

Advance Demand Information, Price Discrimination, and Pre-order Strategies

Cuihong Li • Fuqiang Zhang

School of Business, University of Connecticut, Storrs, CT 06269
Olin Business School, Washington University in St. Louis, St. Louis, MO 63130
cuihong.li@business.uconn.edu • fzhang22@wustl.edu

This paper studies the pre-order strategy that a seller may use to sell a perishable product in an uncertain market with heterogeneous consumers. By accepting pre-orders, the seller is able to obtain advance demand information for inventory planning and price-discriminate the consumers. Given the pre-order option, the consumers react strategically by optimizing the timing of purchase. We find that accurate demand information improves the availability of the product, which undermines the seller's ability to charge a high pre-order price. As a result, advance demand information may hurt the seller's profit due to its negative impact for the pre-order season. This cautions the seller about a potential conflict between the benefits of advance demand information and price discrimination when facing strategic consumers. A common practice to contain consumers' strategic waiting is to offer price guarantees that compensate pre-order consumers in case of a later price cut. Under price guarantees, the seller will reduce price in the regular season only if the pre-order demand is low; however, such advance information implies weak demand in the regular season as well. Therefore, under price guarantees, advance demand information may still hurt the seller's profit due to its adverse impact for the regular season. We investigate the seller's pre-order strategy choice (i.e., whether the pre-order option should be offered and whether it should be coupled with price guarantees). We find that the answer depends on the relative sizes of the heterogeneous consumer segments. Specifically, we provide a simple threshold structure to characterize the seller's optimal strategy choice.

Keywords: Pre-order, advance demand information, price discrimination, strategic consumer behavior, price guarantee

1. Introduction

Pre-order refers to the practice of a seller accepting customer orders before a product is released. Such a practice has become commonplace for a wide variety of product categories in recent years. To name a few examples: In 2002, Apple reported the successful use of pre-orders for both the original and new iMac computers that revived the fortunes of the then-troubled company (*Wall Street Journal*, 2002). When launching the new iPhone 3G S,

the third generation of the smart phone, Apple allowed customers to pre-order the product to secure the delivery on the release date (Keizer, 2009). In early September 2009, Nokia announced that its first Linux-based smart phone, the N900, was available through pre-orders in the US market (Goldstein, 2009). Game consoles, such as Nintendo's Wii and Sony's Playstation 3, had been put on pre-order before they were formally released (Martin, 2006 and Macarthy, 2007).

The pre-order option is clearly beneficial to consumers because it guarantees prompt delivery on release. This is especially valuable when the product is a big hit and will be hard to find in stores due to its popularity. It is not uncommon for extremely popular gadgets to run out of stock immediately after release. Consumers may have to wait for weeks or even months to get the product. For instance, retailers warned customers in mid-2006 that no Wii units would be available without a pre-order until 2007 (Martin, 2006). Pre-orders are often placed by enthusiastic consumers who are eager to be the first to get their hands on a new product.

The pre-order strategy may bring significant benefits to the seller as well. First, the seller can gauge how much demand there will be for his product from the pre-order sales. Pre-order is mostly used for new products with relatively short life cycles. It is generally hard to obtain accurate demand forecast for such products. Thus the advance demand information from pre-orders will be very useful in procurement, production, and inventory planning. This has been further promoted by the advent of the Internet and other information technologies that have greatly reduced the costs associated with data collection and processing. It has been reported that by accepting pre-orders for iPhone 3G S, Apple did not experience the same kind of stockout problems that occurred with its iPhone 3G due to a mismatch between supply and demand (Keizer, 2009). Second, the pre-order strategy provides leverage for the seller to charge different prices based on the timing of a customer's purchase. By accepting pre-orders, the seller can identify the consumer segment that is willing to pay a premium price for guaranteed early delivery. These consumers are either new technology lovers or simply loyal fans of the brand. They are often referred to as "early adopters" and the extra price they pay is called an "early adopter tax". In fact, recently the early adopter tax has become controversial in various industries (see Nair, 2007 for video games; Jack, 2007 for Apple's iPhone; and Falcon, 2009 for Sony's latest gaming gadget, PSP Go).

With the pre-order option, a forward-looking customer may choose the timing of purchase: Placing a pre-order guarantees the availability of the product, but this is probably at the

expense of a higher price; waiting for a lowered price (e.g., after the product is released) sounds quite attractive, but meanwhile the consumer has to face the risk of stockout. How much a consumer is willing to pay for a pre-order depends on her valuation of the product and the expectation of future price and availability of the product. The seller needs to base inventory and pricing decisions on the presence of advance demand information and consumers' strategic behavior. Despite the prevalence of the pre-order practice, there has been surprisingly little research that analyzes the seller's optimal decisions while taking both demand information updating and forward-looking consumers explicitly into account. In this paper, we study the pre-order strategy by developing a modeling framework that incorporates all the important elements mentioned above: the seller's inventory and pricing decisions, consumers' strategic response, and advance demand information. Specifically, we aim to address the following research questions:

What is the value of advance demand information? From the operations point of view, a major benefit of accepting pre-orders is that the seller can obtain advance demand information. This information helps improve the seller's decision on initial production runs to satisfy demand, and its effectiveness has been frequently lauded in the literature. However, a better match between supply and demand implies that availability of the product becomes less a concern, which may affect consumers' willingness to pay for a pre-order. Thus, it would be interesting to study when advance demand information is valuable to the seller in the presence of strategic consumer behavior.

What is the impact of price guarantees? Many products on pre-order exhibit patterns of price cutting. For example, Nokia started accepting pre-orders for its N900 smart phone at \$649 and then quickly slashed it to \$589 (Goldstein, 2009 and King, 2009). Amazon charged a pre-order price \$359 for its Kindle 2 and then dropped the price to \$299 after the release (Carnoy, 2009). Consumers may be reluctant to place pre-orders when expecting future price reduction. To encourage early purchases, the seller may offer a price guarantee along with the pre-order option. That is, a customer would receive a refund if the price declines over time. This raises the question of how price guarantees affect the value of advance demand information.

When and how should a seller use the pre-order strategy? A seller may choose to or not to use the pre-order strategy. For example, Apple adopted the pre-order strategy for its iMac computers and iPhone 3G S, but not for its iPhone 3G. Furthermore, when a pre-order strategy is used, the seller may choose to or not to offer the price guarantee. When and how

to use the pre-order strategy is an important question for the seller. To answer this question, we investigate how product and demand characteristics affect the seller's pre-order decisions, and characterize the conditions under which the pre-order strategy (with or without a price guarantee) is most beneficial.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model setting and Section 4 presents the analysis of the pre-order strategy. Section 5 compares the pre-order strategy and the no-pre-order strategy. The impact of price guarantees is studied in Section 6. Section 7 discusses some potential extensions and the paper concludes with Section 8. All proofs are given in the Appendix.

2. Literature Review

This paper is closely related to the literature on the value of using advance purchase orders to obtain more accurate demand forecast. Tang et al. (2004) model the benefits of an advance booking discount program where a retailer offers a price discount to consumers who make early order commitments prior to the selling season. In a follow-up study, McCardle et al. (2004) consider the advance booking discount program under retail competition. They characterize the conditions under which two competing retailers will both adopt the advance booking discount program. Fisher and Raman (1996) propose a quick response strategy under which a retailer utilizes early sales for estimating demand distribution. Eppen and Iyer (1997) study a fashion-buying problem for retailers who can divert inventory to outlet stores using updated demand information. Moe and Fader (2002) put forth a diffusion model to forecast new product sales using advance purchase orders. Specifically, they consider two market segments (i.e., innovators and followers) and use actual sales data to estimate the parameters of the distributions for these two demand segments. We also study the impact of advance demand information in this research. However, our model differs from the above studies in that we incorporate price discrimination and model consumers' strategic behavior explicitly.

There are papers studying advance selling with strategic consumers; see, for example, Dana (1998); Xie and Shugan (2001); Yu, Kapuscinski, and Ahn (2007); Chu and Zhang (2009). This line of research is motivated by situations where the consumption of a service will happen at a future time, and thus consumers are uncertain about its true value when making an advance purchase (e.g., early registration for conferences, booking tickets for sport

events, etc.). It has been shown that advance selling is beneficial because it allows the seller to price-discriminate consumers and increase market participation. These papers focus on the seller's pricing strategy and thus assume that the product quantity (service capacity) is exogenously given (it could be infinite). Alexandrov and Lariviere (2008) study whether a service provider should accept reservations when selling a fixed amount of capacity. The notion of reservation is similar to advance selling and their model is also based on uncertain consumer valuations *ex ante*. The focus of our paper is quite different: We endogenize the seller's inventory decision to study the effect of advance demand information on operations planning. In addition, we study the interaction between price discrimination and advance demand information, which is new in the literature.

This paper is related to the literature that studies the use of price guarantees in intertemporal pricing. Png (1991) is one of the first to study the impact of offering most-favored-customer protection (i.e., price guarantee) in a two-period pricing problem. In Png's model, a seller wishes to sell a fixed inventory to two types of consumers. The size of the total consumer pool is fixed, but the relative sizes of the two types are uncertain (so there is a perfect negative correlation between the demand segments). We differ from Png (1991) in important ways: First, we endogenize the seller's inventory decision, which takes the pre-order sales as an input. Second, we study the effect of advance demand information by allowing general correlation between demand segments. Lai et al. (2009) examine the value of using price guarantees for a newsvendor seller who has the opportunity to mark down the product at the end of the selling season. In their problem, the seller determines the inventory before accepting consumer orders, and the low-type consumers represent an infinite salvage market. Therefore, they do not include the element of advance demand information. Levin et al. (2007) analyze a dynamic pricing problem with fixed capacity and price guarantees. They focus on determining the optimal dynamic price and guarantee policies by assuming myopic consumers.

Dynamic pricing problems have been widely studied in the revenue management literature. The objective of these studies is to find the optimal pricing and capacity (inventory) allocation rules to maximize a seller's revenue/profit. Elmaghraby and Keskinocak (2003) and Talluri and van Ryzin (2004) provide comprehensive reviews of these studies. Recently, there have been an increasing number of papers that explicitly incorporate an individual consumer's response to a seller's pricing and capacity decisions. Su (2007), Aviv and Pazgal (2008), and Elmaghraby et al. (2008) analyze a seller's pricing strategy in the presence of

consumers' strategic waiting behavior. Liu and van Ryzin (2008) study a capacity-rationing strategy for a seller to induce early purchases in a setting with a fixed price path. Yin et al. (2009) study the implications of store display formats when consumers can strategically choose their purchase timing. Levin et al. (2009) consider a dynamic pricing model where firms sell perishable goods to strategic consumers under oligopolistic competition. Jerath et al. (2009) study last-minute sales through an opaque intermediary, which mitigates consumers' strategic waiting. Mersereau and Zhang (2009) analyzes the value of knowledge on strategic customer behavior in a two-period markdown pricing setting. The contribution of our paper is to consider the capacity decision along with demand updating in a dynamic pricing problem, which, to the best of our knowledge, has not yet been addressed.

There has also been a growing interest in studying the impact of strategic consumer behavior on a seller's inventory decision. Su and Zhang (2008) investigate the implication of forward-looking consumers on supply chain performance. They show that under the presence of strategic consumers, double marginalization may actually improve supply chain efficiency. Su and Zhang (2009) propose the use of availability guarantees to placate consumers who are concerned with stockout-related costs. Cachon and Swinney (2009) examine the quick response strategy that allows the seller to replenish inventory after the selling season starts. They find that the quick response strategy could be more valuable from the seller's perspective when facing forward-looking consumers. This is mainly because quick response enables the seller to minimize the mismatch between supply and demand, and thus decrease the need for price markdowns (note that the root cause of strategic waiting is price markdowns). Swinney (2009) further shows that quick response may hurt a seller's profit when consumers face uncertain product valuations. In this case, quick response improves future product availability and thus encourages consumers to wait until they learn their valuations. Essentially, Cachon and Swinney (2009) and Swinney (2009) focus on the value of a late ordering opportunity for a seller (after demand is realized); while we concentrate on the value of an early selling opportunity (before inventory is ordered). In addition, we consider the impact of price guarantees, which is absent in these two papers. Huang and Van Mieghem (2009) study the value of online click tracking, which allows a seller to collect advance demand information for pricing and inventory planning. In their model, as the product is sold only after consumer clicking, advance demand information always benefits the seller. In contrast, by studying the pre-order strategy, we show that advance demand information collected from advance selling may hurt the seller.

3. Model

A seller sells a perishable product in a two-period time horizon. The product is released at the beginning of the second period (i.e., the regular selling season), but the seller may accept pre-orders in the first period (i.e., the pre-order season). There are two types of consumers in the market: The high type has a valuation v_H and the low type has a valuation v_L for the product, where $v_H > v_L$. That is, the high type refers to technology-savvy consumers. Define $\Delta = v_H - v_L$ to be the difference between these two valuations.¹ Each consumer is infinitesimal and demands one unit of the product. Usually, the technology-savvy consumers are early adopters who follow the market trend closely. So we assume all the high-type consumers arrive in the first period whereas all the low-type consumers arrive in the second period (see Moe and Fader, 2002 for a similar assumption). We discuss more general arrival patterns in Section 7. Throughout the paper we assume all players are risk-neutral: The seller aims to optimize his total expected profit, and the consumers maximize their expected net utility.

Market demand is uncertain in both periods. Let X_i denote the demand of type i ($i = H, L$), where X_i follows a normal distribution Φ_i (density ϕ_i) with mean μ_i and standard deviation σ_i . Let $\lambda_i = \frac{\mu_i}{\sigma_i}$ be the mean-standard deviation ratio of X_i . Bivariate normal distribution has been commonly used in the literature, especially when modeling demand information updating (see, for example, Fisher and Raman, 1996 and Tang et al., 2004). Let $\rho \in [-1, 1]$ be the correlation coefficient between X_H and X_L . A positive correlation (e.g., $\rho = 1$) corresponds to situations where the primary uncertainty is on the total size of the market but not on the portion of each market segment, while a negative correlation (e.g., $\rho = -1$) relates to situations where the total size of the market is relatively certain but the relative sizes of the two demand types are uncertain. For ease of exposition, we will focus on $\rho \geq 0$ in this paper (the analysis of the case $\rho < 0$ is similar and will be discussed later in Section 7). We may view ρ as an indicator of the accuracy of advance demand information: A large ρ means less uncertainty in the second-period demand (if $\rho = 1$, then the seller can be certain of the low-type demand after observing the high-type demand from pre-order

¹We assume there is no uncertainty in consumer valuations in our problem setting. This assumption is reasonable when there is sufficient information for consumers to evaluate the product before its release. Nowadays many firms use exhibitions, advertising, and their websites to offer demonstration and detailed product information to consumers. In this paper, we focus on the impact of information updating of random demand. Incorporating valuation uncertainty makes our analysis less transparent and is beyond the scope of this paper.

sales).

