \qquad \textbf{(12)}$$

$$\sum_{p \in P} \xi_{cir,i,p,i'} \ \ xi_{cir,i,p,i'} \leq F^{U}_{cir,i,i'} \ \ yt_{cir,i,i'} \qquad \forall \begin{array}{l} cir \in CIR, \\ i' \,\&\, i \in I, \text{where } i \neq i' \end{array} \qquad \textbf{(13)}$$

$$\sum_{i \in I} \left(xf_{cfr,i} - e_{crpex,i} - DEM_{cfr}\, x_{cfr',i} \right) \geq \lambda_{dem} \Gamma_{dem} + \mu^{dem}_{cfrex} \qquad \forall \begin{array}{l} cfr, crpex \,\&\, cfr', \\ \text{where } cfr \in CFR, \\ crpex \in PEX, \\ cfr' = |PEX| + 1 \end{array} \qquad \textbf{(14)}$$

$$IM^{L}_{cr} \leq \sum_{i \in I} sr_{cr,i} \leq IM^{U}_{cr} \qquad \forall\ cr \in CR \qquad \textbf{(15)}$$

$$\lambda_{price} + \mu^{price}_{cfrex} \geq d_{price} e_{cfrex,i} \qquad \forall\ cfrex \in PEX, i \in I \qquad \textbf{(16)}$$

$$\lambda_{\cos t} + \mu^{\cos t}_{cr} \geq d_{\cos t} sr_{cr,i} \qquad \forall\ cr \in CR, i \in I \qquad \textbf{(17)}$$

$$\lambda_{dem} + \mu^{dem}_{cfrex} \geq d_{dem} x_{cfr',i} \qquad \forall \begin{array}{l} cfrex \in PEX, i \in I \\ cfr'' = |PEX| + 1 \end{array}, \qquad \textbf{(18)}$$

$$x_{cfr',i} = 1 \qquad \forall \begin{array}{l} i \in I \\ cfr'' = |PEX| + 1 \end{array} \qquad \textbf{(19)}$$

$$\mu^{price}_{cfrex} \geq 0 \qquad \forall\ cfrex \in PEX \qquad \textbf{(20)}$$

$$\mu^{\cos t}_{cr} \geq 0 \qquad \forall\ cr \in CR \qquad \textbf{(21)}$$

$$\mu^{dem}_{cfrex} \geq 0 \qquad \forall\ cfrex \in PEX \qquad \textbf{(22)}$$

$$\lambda_{price}, \lambda_{\cos t}, \lambda_{dem} \geq 0 \qquad \textbf{(23)}$$

The robust objective function (4) aims to minimize the total cost represented by the sum of the crude oil supply cost, operating cost, intermediate exchange piping cost, expansion cost, less the export revenue. The parameters Γ_{price} and $\Gamma_{\cos t}$ control the uncertainty in the objective function and assume values in the intervals $[0, | J_{price} |]$ and

$[0,|J_{\cos t}|]$, where J is the set of uncertain coefficients with deviations d_{price} and $d_{\cos t}$ (for prices and costs, respectively).

Constraint (5) represents the raw material balance in which the crude oil supply $sr_{cr,i}$ at plant $i \in I$ from each crude type $cr \in CR$ is equal to the sum of input flowrates at distillation units $p \in P'$. Constraint (6) limits the intermediate material balances within and across refineries, where the coefficient $\alpha_{cr,cir,i,p}$ can assume either a positive sign if it is an input to a unit or a negative sign if it is an output from a unit; the parameter $\xi_{cir,i,p,i'}$ defines all possible alternatives of connecting intermediate streams; and the variable $xi_{cir,i',p,i}$ represents the transshipment flowrate from one plant to another. The difference between intermediate streams $w_{cr,cir,cfr,i}$ that contribute to the final products and intermediate streams that contribute to the fuel system defines the product material balance, as shown in constraint (7). In order to express the quality attributes that blend by volume, the mass flowrate is converted to volumetric flowrate by dividing it by the specific gravity $SG_{cr,cb}$ in constraint (8). Constraint (9) corresponds to the fuel system material balance where the term $CV_{rf,cir,i}$ represents the caloric value equivalent for each intermediate $cir \in FUEL$ used in the fuel system at plant $i \in I$. The matrix $\beta_{cr,rf,i,p}$ corresponds to the consumption of each processing unit $p \in P$ at plant $i \in I$ as a percentage of unit throughputs.