We first describe the seller’s problem. The seller sets a pre-order price p_1 in the first period. Given this price, the high-type consumers make their pre-order decisions. After the number of pre-orders q is observed, the seller decides the total production quantity $q + Q$, where Q is the quantity used to satisfy the second-period demand. In addition, the seller sets the second-period price p_2 . The seller incurs a constant production cost c ($c < v_L$) for each unit of the product. Note that the seller observes the pre-order quantity (from the first period) but not the second-period demand when making the production decision. Assume unsold products at the end of the problem horizon have negligible salvage value, and there is no penalty for unsatisfied demand. For simplicity, we assume the seller does not discount the second-period profit.

Next we describe the consumers’ problem. The problem for a low-type consumer is trivial: As she arrives in the second period, she buys the product if and only if $p_2 \leq v_L$. A high-type consumer, arriving in the first period, needs to decide whether she should place a pre-order or delay the purchasing to the second period. If she places a pre-order, then she pays the price p_1 and will definitely receive the product in the second period; otherwise, she may purchase the product at a possibly lower price p_2 in the regular season, but under the risk that the product is no longer available. Again, we assume there is no discounting of the high type’s valuation in the second period. Whether a consumer is willing to pre-order depends on her belief that the product will be available in the second period. We model such a belief using a parameter $\theta \in [0, 1]$: A consumer believes that a θ portion of consumers remaining in the market will get the product before her. On one extreme, $\theta = 0$ means a consumer believes that she will be the first in the waiting line; on the other extreme, $\theta = 1$ means that the consumer will be the last in the waiting line. Cachon and Swinney (2009) provide a detailed discussion of this modeling approach and focus on specific θ values for tractability. Similarly, we assume $\theta = 1/2$ in this paper. The qualitative insights will remain unchanged if other θ values are considered.

We introduce the following notations for later use. Let $\Phi(\phi)$ denote the distribution (density) function for the standard normal distribution. We will use capital letters (e.g., X) for random variables and the corresponding small letters (e.g., x) for their realizations. We follow the literature to assume $\Phi_i(0) = 0$ ($i = H, L$) for technical convenience, i.e., the probability of negative demand is negligible (see, for example, Fisher and Raman, 1996). Since $\Phi_i(0) = \Phi(-\lambda_i)$, this assumption is equivalent to λ_i sufficiently high such that $\Phi(-\lambda_i) = 0$,

$i = H, L$. Additionally, we assume $\lambda_L^2 \geq 2\lambda_H^2$, which implies that the low-type demand conditional on any high-type demand realization will have a negligible probability falling into the negative domain.² Throughout the paper, we define “decreasing” (“increasing”) in the weak sense, i.e., decreasing (increasing) means non-increasing (non-decreasing).

4. Pre-order Strategy

In this section we analyze the seller’s pre-order strategy. We start with the analysis of the seller’s price and quantity decisions in the second period, assuming that all high-type consumers have chosen to pre-order given the first-period price. Then we analyze the seller’s first-period price decision, which induces the high-type consumers to pre-order under a rational expectations equilibrium based on the seller’s second-period decisions.

In the second period, with the number of pre-orders q received in the first period, the seller must determine the price p_2 and the order quantity Q to satisfy the second-period demand (besides the quantity q to be ordered for the pre-order demand). Since all high-type consumers are assumed to choose pre-order, it is clear that the number of pre-orders is equal to the high-type demand, $q = x_H$, and only the low-type consumers are present in the second period. Thus it is optimal for the seller to set $p_2 = v_L$. The seller’s order quantity Q depends on the high-type demand realization x_H . Define $X \equiv \frac{X_H - \mu_H}{\sigma_H}$, which follows standard normal distribution. We will work with X instead of X_H due to their one-to-one relationship. For a given realization x of X , the updated low-type demand $\tilde{X}_L(x)$ follows a normal distribution with mean $\tilde{\mu}_L(x) = \mu_L + \rho\sigma_L x$ and standard deviation $\tilde{\sigma}_L = \sigma_L\sqrt{1 - \rho^2}$. Define $z_L \equiv \Phi^{-1}\left(\frac{v_L - c}{v_L}\right)$, where $\Phi(z_L)$ is the critical fractile for the low-type demand. Then the optimal order quantity for the second period is given by

$$Q(x) = \tilde{\mu}_L(x) + z_L\tilde{\sigma}_L, \quad (1)$$

where $x \equiv \frac{x_H - \mu_H}{\sigma_H}$, and the resulting profit for the second period is

$$\Pi_L(x) = (v_L - c)(\mu_L + \rho\sigma_L x) - v_L\phi(z_L)\sigma_L\sqrt{1 - \rho^2}. \quad (2)$$

Note that the expected second-period profit is $\mathbb{E}_X[\Pi_L(X)] = \Pi_L(0)$ because $\mathbb{E}_X[X] = 0$.

²Note $\Pr(X_L \leq 0 | X_H = \mu_H + \sigma_H x) = \Phi\left(-\frac{\mu_L + \rho\sigma_L x}{\sigma_L\sqrt{1 - \rho^2}}\right) = \Phi\left(-\frac{\lambda_L + \rho x}{\sqrt{1 - \rho^2}}\right)$. We need $\frac{\lambda_L + \rho x}{\sqrt{1 - \rho^2}} \geq \lambda_H$ for any $x \in [-\lambda_H, \lambda_H]$. This is equivalent to $\lambda_L \geq (\sqrt{1 - \rho^2} + \rho)\lambda_H$, where the maximum of $\sqrt{1 - \rho^2} + \rho$ is $\sqrt{2}$ at $\rho = 1/\sqrt{2}$.

In the first period, the seller sets the price p_1 to induce pre-order from high-type consumers. Given p_1 , a high-type consumer will choose between pre-order and wait. If she pre-orders, then she receives a net utility $v_H - p_1$; if she waits, then her expected utility is $(v_H - p_2)\xi$, where ξ is the consumer's belief of the availability of the product in the second period if she chooses to wait. The consumer will pre-order if and only if $v_H - p_1 \geq (v_H - p_2)\xi$, or $p_1 \leq v_H - (v_H - p_2)\xi$ (we assume a consumer always pre-orders if there is a tie, since the seller can always cut price by a negligible amount to induce pre-orders). Thus the optimal pre-order price for the seller is

$$p_1 = v_H - (v_H - p_2)\xi. \quad (3)$$

We focus on the rational expectations equilibrium of the above game. In any equilibrium, the players' beliefs must be consistent with the actual outcome. Given $\theta = 1/2$ (each consumer believes that a θ portion of consumers remaining in the market will get the product before her), we know that in the equilibrium, there must be

$$\xi = \mathbb{E}_X \left[\Pr \left(\tilde{X}_L(X)/2 < Q(X) \right) \right] = \mathbb{E}_X \left[\Phi \left(\frac{\lambda_L + \rho X}{\sqrt{1 - \rho^2}} + 2z_L \right) \right], \quad (4)$$

where the second equality is by plugging $\tilde{X}_L(x)$ and $Q(x)$ into the expression. Note $\xi = \Phi(\lambda_L + 2z_L)$ for $\rho = 0$, and $\xi = 1$ for $\rho = 1$.

Thus we have shown that in the rational expectations equilibrium, the seller will set p_1 as in (3), $p_2 = v_L$, and Q as in (1); all high-type consumers will pre-order, i.e., $q = x_H$, and their belief in product availability in the second period is given by (4). In such an equilibrium, the seller's total profit from the pre-order strategy is (we use superscript p for pre-order strategy):

$$\begin{aligned} \Pi^p &= (p_1 - c)\mu_H + \mathbb{E}_X [\Pi_L(X)] \\ &= (v_H - \Delta\xi - c)\mu_H + \Pi_L(0), \end{aligned} \quad (5)$$

where $\Pi_L(X)$ is defined in (2). Note $(v_H - \Delta\xi - c)\mu_H$ is the seller's first-period profit, and $\Pi_L(0)$ is the second-period profit.

4.1 Impact of Demand Correlation (ρ)

What is the value of advance demand information? The accuracy of the advance information can be measured by the demand correlation, ρ . For the positive domain, a higher ρ corresponds to more accurate advance information (i.e., the uncertainty of the low-type demand

is lower after updating). In this subsection, we investigate how the seller's profit depends on ρ .

Based on Equation (5), the derivative of Π^p with respect to ρ can be written as

$$\frac{d\Pi^p}{d\rho} = -\Delta\mu_H \frac{d}{d\rho}\xi + \frac{d}{d\rho}\Pi_L(0). \quad (6)$$

The demand correlation ρ affects both the first-period profit (through its influence on ξ , the equilibrium belief of product availability in the second period) and the second-period profit $\Pi_L(0)$. It is easy to see that $\frac{d}{d\rho}\Pi_L(0) = v_L\phi(z_L)\sigma_L\frac{\rho}{\sqrt{1-\rho^2}}$ is positive—more accurate advance demand information helps the seller better match supply with demand, thus improving the second-period profit. The effect of ρ on the first-period profit depends on its impact on ξ , which is characterized in Lemma 1.

Lemma 1 (i) For $v_L \geq 2c$ (i.e., $z_L \geq 0$), ξ is independent of ρ .

(ii) For $c < v_L < 2c$ (i.e., $z_L < 0$), ξ is increasing in ρ .

Lemma 1 suggests that the availability probability ξ increases in the demand correlation ρ . The reason is as follows. If $v_L \geq 2c$, then the critical fractile is greater than 1/2, meaning that the order quantity Q always covers at least half of the low-type demand. Thus the consumers believe $\xi = 1$ regardless of the ρ value. This resembles the situation studied in Cachon and Swinney (2009), where the θ value is such that a waiting consumer can always secure the product when there is a sale. If $v_L < 2c$, then the order quantity decreases in the updated demand variance based on Equation (1). Since a greater ρ reduces the variance of the updated demand, both the order quantity Q and its associated product availability increase in ρ .

Now we are ready to analyze the influence of ρ on the seller's total profit in the pre-order strategy. A higher ρ means a lower uncertainty about the low-type demand in the second period, which helps improve the second-period profit. However, more accurate demand information may reduce the first-period profit at the same time: Based on Lemma 1, a higher ρ may lead to greater availability probability in the second period, which prevents the seller from charging a high pre-order price (see Equation 3). Therefore, the seller's total profit may not necessarily increase with ρ . Define

$$\tilde{\mu}(\rho) \equiv -\frac{v_L\phi(z_L)\sigma_L}{2\Delta z_L\phi\left(2z_L\sqrt{1-\rho^2} + \lambda_L\right)} \quad (7)$$

with the following property:

Lemma 2 (i) For $z_L \in (-\lambda_L/2, 0)$, $\tilde{\mu}(\rho)$ increases in ρ .

(ii) For $z_L \leq -\lambda_L/2$, $\tilde{\mu}(\rho)$ is quasi-convex with a minimizer $\tilde{\rho} \equiv \sqrt{1 - (\lambda_L/(2z_L))^2}$.

The following result characterizes the seller's total profit in the pre-order strategy.

Proposition 1 (i) If $v_L \geq 2c$ ($z_L \geq 0$), Π^p always increases in ρ .

(ii) If $c < v_L < 2c$ ($z_L < 0$), Π^p increases in ρ when $\mu_H < \tilde{\mu}(\rho)$, and decreases in ρ when $\mu_H \geq \tilde{\mu}(\rho)$. In particular, if $v_L \rightarrow c$, Π^p always decreases in ρ .

Proposition 1 suggests that the seller's profit increases in ρ only when the low-type valuation is sufficiently large ($v_L \geq 2c$), or the high-type demand is relatively small ($\mu_H < \tilde{\mu}(\rho)$). The intuition is as follows: When v_L is large, a greater ρ improves the second-period profit, but does not affect the availability probability and pre-order price (Lemma 1). When μ_H is sufficiently low, the negative impact of ρ on the seller's pre-order profit is small, and it is dominated by the positive effect on the second-period profit. Thus more accurate advance information benefits the seller either when v_L is large or when μ_H is small.

Figure 1 illustrates the situations in which the profit may decrease in ρ . For $v_L < 2c$ ($z_L < 0$), it draws the curve $\tilde{\mu}(\rho)$. If v_L takes an intermediate value ($-\lambda_L/2 < z_L < 0$), $\tilde{\mu}(\rho)$ is increasing (Lemma 2(i)). Thus Π^p decreases in ρ only when ρ is relatively low (for $\mu_H \geq \tilde{\mu}(0)$). This case is shown in the left plot in Figure 1. When v_L is very small ($z_L \leq -\lambda_L/2$), $\tilde{\mu}(\rho)$ is quasi-convex (Lemma 2(ii)). Thus Π^p decreases in ρ when ρ is either intermediate (for $\mu_H \in (\tilde{\mu}(\tilde{\rho}), \tilde{\mu}(0))$) or small (for $\mu_H \geq \tilde{\mu}(0)$). This case is shown in the right plot in Figure 1.

The above analysis delivers a message that more accurate advance demand information does not necessarily improve a seller's profitability. This is in contrast with the traditional wisdom that early sales information (e.g., test sales, pre-orders) helps a firm better forecast demand and hence increase profit (see, for example, Fisher and Raman, 1996). The new driving force underlying our model is the counteracting effects of advance demand information and price discrimination: Due to strategic consumer behavior, a better ability to match supply with demand actually may prevent the firm from charging disparate prices to extract consumer surplus. Practically speaking, the above results imply that the pre-order strategy could be less attractive when pre-orders are highly predictive of the regular-season demand. In situations where the seller can invest to improve the accuracy of demand forecasts, this result suggests that such investment may hurt the seller's profit when facing strategic consumers.

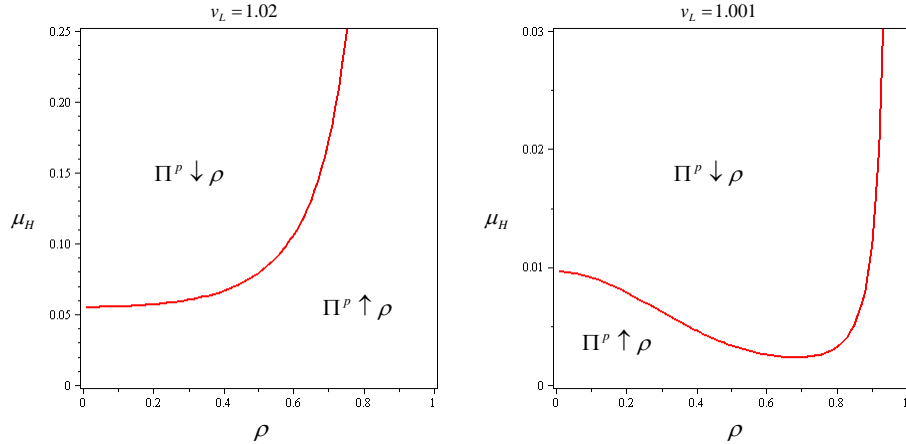


Figure 1: *Impact of ρ on the pre-order profit. The curve shown in each plot is $\tilde{\mu}(\rho)$. $\mu_L = 10$, $\lambda_H = 3$, $\lambda_L = 4.5$, $v_H = 2.3$, $c = 1$.*

4.2 Impact of Economic Parameters (v_H, v_L, c)

This subsection considers the impact of the economic parameters on the seller's performance. It is clear from Equation (5) that Π^P increases with v_H . Next we focus on the impact of v_L and c .

Proposition 2 (i) Π^P increases with v_L when v_L is sufficiently large, and decreases with v_L when v_L is sufficiently small.

(ii) There exists a \hat{v}_H such that Π^P increases in c if and only if $v_H > \hat{v}_H$. In addition, \hat{v}_H decreases in μ_H .

Proposition 2 states that the seller's profit may decrease in the low-type consumers' valuation v_L , or increase in the production cost c , which is counter-intuitive. Why does a higher consumer valuation lead to a lower profit for the seller, or a higher cost benefit the seller? To first explain the effect of v_L , notice that although increasing v_L improves the buyer's second-period profit, its impact on the pre-order price, $p_1 = v_H - \Delta\xi$, is ambiguous (recall $\Delta = v_H - v_L$). On one hand, it leads to a higher price in the second period, making the high-type consumers willing to pay more (with Δ smaller) in the first period. On the other hand, it induces the seller to order more in the second period, which may improve the second-period availability probability ξ and thus reduce the pre-order price. Specifically, when v_L is large, increasing v_L benefits the seller because its impact on the first-period price is negligible (with both Δ small and ξ insensitive to v_L). When v_L is small, the impact of v_L

on the second-period profit is small, but increasing v_L significantly increases the availability probability, thus reducing the pre-order price. In this case, increasing v_L hurts the seller's profit.

The reason a higher cost may benefit the seller is similar: Since a higher cost results in a lower quantity in the second period, it lowers the availability probability, leading to a higher pre-order price. Such an effect on the pre-order price is more significant if v_H is large. In addition, it has a greater impact on the first-period profit if the mean pre-order demand, μ_H , is large. Thus, a higher c may benefit the seller if v_H or μ_H is large.

5. Pre-order versus No-pre-order

When should the seller offer the pre-order option? In this section, we compare the pre-order strategy with the no-pre-order strategy. With no-pre-order, the product is sold only in the regular selling season (i.e., after it is released). Due to a short product life cycle, the seller cannot change price over the regular season; the seller charges either a high price, v_H , or a low price, v_L , throughout the period. Therefore, under the no-pre-order strategy, the seller does not have advance demand information and cannot implement price discrimination anymore. It is clear that charging a price v_L is worse than the pre-order strategy. Hence we consider only the price equal to v_H . The seller's profit from no-pre-order is (we use superscript n for the no-pre-order strategy):

$$\Pi^n = (v_H - c) \mu_H - v_H \phi(z_H) \sigma_H = \mu_H (v_H - c - v_H \phi(z_H) \lambda_H^{-1}), \quad (8)$$

where $z_H = \Phi^{-1}\left(\frac{v_H - c}{v_H}\right)$. We can see that Π^n is independent of ρ . Suppose $\lambda_H = \mu_H / \sigma_H$ is fixed, i.e., the high-type demand X_H changes proportionally with μ_H . Taking derivatives of Π^n and Π^p (Equation 5) with respect to μ_H gives

$$\begin{aligned} \frac{d}{d\mu_H} \Pi^n &= v_H - c - v_H \phi(z_H) \lambda_H^{-1}, \\ \frac{d}{d\mu_H} \Pi^p &= v_H - c - \Delta\xi. \end{aligned}$$

If $\Delta\xi \leq v_H \phi(z_H) \lambda_H^{-1}$, then $\frac{d}{d\mu_H} (\Pi^n - \Pi^p) \leq 0$, and we know $\Pi^n < \Pi^p$ (pre-order outperforms no-pre-order) always holds. Otherwise, if $\Delta\xi > v_H \phi(z_H) \lambda_H^{-1}$, then $\frac{d}{d\mu_H} (\Pi^n - \Pi^p) > 0$, and $\Pi^n < \Pi^p$ only when μ_H is sufficiently small. Note $\Pi^p = \Pi^n$ at $\mu_H = \frac{\Pi_L(0)}{\Delta\xi - v_H \phi(z_H) \lambda_H^{-1}}$, where $\Pi_L(0) = \mathbb{E}_X [\Pi_L(X)]$ is the expected second-period profit in the pre-order strategy. Thus,

define

$$\hat{\mu}_H^n = \begin{cases} \frac{\Pi_L(0)}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}} & \text{if } \Delta\xi > v_H\phi(z_H)\lambda_H^{-1} \\ +\infty & \text{otherwise} \end{cases}. \quad (9)$$

Then we have the following result.

Proposition 3 *Suppose λ_H is fixed.*

- (i) $\Pi^n < \Pi^p$ if and only if $\mu_H < \hat{\mu}_H^n$.
- (ii) There exists $\kappa \in (-\lambda_L/2, 0)$ such that $\hat{\mu}_H^n$ increases in ρ if $z_L > \kappa$, and $\hat{\mu}_H^n$ is quasi-convex in ρ (first decreasing and then increasing in ρ) otherwise.

Proposition 3(i) states that the pre-order strategy should be used if and only if the size of the high-type demand is less than a threshold value. A couple of points are worth mentioning about this threshold result: First, we find that a similar result holds when σ_H , rather than λ_H , is fixed. However, assuming σ_H constant with a varying μ_H seems less realistic, so we omit this result to save space. Second, the threshold $\hat{\mu}_H^n$ depends on the relative sizes of the two consumer segments. Analogously, it can be shown that for a fixed λ_L , there is a threshold $\hat{\mu}_L^n$ such that the no-pre-order strategy is preferred if and only if $\mu_L < \hat{\mu}_L^n$. This is the counterpart of Proposition 3 and therefore is also omitted.

Proposition 3(ii) explains the impact of ρ on the comparison of the two strategies. Specifically, when v_L is relatively high (i.e., $z_L > \kappa$), pre-order is increasingly preferred to no-pre-order as the demand correlation rises. However, when v_L is low (i.e., $z_L \leq \kappa$), then pre-order is preferred not only for large ρ , but also for small ρ ; only intermediate ρ favors no-pre-order. This result is because, when v_L is low and also ρ is small, the positive effect of a higher ρ on the second-period profit is insignificant and dominated by the negative effect of ρ on the first-period profit.

When the seller can optimally choose between the pre-order and no-pre-order strategies, how does his profit $\Pi = \max(\Pi^p, \Pi^n)$ change with ρ ? Since $\hat{\mu}_H^n$ determines the choice between the two strategies, and $\tilde{\mu}(\rho)$ determines the influence of ρ on Π^p , we know that $\hat{\mu}_H^n$ and $\tilde{\mu}(\rho)$ jointly define the behavior of Π .

Proposition 4 *If $\mu_H > \hat{\mu}_H^n$, then Π is independent of ρ . Otherwise, if $\mu_H \leq \hat{\mu}_H^n$, then:*

- (i) For $z_L \geq \kappa$, Π is increasing in ρ (κ is given in Proposition 3(ii));
- (ii) For $z_L < \kappa$, Π is increasing (decreasing) in ρ when $\mu_H < \tilde{\mu}(\rho)$ ($\mu_H \geq \tilde{\mu}(\rho)$).

The seller's optimal choice and the influence of ρ on the optimal profit is illustrated in Figure 2. It draws the two curves, $\hat{\mu}_H^n$ and $\tilde{\mu}(\rho)$ (the curve with a dotted segment). Recall from Proposition 1 that Π^p is always increasing in ρ if $z_L \geq 0$. Now with the option of no-pre-order, the seller's profit Π is increasing in ρ as long as $z_L \geq \kappa$ (note $\kappa < 0$). Thus, the no-pre-order strategy is adopted in a region where more advance demand information would hurt the first-period profit from pre-orders. While the inclusion of no-pre-order mitigates the negative effect of ρ , it may not completely eliminate it. Only when v_L is sufficiently large (i.e., $z_L \geq \kappa$), we will have the result that Π always increases in ρ , as shown in the first plot of Figure 2. If v_L is small (i.e., $z_L < \kappa$), then including the no-pre-order option only reduces (but does not remove) the whole region in which more advance demand information hurts the seller's profit. This is shown in the last two plots in Figure 2. Between these two plots, the left one shows an example in which the seller's profit Π strictly decreases in ρ only when ρ is relatively small; while the right plot shows an example in which Π strictly decreases in ρ when ρ is either small (for μ_H relatively high) or intermediate (for μ_H relatively low).

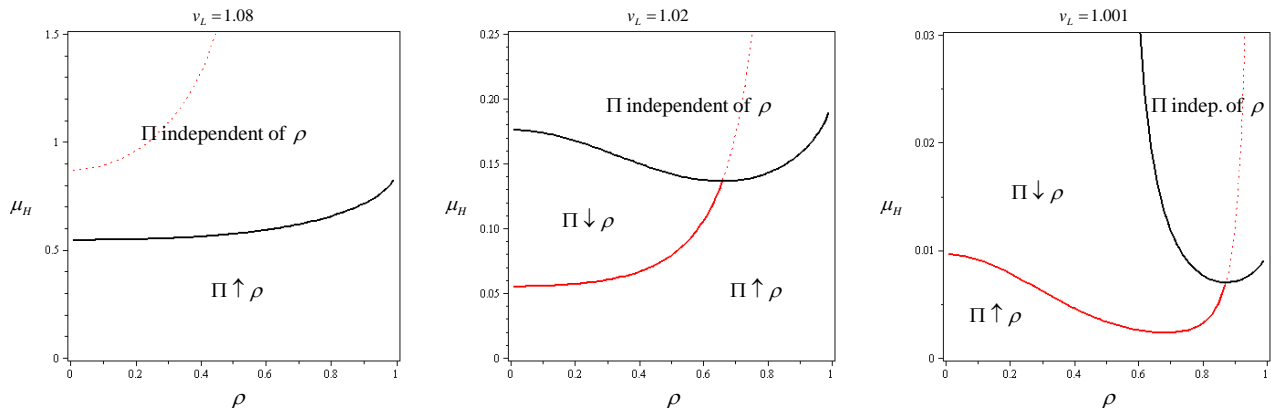


Figure 2: *Impact of ρ on $\Pi = \max(\Pi^p, \Pi^n)$. The curve with a dotted segment is $\tilde{\mu}(\rho)$. The other curve is $\hat{\mu}_H^n$. The parameter values follow those for Figure 1.*

6. Pre-order with Price Guarantee

Price guarantee has become a common industry practice during the past decade (see Lai et al., 2009 for more discussion about price guarantees). Under a price guarantee, a consumer will be compensated if the product price declines over time. It is intended to eliminate consumers' incentives to wait for markdowns so the seller can enjoy quicker sales at a relatively

high price. Generally, there should be some technological requirements for implementing a price guarantee, and it might be costly to satisfy these requirements (e.g., the seller has to invest in hardware and manpower to monitor transactions and manage the refunding process). For simplicity, we assume that all necessary technologies are already in place and there is a zero implementation cost. Under this assumption, in this section we study the impact of the price guarantee on the seller's pre-order strategy. We aim to answer the following two questions: First, when should a seller offer a price guarantee along with the pre-order option? Second, how does the introduction of the price guarantee affect the value of advance demand information?

6.1 Value of Price Guarantee

In a price guarantee mechanism, the seller needs to determine not only the prices p_1 and p_2 in the two periods, but also the refund η paid to each pre-order consumer if the price drops from p_1 to p_2 in the second period. An intuitive special case is $\eta = p_1 - p_2$, which we call a full-price guarantee. Most of the existing literature focuses on this special case (see, for example, Png, 1991 and Lai et al., 2009). Our analysis is more general because we do not impose any restriction on η . Similarly as for the pre-order mechanism without a price guarantee, we first analyze the seller's decisions in the second period, assuming all high-type consumers have chosen to pre-order. Then we analyze the seller's decisions in the first period that induce consumers to pre-order under a rational expectations equilibrium. As a high-type consumer will pre-order for any price below v_L , it is clear that the pre-order price should be greater than v_L .

At the start of the second period, the seller needs to decide whether to lower the price to $p_2 = v_L$ to serve the low-type consumers. This decision depends on the realization of the high-type demand in the first period, x_H , or equivalently, $x = \frac{x_H - \mu_H}{\sigma_H}$, which is observed in the pre-order sales. In addition, it depends on the promised refund η . The price will be reduced if and only if the profit from the low-type consumers, $\Pi_L(x)$, is greater than the total refund, $\eta(\mu_H + \sigma_H x)$:

$$\Pi_L(x) > \eta(\mu_H + \sigma_H x), \quad (10)$$

where $\Pi_L(x)$ is given in (2). Define

$$\hat{x}(\eta) \equiv \frac{\Pi_L(0) - \eta\mu_H}{\eta\sigma_H - (v_L - c)\rho\sigma_L}. \quad (11)$$

Then the condition (10) reduces to $x < \hat{x}$ if $\eta \geq (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$, or $x > \hat{x}$ if $\eta < (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$. The impact of a higher x on the seller's price-reduction decision is two-fold: On one hand, it implies a greater amount of total refund if the price is reduced; on the other hand, it means a higher low-type demand and thus a greater second-period profit. The first effect dominates when η is large, while the second effect dominates when η is small. Therefore, depending on the magnitude of the refund η , the seller will reduce the price when the high-type demand is either lower than or higher than a threshold. The next lemma proves a stronger version of this observation.