Constraints (10) and (11) specify quality bounds for products that either blend by mass $qw \in Q_w$ or by volume $qv \in Q_v$. Constraint (12) limits capacity of each processing unit. The coefficient $\gamma_{m,p}$ is a zero-one matrix for the assignment of production unit $m \in M$ to process operating mode $p \in P$. The term $AddC_{m,i}$ accounts for the additional capacity expansion and the integer variable $ye_{m,i}$ represents the decision of expanding a production unit. Constraint (13) sets the upper bound $F^{U}_{cir,i,i'}$ on intermediate stream flowrates between the different refineries. The integer variable $yt_{cir,i,i'}$ represents the decision of exchanging intermediate products among the refineries.

Constraint (14) stipulates that the total production from each refinery $xf_{cfr,i}$ less the amount exported $e_{crpex,i}$ must satisfy the domestic demand. The parameter Γ_{dem} adjusts the demand robustness and adopts values in the interval $[0,1]$. Constraint (15) limits the available feedstock $cr \in CR$ to the refineries.

Constraints (16), (17) and (18) represent the minimum deviation necessary to deal with uncertain in the parameters. The auxiliary variable $x_{cfr',i}$ is defined at equation (19). Finally, the other constraints define non negativity of robust variables.

5. Case study

The industrial scale study is based on the numerical example of Al-Qahtani and Elkamel (2008). This example consists of three large industrial scale refineries and represents a general set up that can be found in many industrial sites around the world. The model decisions include the selection of oil blend combination, production units' expansion options and operating levels. The refineries follow a centralized coordination, in which the feedstock supply of two groups of oils (Arabian Light and Kuwait) is shared and the refineries collaborate to meet a given local market demand for the following refined products: liquefied petroleum gas (LPG), light naphtha (LN), two grades of gasoline (PG98 and ES95), jet fuel (JP4), military jet fuel (ATKP), gas oil (AHGO), diesel fuel (Diesel), heating fuel oil (HFO), and petroleum coke (Coke). Production that exceeds the local market demand is either sold in the spot market or exported.

The optimum crude oil blend is used to feed the atmospheric crude unit, which separates crude oil into several fractions. Depending on the production targets, different conversion and treatment processes are applied to the crude fractions. Conversion processes (cocker, fluid catalytic cracker, and catalytic reforming) transform a fraction into another or change the molecular structure of the fraction. Treatment processes (hydrodesulphurization and hydrotreatment) provide better cutting of semi-finished products by reducing contaminants, such as sulfur, nitrogen and metals, or removing them from their structure.

The overall objective of minimizing total annualized cost for the numerical example is considered in two ways: with and without integration among the refineries. Then, the integration benefits in a stochastic environment are discussed.

The robust formulation to account for uncertainty in the objective function coefficients considers 10% of the positive deviation related to cost coefficients and 10% of the negative deviation related to price coefficients from the expected values. The parameters Γ_{price} and $\Gamma_{\cos t}$ take values at the intervals $[0,7]$ and $[0,2]$, respectively. In addition, the robust dimension related to demand is constructed based on 5% of negative deviation from the nominal value. For this approach, $|J|$ is equal to 1.

In the next section, this numerical example is presented and solved to optimality to demonstrate the effectiveness of the robust approach.

5.1. Computational results and discussion

The model was implemented using the Advanced Integrated Multidimensional Modeling Software - AIMMS (Bisschop and Roelofs, 2007) and solved with CPLEX 11.1. The deterministic solution value (minimum cost) for the integrated and nonintegrated network are equal to $7,477,978.57/year and $7,623,151.59/year, respectively.

First, experiments were conducted for each of the uncertain parameters (price, cost, and demand) separately. The robust parameter, Γ, was varied in integer values over the interval $[0,|J|]$. Subsequently, the three uncertain parameters were combined.

From Figures 1 and 2, it can be concluded that cost of crude oil uncertainty exerts greater impact on total cost than product price uncertainty. Actually, comparing the total cost in the Soyster case ($\Gamma_i = |J_i|$), the objective function value considering cost deviation was 6.53% higher than the deterministic solution, whereas the solution with price deviation achieved only 1.24%. Additionally, the computational results for $\Gamma_{price} = 6$ and $\Gamma_{price} = 7$ were the same, because the gasoline ES95 had not been exported, so deviation in its export price does not affect the final result.