Lemma 3 (i) *If $\eta < (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$, then $\hat{x}(\eta) < -\lambda_H$, i.e., price reduction will always happen.*

(ii) *If $\eta \geq (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$, then price reduction happens when $x < \hat{x}(\eta)$, and $\hat{x}(\eta)$ decreases in η .*

Lemma 3(i) shows that if η is sufficiently small (i.e., $\eta < (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$), then price reduction will always happen (recall $\Phi(-\lambda_H) = 0$). This is equivalent to the original pre-order strategy without a price guarantee. Now consider the case $\eta \geq (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$, where price reduction will occur if and only if $x < \hat{x}(\eta)$. Lemma 3(ii) states that $\hat{x}(\eta)$ is a decreasing function, so the probability of price reduction becomes lower when η increases. In the limit, from Equation (11) $\hat{x}(\eta) \rightarrow -\lambda_H$ corresponds to $\eta \rightarrow \infty$, i.e., if the refund is extremely large, then price reduction will never happen. This is equivalent to selling only to the high-type consumers through pre-orders. On the other end, again from Equation (11), $\hat{x}(\eta) \geq \lambda_H$ implies $\Pi_L(0) + (v_L - c) \rho \sigma_L \lambda_H \geq \eta (\mu_H + \sigma_H \lambda_H)$, i.e., η is sufficiently small ($\eta \leq \frac{\Pi_L(\lambda_H)}{2\mu_H}$). This is the case when price reduction will always happen, which corresponds to the original pre-order strategy without a price guarantee.

Next we analyze the seller's first-period decision on the pre-order price p_1 and refund η . Note that the price reduction threshold, $\hat{x}(\eta)$, depends on η but not on p_1 . If a high-type consumer pre-orders, her expected utility is $u_1(p_1, \eta) = v_H - p_1 + \eta \Phi(\hat{x}(\eta))$; if she waits till the second period, her expected utility is $u_2(p_1, \eta) = \Delta \hat{\xi}(\hat{x}(\eta))$, where

$$\hat{\xi}(\hat{x}) \equiv \Pr(X_L/2 < Q(x), x < \hat{x}) = \mathbb{E} \left[\Phi \left(\frac{\lambda_L + \rho X}{\sqrt{1 - \rho^2}} + 2z_L \right) \middle| X \leq \hat{x} \right] \Phi(\hat{x}) \quad (12)$$

is the probability that the product is available at the low price ($p_2 = v_L$) in the second period. Given η , the optimal p_1 must satisfy $u_1(p_1, \eta) = u_2(p_1, \eta)$ so that the high-type

consumers will pre-order. This gives

$$p_1(\eta) = v_H + \eta\Phi(\hat{x}(\eta)) - \Delta\hat{\xi}(\hat{x}(\eta)). \quad (13)$$

Since there is a one-to-one relationship between \hat{x} and η , we will work with the decision variable \hat{x} instead of η , which is more convenient. For any given \hat{x} , we have $\eta(\hat{x}) = \Pi_L(\hat{x})/(\mu_H + \sigma_H\hat{x})$ and p_1 given by Equation (13). Thus the seller's profit can be written as (we use the superscript g for price guarantee):

$$\begin{aligned} \Pi^g(\hat{x}) &= (p_1(\hat{x}) - c)\mu_H + \mathbb{E}[\Pi_L(X) - \eta(\hat{x})(\mu_H + \sigma_H X) | X < \hat{x}]\Phi(\hat{x}) \\ &= \left(v_H - \Delta\hat{\xi}(\hat{x}) - c\right)\mu_H + \Pi_L(0)\Phi(\hat{x}) + ((v_L - c)\rho\sigma_L - \eta(\hat{x})\sigma_H)\mathbb{E}[X | X < \hat{x}]\Phi(\hat{x}) \\ &= \left(v_H - \Delta\hat{\xi}(\hat{x}) - c\right)\mu_H + \Pi_L(0)\Phi(\hat{x}) + \frac{\Pi_L(-\lambda_H)}{\hat{x} + \lambda_H}\phi(\hat{x}). \end{aligned} \quad (14)$$

We may compare this profit function to Π^p given in (5). The first term in (14) is similar to that in (5), where $v_H - \Delta\hat{\xi}(\hat{x}) = p_1(\hat{x}) - \eta(\hat{x})\Phi(\hat{x})$ is the effective pre-order price after taking the expected refund $\eta(\hat{x})\Phi(\hat{x})$ into account. The second term in (14) is also similar to that in (5), where $\Pi_L(0)$ is the expected profit from the low-type demand. It is multiplied by $\Phi(\hat{x})$ because the product will be sold to low-type consumers (with the price reduced to v_L) only if $X < \hat{x}$. However, since the refund is paid to the high-type consumers only when the high-type demand is relatively small ($X < \hat{x}$), the mean of the high-type demand for which the refund is executed is lower than μ_H . In addition, $\Pi_L(0)$ contains the updated variance but not the updated mean of the low-type demand—due to positive correlation between the high-type and low-type demands, the mean of the low-type demand is also lower than μ_L when price reduction occurs. The third term in (5) modifies the profits by incorporating such changes of the demand means caused by the price reduction condition: For any realized $x = \frac{x_H - \mu_H}{\sigma_H}$, the mean of the high-type demand shifts from μ_H by $\sigma_H x$, and the mean of the low-type demand deviates from μ_L by $\rho\sigma_L x$. Thus, the third term can be written as $((v_L - c)\rho\sigma_L - \eta(\hat{x})\sigma_H)\mathbb{E}[X | X < \hat{x}]\Phi(\hat{x})$, or $\frac{\Pi_L(-\lambda_H)}{\hat{x} + \lambda_H}\phi(\hat{x})$ after simplification.

Let \hat{x}^* denote the seller's optimal (profit-maximizing) \hat{x} . For concision, we use $\Pi^g = \Pi^g(\hat{x}^*)$ to denote the seller's optimal profit. Note, when $\hat{x} = \lambda_H$, we have $\hat{\xi}(\lambda_H) = \xi$, and thus $\Pi^g(\lambda_H) = \Pi^p$; that is, pre-order without a price guarantee is a special case of the price guarantee mechanism in which the price reduction threshold \hat{x} is very large so that price reduction will always occur.

Proposition 5 *Suppose λ_H is fixed. Then there exists a $\hat{\mu}_H^g$ such that the seller should offer a price guarantee in the pre-order strategy if and only if $\mu_H > \hat{\mu}_H^g$. In addition, $\hat{\mu}_H^g < \hat{\mu}_H^n$.*

Proposition 5 suggests that a price guarantee is useful only when μ_H is above a threshold value, $\hat{\mu}_H^g$. In other words, the size of the high-type consumer segment must be large enough to warrant the use of price guarantees. This is because a price guarantee essentially enables the seller to charge a relatively high pre-order price by removing consumers' incentives for waiting; thus it is most effective when the size of the high-type consumers is sufficiently high. Otherwise the seller can simply offer pre-order without a price guarantee.

Combining Propositions 3 and 5 gives us the whole picture about the seller's optimal strategy choice. First, if price guarantees are not available (e.g., due to technological constraints), then a seller should use the no-pre-order strategy if and only if the size of the high-type consumer segment is greater than a threshold $\hat{\mu}_H^n$ (Proposition 3). Second, if price guarantees are available, then it is clear that the pre-order strategy with a price guarantee always dominates the no-pre-order strategy. (To see this, imagine that the seller sets $p_1 = v_H$ and promises an extremely large refund so that price reduction will never happen. Then the seller sells only to the high-type consumers at a price v_H in pre-orders.) Thus, the seller only needs to decide whether a price guarantee should be offered together with the pre-order option. Proposition 5 shows that a price guarantee should be offered if and only if $\mu_H > \hat{\mu}_H^g$. In summary, the seller's optimal strategy choice depends on two thresholds $\hat{\mu}_H^g$ and $\hat{\mu}_H^n$ (with $\hat{\mu}_H^g < \hat{\mu}_H^n$), as illustrated in Figure 3. In the next subsection, we investigate the impact of ρ on the seller's profit under price guarantees and the threshold $\hat{\mu}_H^g$.

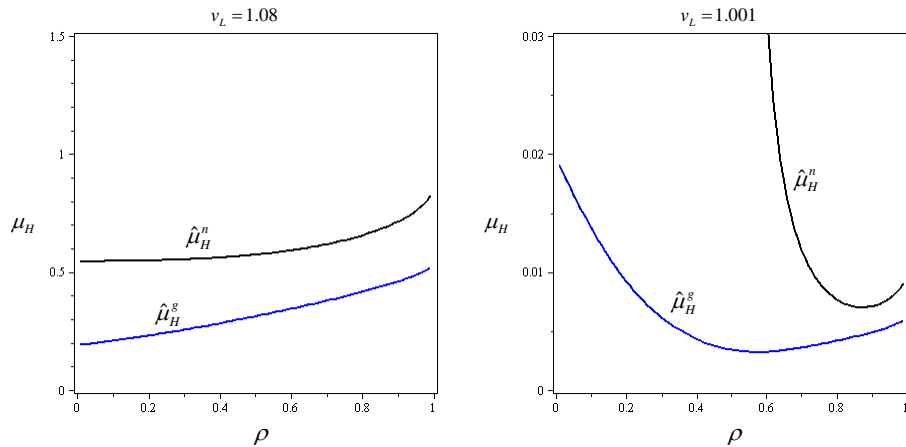


Figure 3: Illustration of $\hat{\mu}_H^g$ and $\hat{\mu}_H^n$. The parameter values follow those for Figure 1.

6.2 Impact of ρ under Price Guarantee

We have shown in Section 4.1 that a higher ρ may decrease the seller's profit in the pre-order strategy. The underlying reason is that a higher ρ means a higher availability of the product in the second period, and thus the seller has to charge a lower pre-order price. How does this result change when price guarantees are used? Here we investigate the impact of ρ under price guarantee. Based on Equation (14), taking derivative of the seller's optimal profit Π^g with respect to ρ yields

$$\frac{d\Pi^g}{d\rho} = -\Delta\mu_H \frac{d}{d\rho} \hat{\xi}(\hat{x}^*) + \Phi(\hat{x}^*) \frac{d}{d\rho} \Pi_L(0) + \left(\frac{\phi(\hat{x}^*)}{\lambda_H + \hat{x}^*} \right) \frac{d}{d\rho} \Pi_L(-\lambda_H). \quad (15)$$

Equation (15) can be compared with Equation (6) for the impact of ρ in the pre-order mechanisms with and without a price guarantee. With a price guarantee, the impact of ρ is three-fold and corresponds to the three terms in (15). First, as shown by the first term, it affects the pre-order price through its influence on $\hat{\xi}(\hat{x}^*)$, the probability that the product is available at a lower price in the second period. Recall that a similar effect exists in the pre-order mechanism without a price guarantee, where this effect is always negative because ξ is increasing in ρ (Lemma 1). With a price guarantee, the influence of ρ on the product availability probability applies only when price reduction happens. Thus, the price guarantee mitigates such a negative effect of ρ on the pre-order price. In fact, a higher ρ even *increases* the pre-order price when ρ is small: Since the price is reduced only when the high-type demand is lower than a threshold ($x < \hat{x}^*$), the low-type demand tends to be low as well when price reduction occurs. Thus a higher ρ predicts an even smaller *mean* of the low-type demand in the case of price reduction. This leads to a smaller order quantity for the second period and hence reduces the product availability probability. Although a higher ρ also decreases the *variance* of the low-type demand, which helps improve the product availability, this effect on the variance is negligible and is dominated by the effect on the mean when ρ is small. Therefore, for small ρ , $\hat{\xi}$ is decreasing, and accordingly, the pre-order price is increasing, in ρ . Lemma 4 formally characterizes the impact of ρ on $\hat{\xi}$.

Lemma 4 (i) For $v_L \geq 2c$ (i.e., $z_L \geq 0$) and a given \hat{x} , $\hat{\xi}(\hat{x})$ is independent of ρ .

(ii) For $c < v_L < 2c$ (i.e., $z_L < 0$) and a given \hat{x} , $\hat{\xi}(\hat{x})$ is decreasing in ρ at $\rho = 0$, and increasing in ρ at $\rho = 1$.

Second, as shown by the second term in (15), the demand correlation ρ increases $\Pi_L(0)$, the expected profit from the low-type consumers. Such a positive effect also exists in the

mechanism without a price guarantee. With a price guarantee, this positive effect is discounted by the probability of price reduction $\Phi(\hat{x}^*)$.

Third, as shown by the last term in (15), the price guarantee introduces a new effect of ρ that does not exist in the mechanism without a price guarantee: As explained before, a higher ρ reduces the mean of the low-type demand under price reduction, which hurts the second-period profit. Note this new negative effect of ρ is associated with the second-period profit (through its impact on the low-type demand); while the former negative effect of ρ is associated with the first-period profit (through its impact on the pre-order price).

In order to better understand the impact of ρ on the seller's total profit, we focus on two special cases: the case when v_L is large (greater than $2c$) and the case when v_L is small (close to c). When $v_L \geq 2c$, in both mechanisms with and without a price guarantee, the effect of ρ on the pre-order price is negligible as the belief of product availability (when price reduction occurs) in the second period is nearly one. So the effect of ρ on the second-period profit dominates. When $v_L \rightarrow c$, the second-period profit is negligible, and the effect of ρ focuses on the pre-order price in both mechanisms.

Proposition 6 *When $v_L \geq 2c$ (i.e., $z_L \geq 0$),*

- (i) Π^g is a quasi-convex (first decreasing and then increasing) in ρ ;
- (ii) $\hat{\mu}_H^g$ is increasing in ρ .

Proposition 6(i) can be compared with Proposition 1(i) for the impact of ρ on the seller's profit with and without a price guarantee when $v_L \geq 2c$. Proposition 1(i) demonstrates that the seller's profit without a price guarantee, Π^p , is always increasing in ρ . Interestingly, Proposition 6(i) shows that, with the optimal price guarantee, the seller's profit may be *decreasing* in ρ for sufficiently small ρ . This result is due to the negative effect of ρ on the second-period demand introduced by the price guarantee (i.e., the third term in (15)). Thus, with $v_L \geq 2c$ and small ρ , increasing ρ may reduce the total profit in the price guarantee mechanism.

Recall from Proposition 5 that the seller prefers to use a price guarantee when $\mu_H > \hat{\mu}_H^g$. Proposition 6(ii) states that, for $v_L \geq 2c$, the seller is less likely to use a price guarantee as ρ increases. This is again due to the explanation above: When v_L is relatively large, the adverse effect of a higher ρ on the second-period profit dominates, making price guarantees less attractive. Such a case with $\hat{\mu}_H^g$ increasing in ρ is illustrated in the left plot of Figure 3.

Proposition 7 *When $v_L \rightarrow c$,*

- i) Π^g is increasing in ρ at $\rho = 0$, and decreasing in ρ at $\rho = 1$;*
- ii) $\hat{\mu}_H^g$ is decreasing in ρ at $\rho = 0$, and is increasing in ρ at $\rho = 1$.*

Proposition 7(i) indicates that, when v_L is very small, the seller's profit under a price guarantee is increasing in ρ for ρ close to 0, and decreasing in ρ for ρ close to 1. Through numerical experiments, we observe that Π^g is a quasi-concave function of ρ on $[0, 1]$. Recall from Proposition 1(ii) that Π^p always decreases in ρ when $v_L \rightarrow c$. This difference results from the fact that the price guarantee mitigates the negative effect of ρ on the pre-order price, and furthermore, it may even reverse the direction, turning the negative effect to a positive one when ρ is small.

From Proposition 7(ii), we know that as ρ increases, $\hat{\mu}_H^g$ decreases at ρ close to 0, and increases at ρ close to 1. Again, numerical experiments show that $\hat{\mu}_H^g$ is quasi-convex in ρ . Thus, when v_L is very small, price guarantee is favored only for intermediate ρ ; when ρ is either high or low, pre-order without a price guarantee is better. Such a case with $\hat{\mu}_H^g$ quasi-convex in ρ is illustrated in the right plot of Figure 3.

6.3 Full-price Guarantee

We have analyzed general price guarantees without any constraint on the refund amount η . In reality, it is common that the refund is set exactly equal to the price drop, i.e., the early purchasers are sure to receive the lowest price in the selling season. We call this a full-price guarantee. Clearly, it is more intuitive and probably is perceived as a fair policy from the consumers' perspective. Thus we also study this special price guarantee. The seller charges a pre-order price p_1 in the first period, and promises to refund the price difference $\eta = p_1 - v_L$ if the price drops to v_L in the second period.

We first analyze a benchmark full-price guarantee mechanism with $p_1 = v_H$. In this mechanism, the promised refund is simply $\eta = \Delta$, and the price will be reduced if and only if $\Pi_L(x) > \Delta(\mu_H + \sigma_H x)$ (recall $x = \frac{x_H - \mu_H}{\sigma_H}$ measures the high-type demand realization). Let \hat{x}^{bg} denote the price reduction threshold and Π^{bg} be the corresponding seller profit (we use superscript *bg* for benchmark guarantee). For a high-type consumer, the expected utility from pre-order is $u_1 = \Delta\Phi(\hat{x}^{bg})$, and the expected utility from waiting is $u_2 = \Delta\hat{\xi}(\hat{x}^{bg})$. Since $\hat{\xi}(\hat{x}^{bg}) < \Phi(\hat{x}^{bg})$ from Equation (12), there is always $u_1 > u_2$; in other words, the high-type consumers will always pre-order. Therefore, under a full-price guarantee, the seller

is able to induce pre-orders by fixing the pre-order price at $p_1 = v_H$. Since the pre-order price is independent of the second-period product availability, the negative effect of a greater ρ on the first-period profit does not exist any more.

Proposition 8 *In the benchmark price guarantee with $p_1 = v_H$ and $\eta = \Delta$, the seller's profit is quasi-convex (first decreasing and then increasing) in ρ .*

Proposition 8 states that the seller's profit may decrease in ρ even *without* the conflict between advance demand information and the pre-order price. This confirms the robustness of the message delivered by Proposition 6: There is an additional negative effect of ρ introduced by a price guarantee on the second-period profit (which is different from the negative effect on the pre-order price).

While having $p_1 = v_H$ is intuitive, it is not optimal for a full-price guarantee. To see this, recall $u_1 > u_2$ under $p_1 = v_H$. Thus, the seller can raise p_1 above v_H to improve her profit while still inducing pre-orders from all high-type consumers.³ Next we analyze the full-price guarantee with an optimal p_1 . Similar to general price guarantees, a full-price guarantee can also be characterized by a threshold \hat{x} such that the price will be reduced if and only if $x = \frac{x_H - \mu_H}{\sigma_H} \leq \hat{x}$. The one-to-one relationship between η and \hat{x} is governed by $\eta(\hat{x}) = \Pi_L(\hat{x}) / (\mu_H + \sigma_H \hat{x})$. To induce pre-orders, the optimal threshold \hat{x} must be such that the utility from pre-order, $u_1 = v_H - p_1 + \eta(\hat{x}) \Phi(\hat{x})$, is equal to the utility from waiting, $u_2 = \eta(\hat{x}) \hat{\xi}(\hat{x})$. This gives

$$p_1 = v_H + \eta(\hat{x}) \left[\Phi(\hat{x}) - \hat{\xi}(\hat{x}) \right] > v_H, \quad (16)$$

where the inequality follows from $\Phi(\hat{x}) > \hat{\xi}(\hat{x})$ ($\Phi(\hat{x})$ is the probability of a price reduction; $\hat{\xi}(\hat{x})$ is the probability that price is reduced *and* the product is available). In addition, the seller's profit under a full-price guarantee can be written as (we use superscript *fg* for full guarantee):

$$\begin{aligned} \Pi^{fg}(\hat{x}) &= (p_1 - c) \mu_H + \mathbb{E}[\Pi_L(X) - \eta(\hat{x}) (\mu_H + \sigma_H X) | X < \hat{x}] \Phi(\hat{x}) \\ &= (v_L + \eta(\hat{x}) (1 - \Phi(\hat{x})) - c) \mu_H + \Pi_L(0) \Phi(\hat{x}) + \frac{\Pi_L(-\lambda_H)}{\hat{x} + \lambda_H} \phi(\hat{x}), \end{aligned} \quad (17)$$

where the second inequality is by using $p_1 = \eta(\hat{x}) + v_L$. The next result is similar to Proposition 5 for general price guarantees.

³The same logic applies to Png (1991) and Lai et al. (2009), who study full-price guarantees in similar settings: There are two consumer segments with deterministic valuations, v_H and v_L , respectively; the seller can set prices in a two-period selling season. Both papers restrict their attention to $p_1 \leq v_H$ (see the 3rd paragraph in page 1016 of Png, 1991 and Proposition 7 in Lai et al., 2009), which is suboptimal.

Proposition 9 *Suppose λ_H is fixed. Then there exists a $\hat{\mu}_H^{fg}$ such that the seller should offer a full-price guarantee under the pre-order strategy if and only if $\mu_H > \hat{\mu}_H^{fg}$.*

The format of full-price guarantees is commonly used in practice despite its sub-optimality. This gives rise to an interesting question: How do full-price guarantees perform compared to the truly optimal price guarantee? We have conducted an extensive numerical study to derive some insights into this question. This numerical study is designed to cover a wide range of possible situations. Specifically, we fix $c = 1$, $\mu_L = 10$ (these two numbers serve as the anchors of the economic and demand parameters, respectively), and vary all the other parameters. Table 1 summarizes the parameter settings.

c :	1
v_H :	2, 3, 4
v_L :	$c + a(v_H - c)$, a varying from 0 to 1 with an increment of 0.1
μ_L :	10
μ_H :	$b\mu_L$, b varying from 0 to 2 with an increment of 0.2
λ_H :	3, 3.5, 4, 4.5, 5
λ_L :	$1.5\lambda_H, 1.75\lambda_H, 2\lambda_H$ (recall $\lambda_L \geq \sqrt{2}\lambda_H$)
ρ :	0 to 1 at an increment of 0.1

Table 1: *Parameter values for the numerical study.*

We find that the two full-price guarantees (with $p_1 = v_H$ and the optimal p_1) are very close in their performance: The minimum of the ratio (Π^{bg}/Π^{fg}) is 97.5% among all tested scenarios. Thus the seller can simply set $p_1 = v_H$ if he wishes to use a full-price guarantee; using the optimal p_1 does not improve the profit much. The numerical study also indicates that both full-price guarantees perform quite well relative to the optimal price guarantee, especially when λ_H and λ_L are large. Table 2 records the minimum and 90 percentile (in the brackets) of the ratio (Π^{bg}/Π^g) for different λ_H and λ_L values (the result for Π^{fg} is very similar and therefore omitted). Each row (column) corresponds to a value of λ_H (λ_L). We can see that the full-price guarantee with $p_1 = v_H$ captures at least 83% of the optimal profit (or 99.5% of the optimal profit for 90 percent of the scenarios), and its performance is much better if λ_H or λ_L is large. Note that a large λ_i ($i = H, L$) means a lower demand variability. So the numerical study suggests that a simple full-price guarantee with $p_1 = v_H$ performs reasonably well as long as the demand variability is not very large for both demand types.

$\lambda_H \backslash \lambda_L$	$1.5\lambda_H$	$1.75\lambda_H$	$2\lambda_H$
3	0.829 (0.998)	0.848 (0.997)	0.938 (0.997)
3.5	0.846 (0.999)	0.955 (0.999)	0.988 (0.999)
4	0.949 (1.000)	0.992 (1.000)	0.998 (0.999)
4.5	0.992 (1.000)	0.998 (1.000)	0.998 (1.000)
5	0.999 (1.000)	0.999 (1.000)	0.999 (1.000)

Table 2: *The minimum (and the 90th percentile) of the ratio (Π^{bg}/Π^g) for different λ_H and λ_L values.*

7. Discussion

The basic model studied in the previous sections is stylized and has some limitations, though it enables us to maintain tractability and reveal the key driving forces underlying the main results. This section relaxes some of the assumptions and discusses their implications to the analysis and results.

7.1 Limited Capacity

In the basic model, the seller has an unconstrained supply of the product when making the ordering decision. This assumption may not hold under certain situations - for instance, if the seller is a manufacturer who has a limited production capacity or a retailer who receives a quota (a cap on the order quantity) from his supplier. In both cases, there is a limit on the quantity the seller can order. Below we briefly discuss how such a limit affects our previous results.

First we consider its impact on the comparison between pre-order and no-pre-order. With an unlimited capacity, the seller can fully cover the pre-order demand and then order additional quantity for the second-period demand. Thus the product availability in the second period is independent of μ_H . In that case, we know that increasing μ_H makes no-pre-order more favorable than pre-order. Now with a limited capacity, the second-period availability decreases in μ_H because a higher pre-order demand means less capacity left for the second period. This effect leads to a higher first-period price and favors the pre-order strategy, which does not exist in the unlimited capacity case. Therefore, a more constrained capacity will make the pre-order strategy more preferable. In other words, the threshold $\hat{\mu}_H^n$ will shift downward as the seller's capacity is more constrained. This has been confirmed in extensive numerical experiments. Additionally, we also find examples in which the seller's

profit increases as the capacity constraint becomes tighter. Again the reason is that a tight supply drives the first-period price up and hence is beneficial to the seller.

Second, for the impact of ρ , we find that the negative effect of ρ in the pre-order strategy still exists but is weakened by the limited capacity. This is because the negative effect hinges on the product availability in the second period. Given a constrained capacity, the advance demand information has a smaller impact on the second-period availability (imagine a situation where the seller observes a strong pre-order signal but is unable to order the desired quantity for the second period due to the limited capacity).

7.2 Consumer Arrivals

It has been assumed in the basic model that all the high-type consumers arrive in the first period while all the low-type consumers arrive in the second period. This assumption can be relaxed by allowing mixed arrivals in each period (all else remains the same as before): An α (β) fraction of low-type (high-type) consumers arrive in the first (second) period. Our basic model represents a special case with $\alpha = \beta = 0$. Now we consider positive values for these two fractions.

When $\alpha > 0$, i.e., some low-type consumers are present in the first period, and the seller may charge a pre-order price $p_1 = v_L$, which would not happen when $\alpha = 0$. This allows both types of customers to pre-order. Then pre-order is used only as a mechanism to obtain advance demand information, but not to achieve price discrimination. Such a strategy sacrifices the potential surplus the seller can extract from the high-type consumers. It would be optimal only under extreme conditions: The low-type consumer pool is very large (relative to the high-type consumer pool), the valuation difference $\Delta = v_H - v_L$ is very small, and α is sufficiently high; otherwise, charging $p_1 > v_L$ would still be optimal. With $p_1 > v_L$ (which seem to be reasonable for most practical situations), all the low-type customers, regardless of the arriving time, will purchase in the second period, thus both the analysis and results in the basic model should remain unchanged.

Analogously, some high-type consumers may arrive in the second period, i.e., $\beta > 0$. This may cause the seller to target these consumers by charging a high price $p_2 = v_H$ in the second period. Clearly, such a strategy abandons the low-type customers, and is optimal only if β is sufficiently high or Δ is large. Since the high-type refers to early adopters who follow new product trends closely, it is reasonable to assume that β is generally small in our pre-order setting. Then with $p_2 = v_L$, those high-type consumers who arrive in the second

period will join the low-type consumers to purchase in the second period, and again, the analysis and results should remain unchanged.

7.3 Negative Demand Correlation ($\rho < 0$)

We have focused on $\rho \geq 0$ in the basic model. This is appropriate when modeling new product introductions, of which the total market size is variable and each consumer segment expands with the total market. There may be situations where the total market size is relatively certain, but the portion of each consumer segment is variable. These situations can be captured by a negative demand correlation (see, for example, a model with $\rho = -1$ in Png, 1991). With a negative ρ , a small ρ (thus a large $|\rho|$) means more accurate advance information. A negative ρ does not change the analysis of the pre-order strategy without a price guarantee in Section 4. By replacing ρ with $-\rho$, it can be readily shown that all results in Section 4 apply to $\rho \in [-1, 0]$ as well. Next we discuss the pre-order strategy with a price guarantee.

Under a price guarantee, the seller will lower the price in the second period if and only if the realized high-type demand is less than a threshold. In contrast to the case of positive ρ , now $\rho < 0$ means that price reduction occurs when the low-type demand is relatively high. That is, as $|\rho|$ increases, the variance of the low-type demand under price reduction becomes smaller while the mean becomes stochastically larger. Therefore, under a negative demand correlation, the negative effect of advance information introduced by price guarantees on the second-period profit does not exist anymore.