When Γ_{price} and $\Gamma_{\cos t}$ are equal to zero, the additional production to protect against the major impact in the objective function are attributed to λ_{price} and $\lambda_{\cos t}$, respectively. Since these variables are multiplied by zero in the objective function, they can take any value to satisfy the minimum deviation constraint that the optimal objective function value does not change. A similar analysis can also be performed for demand uncertainty.

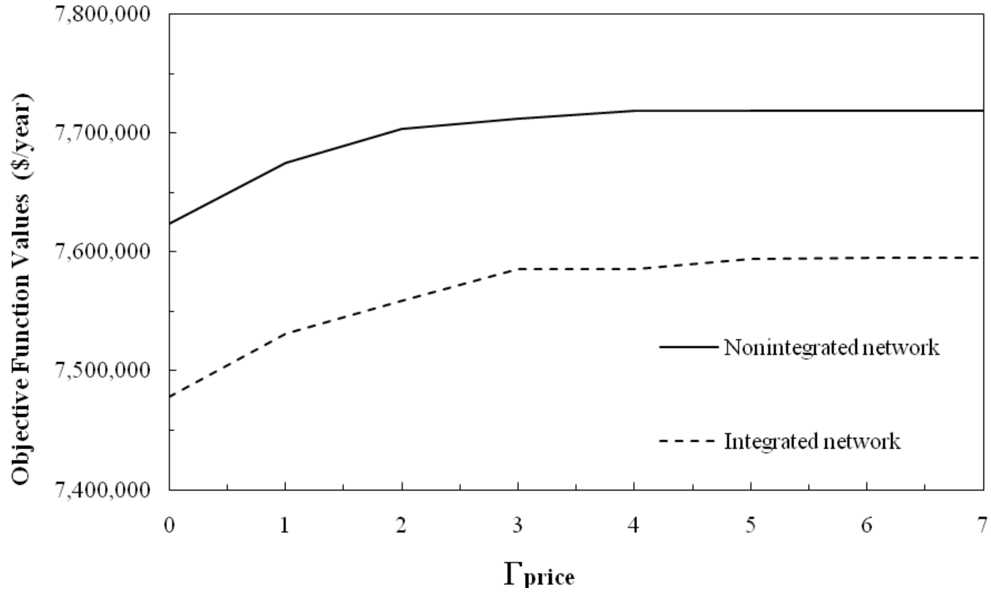

Figure 1. Objective function variation consistent with Γ_{price}

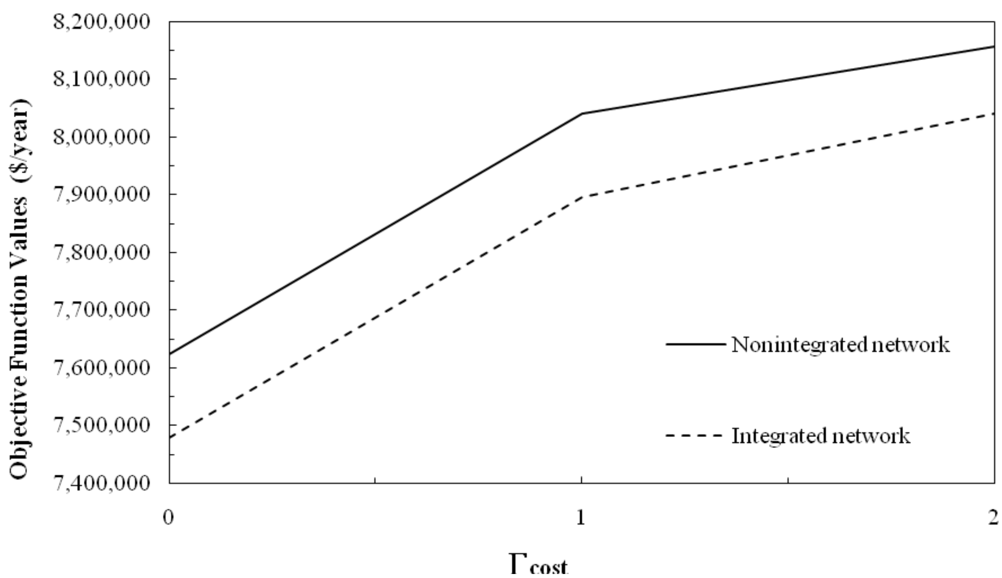

Figure 2. Objective function variation consistent with $\Gamma_{\cos t}$

Moreover, by allowing integration among the refineries, the annual savings exceed $100,000 for both price and cost deviations in the Soyster case ($\Gamma_i = |J_i|$). These savings tend to increase as the number of plants, production units and integration alternatives across the enterprise increase.