While $\rho < 0$ removes the negative impact of increasing $|\rho|$ on the second-period profit, interestingly, it may aggravate the negative effect on the first-period profit. This is because, with a higher $|\rho|$, the low-type demand in case of price reduction tends to be larger, which motivates the seller to order more to satisfy demand in the regular season. This improves the product availability in the regular season and leads to a lower pre-order price. Thus, for $\rho < 0$, a higher $|\rho|$ will have a stronger adverse impact on the pre-order price under a price guarantee; as a result, the seller's optimal pre-order price always decreases in $|\rho|$.

To summarize, under a price guarantee and $\rho < 0$, a larger $|\rho|$ hurts the first-period profit (in a stronger way compared to $\rho \geq 0$) but improves the second-period profit. Specifically, we can show that Π^g increases in $|\rho|$ for $v_L \geq 2c$ (recall the effect of ρ on the first-period price is negligible in this case), and Π^g decreases in $|\rho|$ when $v_L \rightarrow c$ (recall the effect of ρ on the second-period profit is negligible in this case). Therefore, with $\rho < 0$, we still have the result

that more advance demand information may hurt a seller's profit under a price guarantee, although this result is solely caused by the conflict between advance demand information and price discrimination.

8. Conclusion

This paper studies the prevailing pre-order practice in which a seller accepts consumer orders before the release of a product. Consumers who are eager to obtain the product will benefit from pre-order because it guarantees immediate product availability on release. Pre-order is also beneficial to the seller because it allows the seller to gauge market demand from pre-order sales, and to charge distinct prices to different consumer segments. In this paper, we develop a modeling framework to analyze the pre-order strategy a seller may use to sell a perishable product in a short selling season. The market consists of two consumer segments, those who arrive in the pre-order season with valuation v_H and those in the regular selling season with valuation v_L , respectively. There is a correlation between these two random demand segments, which we use to measure the accuracy of advance demand information. To the best of our knowledge, this modeling framework is the first to simultaneously incorporate the following three important elements: seller's inventory and pricing decisions, consumers' forward-looking behavior, and advance demand information.

The value of advance demand information has been widely studied in the operations literature. Most studies emphasize the benefit of advance demand information, i.e., it helps improve a firm's inventory decision when facing uncertain market demand. However, we find that the seller's profit may *decrease* with the accuracy of advance demand information obtained from the pre-order sales. Specifically, the seller will benefit from more accurate advance demand information only if the low-type valuation is sufficiently large, or the high-type demand is relatively small. This result is due to a conflict between advance demand information and price discrimination: Accurate demand information increases product availability in the regular selling period, which hinders the seller from charging a high price to the high-type customers in the *pre-order season*.

An effective way of resolving such a conflict between advance demand information and price discrimination is to offer a price guarantee: The seller promises to compensate early purchasers in case the price is lowered later. Interestingly, we find that the seller's profit may still *decrease* in the accuracy of advance demand information, even when the seller

would only benefit from more advance demand information if the price guarantee were not offered. The explanation of this surprising result is as follows. Although price guarantees can mitigate (and in some situations may even reverse) the effect of the above conflict, they introduce another adverse effect of advance demand information: With price guarantees, a seller will lower the price to profit from low-type consumers only when the pre-order demand (from high-type consumers) is sufficiently low, but a low pre-order demand implies a weak low-type demand as well. As a result, the use of price guarantees excludes situations where the seller can enjoy a high profit *in the regular season* from low-type consumers, and such an adverse effect is stronger when the demand correlation is higher (which gives more accurate advance demand information).

Therefore, contrary to conventional wisdom, we have demonstrated that advance demand information can be detrimental to firms when facing forward-looking consumers: (1) It may hurt the seller's profit in the pre-order season through its negative effect on the pre-order price. (2) In the presence of price guarantees, it can also hurt the seller's profit in the regular season through its negative effect on the regular-season demand. Due to such negative effects of advance demand information, the seller needs to carefully decide whether to accept pre-orders, and whether to provide price guarantees with pre-orders. We find that the seller's strategy choice depends critically on the relative market sizes of the two types of consumers. To be specific, there are two thresholds $\hat{\mu}_H^n$ and $\hat{\mu}_H^p$ ($\hat{\mu}_H^n > \hat{\mu}_H^p$) that determine the seller's optimal strategy: First, when price guarantee is not available, the seller should not accept pre-orders (i.e., sell only to the high-type consumers in the regular season) if and only if the market size of the high-type consumers is greater than $\hat{\mu}_H^n$. Second, when price guarantee is available, the seller should always accept pre-orders. In addition, a price guarantee should be offered if and only if the market size of the high-type consumers is greater than $\hat{\mu}_H^p$. This simple threshold structure provides a useful guideline for practitioners when making pre-order strategy decisions.

This research can be extended in a couple of directions. First, a more sophisticated consumer valuation model can be introduced. For instance, consumers may have uncertain valuations that unfold over time. Moreover, consumers may update their valuations based on pre-orders (e.g., a highly sought-after product during the pre-order season provides a positive signal about the product value). Incorporating information updating for the consumers in addition to demand updating for the seller is an interesting direction for future research. Second, this paper focuses on a monopolist's selling strategy. A natural extension

is to consider the pre-order strategy in a duopoly setting. It would be interesting to study how competition affects the firms' strategy choice and the associated pricing and inventory decisions.

Acknowledgments

The authors are grateful to participants at the 2009 MSOM Conference in Boston, Massachusetts and the 2009 Informs Annual Meeting in San Diego, California for their helpful comments.

References

- Alexandrov, A., M.A. Lariviere. Are reservations recommended? Working paper, Kellogg School of Management, Northwestern University.
- Aviv, Y., A. Pazgal. 2008. Optimal pricing of seasonal products in the presence of forward-looking consumers. *M&SOM* **10**(3) 339-359.
- Cachon, G.P., R. Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* **55**(3) 497-511.
- Carnoy, D. 2009. Amazon drops price of Kindle 2 to \$299. <http://news.cnet.com>
- Chu, L., H. Zhang. 2009. Optimal pre-order discount and information release. Working paper, University of Southern California.
- Dana, J. 1998. Advance-purchase discounts and price discrimination in competitive markets. *J. of Political Economy* **106**(2) 395-422.
- Elmaghraby, W., P. Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. *Management Science* **49**(10) 1287-1309.
- Elmalghraby, W., A.Gulcu, P. Keskinocak. 2008. Designing optimal preannounced mark-downs in the presence of rational customers with multiunit demands. *M&SOM* **10**(1) 126-148.

- Eppen, G.D., A.V. Iyer. 1997. Improved fashion buying with Bayesian updates. *Operations Research* 45(6) 805-819.
- Falcon, J. 2009. Sony admits early adopter “tax” on PSP Go! <http://www.gamestooge.com/> June 30.
- Fisher, M., A. Raman. 1996. Reducing the cost of demand uncertainty through accurate response to early sales. *Operations Research* 44(1): 87-99.
- Goldstein, P. 2009. Nokia N900 goes on pre-order in U.S. <http://www.fiercewireless.com/> September 4.
- Huang, T., J.A. Van Mieghem. 2009. Are you willing to click? Advance demand information and price discrimination under strategic consumer behavior. Working paper, Kellogg School of Management, Northwestern University.
- Jack, S. 2007. iPhone early adopters: how do you feel about the \$399 iPhone? <http://www.macdailynews.com> September 5.
- Jerath, K., S. Netessine, S.K. Veeraraghavan. 2009. Revenue management with strategic customers: Last minute selling and opaque selling. *Management Science*, forthcoming.
- Keizer, G. 2009. AT&T, Best Buy sell out of iPhone 3G S pre-order supplier. *Computerworld* June 15.
- King, C. 2009. Price drop on Nokia N900 at Amazon.com. <http://www.pocketables.net/> September 18.
- Lai, G., L.G. Debo, K. Sycara. 2009. Buy now and match later: Impact of posterior price matching on profit with strategic consumers. *M&SOM*, forthcoming.
- Levin, Y., J. McGill, M. Nadiak. 2007. Price guarantees in dynamic pricing and revenue management. *Operations Research* 55(1) 75-97.
- Levin, Y., J. McGill, M. Nadiak. 2009. Dynamic pricing in the presence of strategic consumers and oligopolistic competition. *Management Science* 55(1) 32-46.
- Liu, Q., G. van Ryzin. 2008. Strategic capacity rationing induce early purchases. *Management Science* 54(6) 1115-1131.

- Macarthy, A. 2007. Retailers criticise HMV over PS3 pre-order deal.
http://www.gamersquad.com February 7.
- McCardle, K., K. Rajaram, and C. S. Tang. 2004. Advance booking discount programs under retail competition. *Management Science* **50**(5) 701-708.
- Martin, M. 2006. No Wii until 2007 without a pre-order, warns retail.
http://www.gamesindustry.biz/ July 12.
- Mersereau, A. J. and D. Zhang. Estimation, pricing, and strategic customer behavior. Working paper. 2009.
- Mindlin, A. 2005. Price war ensues for latest 'Harry Potter'. *New York Times* July 11.
- Moe, W., P.S. Fader. 2002. Using advance purchase orders to forecast new product sales. *Marketing Science* **21**(3) 347-364.
- Nair, H. 2007. Intertemporal price discrimination with forward-looking consumers: applications to the US market for console video-games. *Quant. Market. Econ.* **5** 239-292.
- Pham, A. 2001. GameCube sold faster than Xbox, Toysrus says. *Los Angeles Times* Oct. 9.
- Png, I. P. L. 1991. Most-favored-customer protection versus price discrimination over time. *J. of Political Economy* **99**(5) 1010-1028.
- Su, X. 2007. Inter-temporal pricing with strategic customer behavior. *Mgt. Sci.* **53**(5): 726-741.
- Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science* **54**(10) 1759-1773.
- Su, X., F. Zhang. 2009. On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science* **55**(5) 713-726.
- Swinney, R. 2009. Selling to Strategic Consumers When Product Value is Uncertain: The Value of Matching Supply and Demand. Working paper, Graduate School of Business, Stanford University.

- Talluri, K.T., G.J. van Ryzin. 2004. *The theory and practice of revenue management*. Springer, New York.
- Tang, C. S., K. Rajaram, A. Alptekinoglu, and J. Ou. 2004. The benefits of advance booking discount programs: Model and analysis. *Management Science* **50**(4) 465-478.
- Wall Street Journal. 2002. Apple breaks its record with 150,000 pre-orders of new iMac computer. January 29, page B6.
- Weng, Z. K. and M. Parlar. 1999. Integrating early sales with production decisions: analysis and insights. *IIE Transactions* **31**(11) 1051-1060.
- Xie, J. and S. M. Shugan. 2001. Electronic tickets, smart cards, and online prepayments: When and how to advance sell. *Marketing Science* **20**(3) 219-243.
- Yin, R., Y. Aviv, A. Pazgal, C.S. Tang. 2009. The implications of customer purchasing behavior and in-store display formats. *Management Science*, forthcoming.
- Yu, M., R. Kapuscinski, H.-S. Ahn. 2007. Advance selling with limited capacity and customers' dependent valuations. Working paper, University of Michigan.

Appendix

Proof of Lemma 1: This is based on

$$\begin{aligned}\frac{d\xi}{d\rho} &= (1 - \rho^2)^{-3/2} \mathbb{E} \left[(\rho\lambda_L + X) \phi \left(\frac{\lambda_L + \rho X}{\sqrt{1 - \rho^2}} + 2z_L \right) \right] \\ &= -\frac{2\rho z_L}{\sqrt{1 - \rho^2}} \phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right).\end{aligned}$$

When $z_L < 0$, $\frac{d\xi}{d\rho} > 0$. When $z_L \geq 0$, $2z_L \sqrt{1 - \rho^2} + \lambda_L \geq \lambda_L$ and thus $\phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right) = 0$, implying $\frac{d\xi}{d\rho} = 0$. ■

Proof of Lemma 2: Note $\phi(x)$ is decreasing (increasing) in x for $x \geq 0$ ($x < 0$), and $2z_L \sqrt{1 - \rho^2} + \lambda_L$ is increasing in ρ for $z_L < 0$.

(i) If $z_L \in (-\lambda_L/2, 0)$, then $2z_L \sqrt{1 - \rho^2} + \lambda_L > \lambda_L \left(1 - \sqrt{1 - \rho^2} \right) \geq 0$. Therefore, $\phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right)$ is decreasing in ρ and thus $\tilde{\mu}$ is increasing in ρ .

(ii) If $z_L \leq -\lambda_L/2$, then $\phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right)$ is quasi-concave and reaches its maximum at $2z_L \sqrt{1 - \rho^2} + \lambda_L = 0$, where $\rho = \tilde{\rho}$. Thus $\tilde{\mu}(\rho)$ is quasi-convex with a minimizer $\rho = \tilde{\rho}$. ■

Proof of Proposition 1: Taking derivative of Π^p with respect to ρ gives

$$\begin{aligned}\frac{d}{d\rho} \Pi^p &= -\frac{d\xi}{d\rho} \Delta\mu_H + v_L \phi(z_L) \sigma_L \frac{\rho}{\sqrt{1 - \rho^2}} \\ &= \frac{2z_L \rho \Delta\mu_H}{\sqrt{1 - \rho^2}} \left(\phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right) + \frac{v_L \phi(z_L) \sigma_L}{2\Delta z_L \mu_H} \right).\end{aligned}\quad (18)$$

(i) Since $2z_L \sqrt{1 - \rho^2} + \lambda_L \geq \lambda_L$ for $z_L \geq 0$, it is clear that $\frac{d}{d\rho} \Pi^p \geq 0$ when $z_L \geq 0$.

(ii) When $z_L < 0$, $\frac{d}{d\rho} \Pi^p > 0$ is equivalent to $\phi \left(2z_L \sqrt{1 - \rho^2} + \lambda_L \right) + \frac{v_L \phi(z_L) \sigma_L}{2\Delta z_L \mu_H} < 0$, which can be simplified to $\mu_H < \tilde{\mu}(\rho)$.