The benefits of integration are not limited to reducing cost, but also improve flexibility of production as well as proper utilization and allocation of resources among the refineries network. In fact, in the scenario with no integration the model becomes infeasible if the demand deviation is considered. This finding results from the variation in diesel demand. Diesel production was barely satisfying local demand of 480,000 tones/year and only a little amount was left for export. With such a thin production margin, the plant did not have enough flexibility to face variations in diesel demand. In the integrated network, however, the benefits were in terms of increasing the production margin and exports of diesel and also as gaining more diesel production flexibility to meet any variations in local market demand. This production flexibility was achieved by increasing the Kuwait crude supply and, consequently, changing the overall utilization of some production units. This fact is because Kuwait crude contains more heavy ends than Arabian light. The selection of crude supply type was in favor of Arabian Light as it was processed up to its maximum availability. The remaining required crude to satisfy local market demand was fulfilled by Kuwait crude, although it yields a higher overall annual production cost.

In addition, considering the demand deviation, the objective function value reaches $8,033,696.41/year, which means that demand uncertainty impact the objective function more strongly than cost and price uncertainties. This pattern can also be seen in Figure 3, in which the randomness in the three uncertain parameters was combined. The computational results presented in Figure 3 considers $\Gamma_{\cos t} = 2$. The solutions with $\Gamma_{\cos t} = 1$ were around 1.7% closer to the nominal solution. This addition of uncertainty gradually deteriorates the robust solution. It is also interesting to note that the objective function value is marginally affected by the increased level of protection. Specifically, the increments decrease each time Γ increases. This finding is a feature of the robust formulation and is entirely independent of the problem treated.

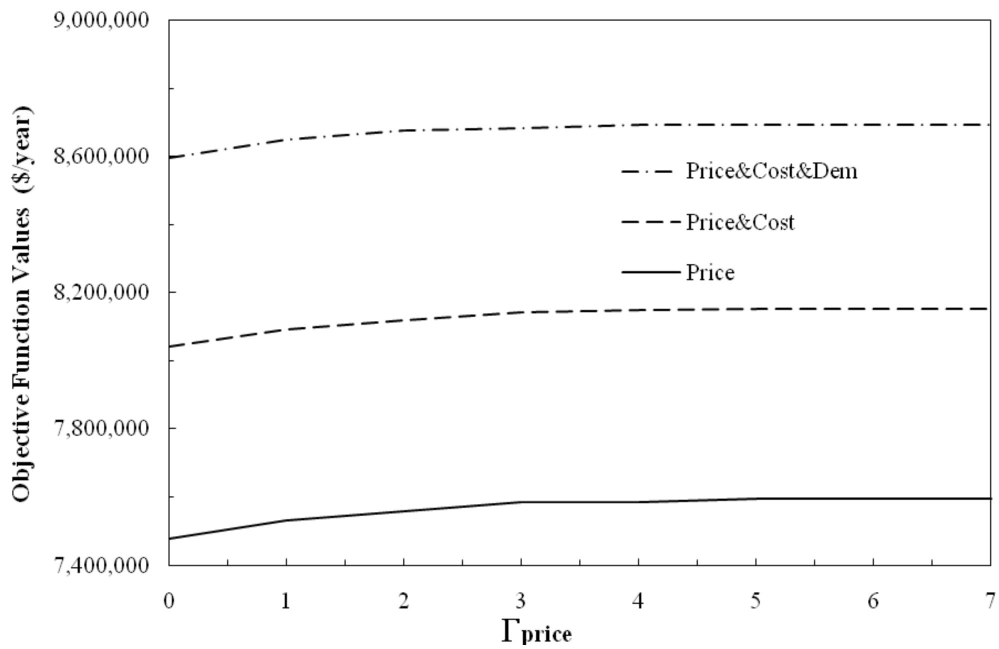

Figure 3. Integrated network – objective function variation consistent with Γ_{price} ($\Gamma_{cost} = 2$)

Figure 4 shows computational results for variations in demand deviations in the case study considering the three different types of randomness – Price&Cost&Dem of Figure 3. These results seem to confirm the model's sensitivity to deviations from the nominal value.

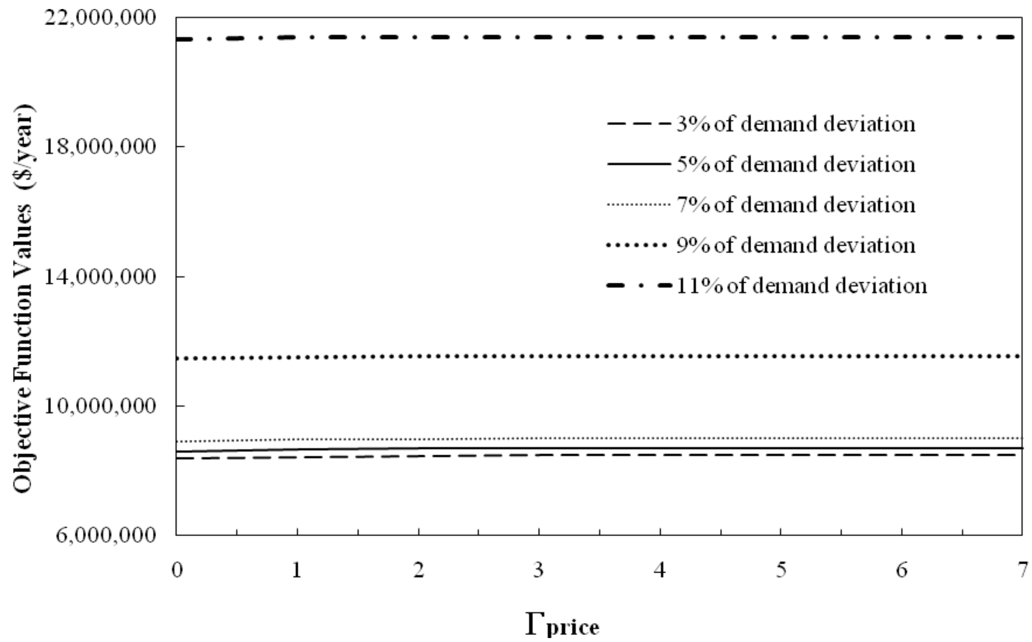

Figure 4. Objective function variation with different demand deviations ($\Gamma_{\cos t} = 2$)

In response to the increase of 9% of demand deviation, the strategic plan suggests the installation of a new isomerization unit in Refinery 3. On the other hand, when 11% of demand deviation is considered, the model suggests installation of three units: a isomerization unit in Refinery 1; a fluid catalytic cracker unit in Refinery 2; and a desulfurization of cycle gas oil unit in Refinery 3. As can be seen in Figure 4, the total annual cost has quite increased in these two scenarios due to the additional units' installation and operating costs.

In spite of many advantages, the robust methodology does not account for a decision-maker's risk-taking behavior. For this reason, a more realistic approach should include a measure of the degree of solution conservatism/reliability. One relevant measure of interest is the probability of constraint violation, which can lead to a more appropriate choice for Γ_{dem}, as shown in expression 3. Our calculations suggests that if the decision-maker only accepts at a maximum of 50% probability of constraint violation, then he/she has to use $\Gamma_{dem} = 1$ to protect himself/herself against price parameter deviations. On the other hand, if the decision-maker accepts up to 60% probability, there is no need to secure additional protection from the nominal problem and Γ_{dem} can be set equal to zero. In this sense, the decision-maker can judge the appropriate tradeoff between conservatism and total profit in order to adopt a more appropriate parameter to control robustness.

6. Conclusions

In this work, a robust mixed-integer linear model for coordination strategy under uncertainty of an integrated multirefinery network was presented. The consideration of uncertainty in price, cost, and demand parameters provided a more robust and practical analysis of the problem, especially in a time when fluctuations in petroleum prices and demand are soaring. A case study considering integration and no integration at the refinery network was solved to illustrate benefits of integrated coordination in a stochastic environment. The results show the economic potential and trade-offs involved in the optimization of such networks. The conclusion is that the Bertsimas and Sim approach may be a useful tool for modeling planning problems without introducing additional computational complexity. In addition, the calculation of probability bounds of constraint violation could help the decision-makers make better choices regarding parameter alternatives to control robustness.