If $v_L \rightarrow c$, then $\phi(z_L) \rightarrow 0$ and thus $\frac{d}{d\rho} \Pi^p$ reduces to $-\frac{d\xi}{d\rho} \Delta\mu_H$, which is non-positive. ■

Proof of Proposition 2: (i) Note $\frac{d}{dv_L} \Pi^p = -\mu_H \left(\Delta \frac{d\xi}{dv_L} - \xi \right) + \frac{d\Pi_L(0)}{dv_L}$. If v_L is sufficiently large so that $\Delta \rightarrow 0$, then $\frac{d}{dv_L} \Pi^p$ reduces to $\mu_H \xi + \frac{d\Pi_L(0)}{dv_L}$. Since

$$\Pi_L(0) = \max_Q \left(v_L \mathbb{E} \left[\min \left(Q, \tilde{X}_L(0) \right) \right] - cQ \right),$$

we know $\frac{d\Pi_L(0)}{dv_L} \geq 0$ from the envelop theorem.

If v_L is sufficiently small so that $v_L \rightarrow c$, then $\phi(z_L) \rightarrow 0$, $\phi'(z_L) \rightarrow 0$. Then $\frac{d\Pi_L(0)}{dv_L} \rightarrow 0$, and $\frac{d}{dv_L}\Pi^p$ reduces to $-\mu_H \left(\Delta \frac{d\xi}{dv_L} - \xi \right)$. This is negative because $\frac{d\xi}{dv_L} \geq 0$, and $\xi \rightarrow 0$ when $v_L \rightarrow c$.

(ii) From $\Phi(z_L) = \frac{v_L - c}{v_L}$ and $\frac{dz_L}{dc} = -\frac{1}{v_L \phi(z_L)} = -\frac{1 - \Phi(z_L)}{c \phi(z_L)}$,

$$\begin{aligned} \frac{d}{dc}\Pi^p &= -\left(\frac{d\xi}{dc}\Delta + 1\right)\mu_H - \mu_L - \frac{dz_L}{dc}v_L\phi'(z_L)\sigma_L\sqrt{1-\rho^2} \\ &= -\left(\frac{d\xi}{dc}\Delta + 1\right)\mu_H - \mu_L + \frac{\phi'(z_L)}{\phi(z_L)}\sigma_L\sqrt{1-\rho^2} \\ &= -\left(\frac{d\xi}{dc}\Delta + 1\right)\mu_H - \mu_L - z_L\sigma_L\sqrt{1-\rho^2}. \end{aligned}$$

Since $\xi = \mathbb{E}_X \left[\Phi \left(\frac{1}{\sqrt{1-\rho^2}} \left(\frac{\mu_L}{\sigma_L} + \rho X \right) + 2z_L \right) \right]$, we have

$$\begin{aligned} \frac{d\xi}{dc} &= -\frac{2(1-\Phi(z_L))}{c\phi(z_L)}\mathbb{E}_X \left[\phi \left(\frac{1}{\sqrt{1-\rho^2}} \left(\frac{\mu_L}{\sigma_L} + \rho X \right) + 2z_L \right) \right] \\ &= -\frac{2\sqrt{1-\rho^2}(1-\Phi(z_L))}{c} \frac{\phi(z_L)}{\phi(z_L)} \phi \left(\frac{\mu_L}{\sigma_L} + 2z_L\sqrt{1-\rho^2} \right). \end{aligned}$$

Then by substituting $\frac{d\xi}{dc}$ in $\frac{d}{dc}\Pi^p$, we have

$$\frac{d}{dc}\Pi^p = \mu_H \left(\frac{2\Delta\sqrt{1-\rho^2}\phi\left(\lambda_L + 2z_L\sqrt{1-\rho^2}\right)}{v_L\phi(z_L)} - \left(1 + \frac{\mu_L + z_L\sigma_L\sqrt{1-\rho^2}}{\mu_H} \right) \right).$$

$\frac{d}{dc}\Pi^p$ is positive when v_H is sufficiently large, and this threshold of v_H decreases with μ_H . ■

(Next we prove a lemma that will be used in the proof of Proposition 3.)

Lemma 5 Define $\Gamma(z_L) \equiv \lambda_L z_L \frac{\Phi(z_L)}{\phi(z_L)} - z_L + \frac{\Phi(\lambda_L + 2z_L) - v_H \phi(z_H) \lambda_H^{-1} \Delta^{-1}}{2\phi(\lambda_L + 2z_L)}$. There exists $\tau \in (-\lambda_L/2, 0)$ such that $\Gamma(z_L) < 0$ for $z_L < \tau$ and $\Gamma(z_L) \geq 0$ for $z_L \in [\tau, 0]$.

Proof of Lemma 5: Define $\Gamma_1(z_L) \equiv \lambda_L z_L \frac{\Phi(z_L)}{\phi(z_L)} - z_L + \frac{\Phi(\lambda_L + 2z_L)}{2\phi(\lambda_L + 2z_L)}$. Both $z_L \frac{\Phi(z_L)}{\phi(z_L)}$ and $\frac{\Phi(\lambda_L + 2z_L)}{\phi(\lambda_L + 2z_L)}$ are increasing convex functions of z_L . Thus $\Gamma_1(z_L)$ is convex.

Since $\phi(-\lambda_L) = 0$, we focus on $z_L > -\lambda_L/\sqrt{2}$. Note $\Gamma_1(-\lambda_L/\sqrt{2}) < 0$ and $\Gamma_1(-\lambda_L/2) < 0$ for large λ_L . Then $\Gamma_1(z_L) < 0$ and thus $\Gamma(z_L) < 0$ for $z_L \in [-\lambda_L/\sqrt{2}, -\lambda_L/2]$.

For $z_L \in [-\lambda_L/2, 0]$, we have

$$\frac{d}{dz_L} \left(\frac{\Phi(\lambda_L + 2z_L) - v_H \phi(z_H) \lambda_H^{-1} \Delta^{-1}}{2\phi(2z_L + \lambda_L)} \right) = 1 + (2z_L + \lambda_L) \frac{\Phi(\lambda_L + 2z_L) - v_H \phi(z_H) \lambda_H^{-1} \Delta^{-1}}{\phi(2z_L + \lambda_L)}$$

is increasing in z_L . Thus $\Gamma(z_L)$ is convex for $z_L \in [-\lambda_L/2, 0]$. Note $\Gamma(0) > 0$ and $\Gamma(-\lambda_L/2) < 0$. Then there exists a unique $\kappa \in (-\lambda_L/2, 0)$ such that $\Gamma(z_L) < 0$ for $z_L \in (-\lambda_L/2, \kappa)$, and $\Gamma(z_L) \geq 0$ for $z_L \in [\kappa, 0]$. ■

Proof of Proposition 3: (i) Note $\frac{d}{d\mu_H}\Pi^n = v_H - c - v_H\phi(z_H)\lambda_H^{-1}$ and $\frac{d}{d\mu_H}\Pi^p = v_H - c - \Delta\xi$. If $\Delta\xi \leq v_H\phi(z_H)\lambda_H^{-1}$, then $\Pi^n < \Pi^p$ because $\Pi^p \geq \mu_H(v_H - c - \Delta\xi)$, $\Pi^n = \mu_H(v_H - c - v_H\phi(z_H)\lambda_H^{-1})$. If $\Delta\xi > v_H\phi(z_H)\lambda_H^{-1}$, then $\frac{d}{d\mu_H}(\Pi^n - \Pi^p) > 0$. Since $\Pi^p = \Pi^n$ at $\mu_H = \frac{\Pi_L(0)}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}$, $\Pi^n < \Pi^p$ if and only if $\mu_H < \frac{\Pi_L(0)}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}$.

(ii) With $\hat{\mu}_H = \frac{\Pi_L(0)}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}$,

$$\begin{aligned} \frac{d}{d\rho}\hat{\mu}_H &= -\frac{\Delta\hat{\mu}_H}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}\frac{d\xi}{d\rho} + \frac{1}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}\frac{d\Pi_L(0)}{d\rho} \\ &= \frac{\Delta\hat{\mu}_H}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}\frac{2\rho z_L}{\sqrt{1-\rho^2}}\phi\left(2z_L\sqrt{1-\rho^2} + \lambda_L\right) + \frac{v_L\phi(z_L)\sigma_L}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}}\frac{\rho}{\sqrt{1-\rho^2}} \\ &= \frac{v_L\phi(z_L)\sigma_L\rho}{(\Delta\xi - v_H\phi(z_H)\lambda_H^{-1})\sqrt{1-\rho^2}}\left(1 + \frac{2z_L\Delta}{v_L\phi(z_L)\sigma_L}\hat{\mu}_H\phi\left(2z_L\sqrt{1-\rho^2} + \lambda_L\right)\right) \\ &= \frac{v_L\phi(z_L)\sigma_L\rho}{(\Delta\xi - v_H\phi(z_H)\lambda_H^{-1})\sqrt{1-\rho^2}}\left(1 - \frac{\hat{\mu}_H}{\tilde{\mu}}\right), \end{aligned}$$

where $\tilde{\mu}$ is defined in (7). From Lemma 2, $\tilde{\mu}$ increases in ρ if $z_L \in (-\lambda_L/2, 0)$, and $\tilde{\mu}$ first decreases and then increases in ρ if $z_L \leq -\lambda_L/2$.

If $z_L \geq 0$, then $\tilde{\mu} \leq 0$ and thus $\frac{d}{d\rho}\hat{\mu}_H \geq 0$. In the following we focus on the case $z_L < 0$. For $z_L < 0$, $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ if and only if $\hat{\mu}_H \leq \tilde{\mu}$. This implies that, as ρ increases, $\hat{\mu}_H$ cannot cross $\tilde{\mu}$ from below while $\tilde{\mu}$ is increasing or cross $\tilde{\mu}$ from above while $\tilde{\mu}$ is decreasing. Below we compare $\hat{\mu}_H$ and $\tilde{\mu}$ at $\rho = 1$ and $\rho = 0$ in order to find out their relative positions over the entire range of ρ .

When $\rho = 1$,

$$\begin{aligned} \hat{\mu}_H - \tilde{\mu} &= \frac{(v_L - c)\mu_L}{\Delta - v_H\phi(z_H)\lambda_H^{-1}} + \frac{v_L\phi(z_L)\sigma_L}{2\Delta z_L\phi(\lambda_L)} \\ &= v_L\sigma_L\lambda_L\left(\frac{\Phi(z_L)}{\Delta - v_H\phi(z_H)\lambda_H^{-1}} + \frac{\phi(z_L)}{2\Delta z_L\lambda_L\phi(\lambda_L)}\right). \end{aligned}$$

Since $\lambda_L\phi(\lambda_L) \rightarrow 0$, we have $\hat{\mu}_H - \tilde{\mu} < 0$ and thus $\frac{d}{d\rho}\hat{\mu}_H > 0$. Therefore, $\hat{\mu}_H$ increases in ρ when ρ is sufficiently large.

When $\rho = 0$,

$$\begin{aligned}
\hat{\mu}_H - \tilde{\mu} &= \frac{\Pi_L(0)}{\Delta\xi - v_H\phi(z_H)\lambda_H^{-1}} + \frac{v_L\phi(z_L)\sigma_L}{2\Delta z_L\phi\left(2z_L\sqrt{1-\rho^2} + \lambda_L\right)} \\
&= v_L\sigma_L \frac{\Phi(z_L)\lambda_L - \phi(z_L)}{\Delta\Phi(\lambda_L + 2z_L) - v_H\phi(z_H)\lambda_H^{-1}} + \frac{v_L\phi(z_L)\sigma_L}{2\Delta z_L\phi(2z_L + \lambda_L)} \\
&= v_L\sigma_L\phi(z_L) \left(\frac{\frac{\Phi(z_L)}{\phi(z_L)}\lambda_L - 1}{\Delta\Phi(\lambda_L + 2z_L) - v_H\phi(z_H)\lambda_H^{-1}} + \frac{1}{2\Delta z_L\phi(2z_L + \lambda_L)} \right) \\
&= \frac{v_L\sigma_L\phi(z_L)\Gamma(z_L)}{z_L(\Delta\Phi(\lambda_L + 2z_L) - v_H\phi(z_H)\lambda_H^{-1})}
\end{aligned}$$

where $\Gamma(z_L)$ is defined in Lemma 5. From Lemma 5, there exists $\kappa \in (-\lambda_L/2, 0)$ such that $\hat{\mu}_H > \tilde{\mu}$ and thus $\frac{d}{d\rho}\hat{\mu}_H < 0$ for $z_L < \kappa$, and, $\hat{\mu}_H \leq \tilde{\mu}$ and thus $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ for $z_L \in [\kappa, 0]$.

Then for $z_L \in [\kappa, 0]$ (recall $\kappa > -\lambda_L/2$), since $\hat{\mu}_H \leq \tilde{\mu}$ and $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ at both $\rho = 0$ and 1, we have $\hat{\mu}_H \leq \tilde{\mu}$ and $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ for the entire range of $\rho \in [0, 1]$.

For $z_L < \kappa$, as ρ increases, $\hat{\mu}_H$ can only cross $\tilde{\mu}$ once from above where $\tilde{\mu}$ is increasing. In other words, there exists $\hat{\rho} \in (0, 1)$ such that $\hat{\mu}_H \geq \tilde{\mu}$ and $\frac{d}{d\rho}\hat{\mu}_H \leq 0$ for $\rho \in [0, \hat{\rho}]$, and $\hat{\mu}_H \leq \tilde{\mu}$ and $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ for $\rho \in [\hat{\rho}, 1]$. ■

Proof of Proposition 4: From Proposition 3 (i), if $\mu_H > \hat{\mu}_H^n$, then $\Pi = \Pi^n$, which is independent of ρ . If $\mu_H \leq \hat{\mu}_H^n$, then $\Pi = \Pi^p$.

(i) From Proposition 3 (i) and (ii), there exists a threshold of ρ such that $\Pi = \Pi^n$ for ρ lower than the threshold, and $\Pi = \Pi^p$ for ρ higher than the threshold. From the proof of Proposition 3(ii), $\frac{d}{d\rho}\hat{\mu}_H \geq 0$ if and only if $\hat{\mu}_H \leq \tilde{\mu}$. Thus when $\Pi = \Pi^p$, we have $\mu_H < \hat{\mu}_H \leq \tilde{\mu}$. But from Proposition 1, Π^p is increasing in ρ if $z_L \geq 0$, or if $z_L < 0$ and $\mu_H < \tilde{\mu}$. Thus Π is increasing in ρ .