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Nomenclature

Sets and indices

C	commodity *c, c'*
I	plant *i, i'*
M	production unit *m*
P	refinery process *p*
Q	quality specification *q*

Subsets

CFR	final products *cfr, subset of C*
CIR	intermediate streams *cir, subset of C*
CR	raw materials *cr, subset of C*
CB	streams for blending refinery products *cb, subset of C*
PEX	products for exports *cfrex, subset of C*
RF	fuel streams *rf, subset of C*
QV	quality of products that blends by volume *qv, subset of Q*
QW	quality of products that blends by weight *qw, subset of Q*
$CFCB_{cfr,cir}$	blending combinations between *cfr* and *cir*
$FUEL_{rf,c}$	streams *c* comprising refinery fuel *rf*

Parameters

$CrCost_{cr}$	*cost of crude cr*
$OpCost_{p}$	*operating cost of process p*
$InsCost_{i,i'}$	*insulation cost of piping to transfer commodity cir from plant i to plant i'*
$InCost_{m}$	*installation cost of a refinery production unit m*
PR_{cfrex}	*export price of product crpex*
$\alpha_{cr,cir,i,p}$	*input-output coefficients of stream cir from crude cr at plant i by process p*
$\xi_{cir,i,p,i'}$	*integration superstructure of all possible alternatives to transfer commodity cir from plant i to process p in plant i'*
$SG_{cr,cir}$	*specific gravity of commodity cir by crude cr*
$CV_{rf,c,i}$	*caloric value equivalent of refinery fuel rf by commodity c at plant i*

$\beta_{cr,rf,i,p}$	*fuel consumption coefficients of refinery fuel rf from crude cr at plant i by process p*
$ATT_{cr,cir,q}$	*attributes of intermediate streams cir produced from crude cr blending of property q*
$Q^{L}_{cfr,q}$	*quality bounds of commodity cfr of property q*
$Q^{U}_{cfr,q}$	*upper level bounds of commodity cfr of property q*
$MinC_{m,i}$	*minimum capacity of production unit m at plant i*
$MaxC_{m,i}$	*maximum capacity of production unit m at plant i*
$AddC_{m,i}$	*additional capacity of production unit m at plant i*
$\gamma_{m,p}$	*assignment of process p to equipment m*
$F^{U}_{cir,i,i'}$	*flowrate upper bound of intermediate cir from plant i to i'*
DEM_{cfr}	*demand of product cfr*
IM^{L}_{cr}	*import lower bound of commodity cr*
IM^{U}_{cr}	*import upper bound of commodity cr*

Variables

$e_{crpex,i}$	exports of final product *crpex* from refinery *i*
$sr_{cr,i}$	supply of raw material *cr* to refinery *i*
$w_{cr,c,c',i}$	blending levels of crude *cr* that produces the stream *c* to yield a stream *c'* at plant *i*
$z_{cr,p,i}$	process input flowrate of crude *cr* to process *p* at plant *i*
$xf_{cfr,i}$	mass flowrate of refinery final product *cfr* by refinery *i*
$xv_{cfr,i}$	volumetric flowrate of refinery final product *cfr* by refinery *i*
$xi_{cir,i,p,i'}$	transshipment level of commodity *cir* from plant *i* to process *p* at plant *i'*
$ye_{m,i}$	binary variable representing refinery expansion of production unit *m* at plant *i*
$yt_{cir,i,i'}$	binary variable representing transshipment of refinery commodity *cir* from plant *i* to plant *i'*

Robust variables

$\mu^{price}_{cfrex}, \mu^{cost}_{cr}, \mu^{dem}_{cfrex}$	quantify sensitivity to changes in cost, price, and demand, respectively
$\lambda_{price}, \lambda_{cost}, \lambda_{dem}$	quantify sensitivity to changes in cost, price, and demand, respectively
$x_{cfr',i}$	auxiliary variable of robust formulation to account for demand randomness

Robust parameters

Γ_{cost}, Γ_{price}, Γ_{dem}	parameters to adjust cost, price, and demand robustness, respectively

$d_{\cos t}$, d_{price} , d_{dem} cost, price, and demand deviations, respectively

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