(ii) From Proposition 3 (ii), Π^p increases in ρ when $\mu_H < \tilde{\mu}(\rho)$, and decreases in ρ when $\mu_H \geq \tilde{\mu}(\rho)$. ■

Proof of Lemma 3: (i) Note $\Pi_L(-\lambda_H) = \Pi_L(0) - (v_L - c)\rho\sigma_L\lambda_H \geq 0$. It implies $\frac{\Pi_L(0)}{\mu_H} - \eta \geq (v_L - c)\rho\frac{\sigma_L}{\sigma_H} - \eta$. With $\eta < (v_L - c)\rho\frac{\sigma_L}{\sigma_H} < 0$, we have

$$\left(\frac{\Pi_L(0)}{\mu_H} - \eta \right) / \left((v_L - c)\rho\frac{\sigma_L}{\sigma_H} - \eta \right) \geq 1.$$

Thus

$$\hat{x}(\eta) = -\lambda_H \left(\frac{\Pi_L(0)}{\mu_H} - \eta \right) / \left((v_L - c)\rho\frac{\sigma_L}{\sigma_H} - \eta \right)$$

is less than $-\lambda_H$.

(ii) If $\eta < \frac{\Pi_L(0)}{\mu_H}$, the the result is straightforward.

If $\eta \geq \frac{\Pi_L(0)}{\mu_H}$, since $\frac{\Pi_L(0)}{\mu_H} \geq (v_L - c) \rho \frac{\sigma_L}{\sigma_H}$ (from $\Pi_L(-\lambda_H) \geq 0$), $\frac{\eta - \Pi_L(0)/\mu_H}{\eta - (v_L - c)\rho\sigma_L/\sigma_H}$ is an increasing function of η . Thus

$$\hat{x}(\eta) = -\lambda_H \left(\eta - \frac{\Pi_L(0)}{\mu_H} \right) / \left(\eta - (v_L - c) \rho \frac{\sigma_L}{\sigma_H} \right)$$

decreases in η . ■

Proof of Proposition 5: Based on Equation (14), $\Pi^g(\hat{x})$ can be transformed to

$$\begin{aligned} \Pi^g(\hat{x}) &= (p_1(\hat{x}) - c) \mu_H - \mathbb{E}[\delta(\hat{x})(\mu_H + \sigma_H x) - \Pi_L(x) | x < \hat{x}] \Phi(\hat{x}) \\ &= (v_H + \delta(\hat{x}) \Phi(\hat{x}) - \Delta \hat{\xi}(\hat{x}) - c) \mu_H + \int_{-\infty}^{\hat{x}} (\Pi_L(x) - \delta(\hat{x})(\mu_H + \sigma_H x)) \phi(x) dx \\ &= (v_H - c - \Delta \hat{\xi}(\hat{x})) \mu_H + \int_{-\infty}^{\hat{x}} \left(\Pi_L(0) - \frac{\Pi_L(-\lambda_H)}{\lambda_H + \hat{x}} x \right) \phi(x) dx \\ &= (v_H - c - \Delta \hat{\xi}(\hat{x})) \mu_H + \Phi(\hat{x}) \Pi_L(0) + \phi(\hat{x}) \frac{\Pi_L(-\lambda_H)}{\hat{x} + \lambda_H}. \end{aligned}$$

Note $\hat{\xi}'(\hat{x}) = \Phi\left(\frac{\lambda_L + \rho \hat{x}}{\sqrt{1 - \rho^2}} + 2z_L\right) \phi(\hat{x})$, and $\phi'(\hat{x}) = -\hat{x} \phi(\hat{x})$. Then

$$\Pi^{g'}(\hat{x}) = \phi(\hat{x}) \Pi_L(-\lambda_H) (A(\hat{x}) - B(\hat{x})),$$

where $A(\hat{x}) \equiv \frac{1}{\Pi_L(-\lambda_H)} \left(\Pi_L(0) - \Delta \mu_H \Phi\left(\frac{\lambda_L + \rho \hat{x}}{\sqrt{1 - \rho^2}} + 2z_L\right) \right)$ and $B(\hat{x}) \equiv \frac{\hat{x}(\hat{x} + \lambda_H) + 1}{(\hat{x} + \lambda_H)^2}$. $A(\hat{x})$ is decreasing, and $B(\hat{x})$ is quasi-convex in \hat{x} . The minimum of $B(\hat{x})$ is equal to $1 - \frac{\lambda_H^2}{4} < 0$, and is achieved at $\hat{x} = \frac{2}{\lambda_H} - \lambda_H < 0$. Also note $B(-\lambda_H) = \infty$ and $B(\lambda_H) = \frac{2\lambda_H^2 + 1}{4\lambda_H^2} < 1$. $B(\hat{x})$ is independent of μ_H and $A(\hat{x})$ is decreasing in μ_H .

If $\mu_H \rightarrow 0$: $A(\hat{x}) \rightarrow \frac{\Pi_L(0)}{\Pi_L(-\lambda_H)}$ with $A(\lambda_H) > B(\lambda_H)$. Thus $\Pi^g(\hat{x})$ is quasi-convex in \hat{x} ; in other words, \hat{x}^* is equal to $-\lambda_H$ or λ_H . Since $\Pi^p > \Pi^n \rightarrow 0$, we know $\hat{x}^* = \lambda_H$ and $\Pi^g = \Pi^p$.

If μ_H is sufficiently large so that $A(\hat{x}) < B(\hat{x})$ for all $\hat{x} \in [-\lambda_H, \lambda_H]$, then $\hat{x}^* = -\lambda_H$ and $\Pi^g = (v_H - c) \mu_H > \Pi^p$.

From $\frac{d}{d\mu_H} \Pi^g = v_H - c - \Delta \hat{\xi}(\hat{x}^*)$ and $\frac{d}{d\mu_H} \Pi^p = v_H - c - \Delta \xi$, where $\xi = \hat{\xi}(\lambda_H) > \hat{\xi}(\hat{x}^*)$, we have $\frac{d}{d\mu_H} \Pi^p \leq \frac{d}{d\mu_H} \Pi^g$. Thus we know $\hat{\mu}_H^g$ exists.

If $\hat{\mu}_H^g > \frac{\Pi_L(0)}{\Delta \xi}$, then there exists $\mu_H \in \left(\frac{\Pi_L(0)}{\Delta \xi}, \hat{\mu}_H^g \right)$ for which $\Pi^g = \Pi^p > (v_H - c) \mu_H$. But $\Pi^p > (v_H - c) \mu_H$ if and only if $\mu_H < \frac{\Pi_L(0)}{\Delta \xi}$. Thus we have $\hat{\mu}_H^g < \frac{\Pi_L(0)}{\Delta \xi}$. Then from $\frac{\Pi_L(0)}{\Delta \xi} < \frac{\Pi_L(0)}{\Delta \xi - v_H \phi(z_H) \lambda_H^{-1}} = \hat{\mu}_H^n$, we have $\hat{\mu}_H^g < \hat{\mu}_H^n$. ■

Proof of Lemma 4: (i) Recall $\lambda_L \geq \sqrt{2}\lambda_H$. Then for any $\hat{x} \in [-\lambda_H, \lambda_H]$, $\frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} \geq \frac{\sqrt{2}-\rho}{\sqrt{1-\rho^2}}\lambda_H \geq \lambda_H$. The latter inequality is because $\frac{\sqrt{2}-\rho}{\sqrt{1-\rho^2}} \geq 1$ for any $\rho \in [0, 1]$. Then with

$$z_L \geq 0, \frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} + 2z_L \geq \lambda_H \text{ and thus } \Phi\left(\frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} + 2z_L\right) = 1, \hat{\xi}(\hat{x}) = \Phi(\hat{x}).$$

(ii)

$$\frac{d}{d\rho}\hat{\xi}(\hat{x}) = \frac{1}{(1-\rho^2)^{3/2}} \int_{-\lambda_H}^{\hat{x}} \left((x + \rho\lambda_L) \phi\left(\frac{\lambda_L + \rho x}{\sqrt{1-\rho^2}} + 2z_L\right) \phi(x) \right) dx.$$

When $\rho = 0$, $\frac{d}{d\rho}\hat{\xi}(\hat{x}) = \phi(\lambda_L + 2z_L) \int_{-\infty}^{\hat{x}} x\phi(x) dx \leq 0$.

When $\rho \rightarrow 1$, $x + \rho\lambda_L > 0$ for any $\hat{x} \in [-\lambda_H, \lambda_H]$ because $\lambda_L \geq \sqrt{2}\lambda_H$. Thus $\frac{d}{d\rho}\hat{\xi}(\hat{x}) \geq 0$.

■

Proof of Proposition 6: (i) From the proof of Lemma 4(i), $\Phi\left(\frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} + 2z_L\right) = 1$ and $\hat{\xi}(\hat{x}) = \Phi(\hat{x})$. Thus

$$\Pi^g(\hat{x}) = (v_H - c - \Delta\Phi(\hat{x}))\mu_H + \Phi(\hat{x})\Pi_L(0) + \phi(\hat{x})\frac{\Pi_L(-\lambda_H)}{\hat{x} + \lambda_H}.$$

For interior $\hat{x}^* \in (-\lambda_H, \lambda_H)$,

$$\begin{aligned} \frac{d}{d\rho}\Pi^g(\hat{x}^*) &= \Phi(\hat{x}^*)\frac{d}{d\rho}\Pi_L(0) + \frac{\phi(\hat{x}^*)}{\hat{x}^* + \lambda_H}\frac{d}{d\rho}\Pi_L(-\lambda_H) \\ &= \frac{\Phi(\hat{x}^*)v_L\phi(z_L)\sigma_L\rho}{\sqrt{1-\rho^2}} \left(1 + \frac{\phi(\hat{x}^*)}{(\hat{x}^* + \lambda_H)\Phi(\hat{x}^*)} \left(1 - \frac{\Phi(z_L)}{\phi(z_L)}\lambda_H\frac{\sqrt{1-\rho^2}}{\rho} \right) \right). \end{aligned}$$

Then $\frac{d}{d\rho}\Pi^g(\hat{x}^*) \geq 0$ if and only if

$$\frac{\sqrt{1-\rho^2}}{\rho} \leq \left(1 + \frac{(\hat{x}^* + \lambda_H)\Phi(\hat{x}^*)}{\phi(\hat{x}^*)} \right) \frac{\phi(z_L)}{\Phi(z_L)\lambda_H}.$$

It is clear that $\Pi^g(\hat{x}^*)$ is quasi-convex in ρ if $\frac{d}{d\rho}\hat{x}^* \geq 0$. In the following, we show that there exists $\gamma \in [0, 1]$ such that $\frac{d}{d\rho}\hat{x}^* \geq 0$ for $\rho \geq \gamma$, and $\frac{d}{d\rho}\Pi^g(\hat{x}^*) < 0$ for $\rho < \gamma$.

From the proof of Proposition 5,

$$\Pi^{g'}(\hat{x}) = \phi(\hat{x})\Pi_L(-\lambda_H)(A(\hat{x}) - B(\hat{x})),$$

where $A(\hat{x}) = \frac{\Pi_L(0) - \Delta\mu_H}{\Pi_L(-\lambda_H)}$ (since $\Phi\left(\frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} + 2z_L\right) = 1$) is independent of \hat{x} and $B(\hat{x}) \equiv \frac{\hat{x}(\hat{x} + \lambda_H) + 1}{(\hat{x} + \lambda_H)^2}$. An interior solution \hat{x}^* is defined by $A = B(\hat{x}^*)$ with $B'(\hat{x}^*) > 0$. Because

$B(\lambda_H) < 1$ and thus $B(\hat{x}^*) < 0$, we focus on $A < 1$. Given

$$\begin{aligned}\frac{dA}{d\rho} &= \frac{1}{\Pi_L(-\lambda_H)} \left(\frac{d\Pi_L(0)}{d\rho} - A \frac{d\Pi_L(-\lambda_H)}{d\rho} \right) \\ &= \frac{v_L \phi(z_L) \sigma_L}{\Pi_L(-\lambda_H)} \left(\frac{\rho}{\sqrt{1-\rho^2}} (1-A) + A \frac{\Phi(z_L)}{\phi(z_L)} \lambda_H \right), \\ \frac{d^2 A}{d\rho^2} &= \frac{1}{\Pi_L(-\lambda_H)} \left(-2 \frac{dA}{d\rho} \frac{d\Pi_L(-\lambda_H)}{d\rho} + \frac{d^2 \Pi_L(0)}{d\rho^2} (1-A) \right),\end{aligned}$$

$\frac{d^2 A}{d\rho^2} > 0$ when $\frac{dA}{d\rho} = 0$. Thus A is quasi-convex on ρ ; there exists $\gamma \in [0, 1]$ such that $\frac{dA}{d\rho} \leq 0$ if and only if $\rho \leq \gamma$.

When $\rho \geq \gamma$, i.e., $\frac{dA}{d\rho} \geq 0$, \hat{x}^* is increasing in ρ because, for any $\hat{x}^* \in (-\lambda_H, \lambda_H)$, $B(\hat{x})$ is increasing at $\hat{x} = \hat{x}^*$.

When $\rho < \gamma$, $\frac{dA}{d\rho} < 0$ implies $\frac{d}{d\rho} \Pi_L(-\lambda_H) < 0$ (otherwise, $\frac{d}{d\rho} \Pi_L(-\lambda_H) \geq 0$ leads to $\frac{dA}{d\rho} \geq 0$). Then from $\Pi^g(\hat{x}^*) = (v_H - c) \mu_H + \int_{-\lambda_H}^{\hat{x}^*} \phi(\hat{x}) \Pi_L(-\lambda_H) (A - B(\hat{x})) d\hat{x}$, we know

$$\begin{aligned}\frac{d}{d\rho} \Pi^g(\hat{x}^*) &= \int_{-\lambda_H}^{\hat{x}^*} \left(\phi(\hat{x}) \left(\frac{d}{d\rho} \Pi_L(-\lambda_H) (A - B(\hat{x})) + \Pi_L(-\lambda_H) \frac{dA}{d\rho} \right) \right) d\hat{x} \\ &= \frac{\frac{d}{d\rho} \Pi_L(-\lambda_H)}{\Pi_L(-\lambda_H)} (\Pi^g(\hat{x}^*) - (v_H - c) \mu_H) + \int_{-\lambda_H}^{\hat{x}^*} \phi(\hat{x}) \Pi_L(-\lambda_H) \frac{dA}{d\rho} d\hat{x}\end{aligned}$$

is negative.

(ii) $\Pi^g(\hat{x}^*) = \Pi^p$ implies $\hat{x}^* = \lambda_H$. This means $A = B(\lambda_H) = \frac{2\lambda_H^2+1}{4\lambda_H^2}$, leading to $\mu_H^g = \frac{1}{\Delta} \left(\Pi_L(0) - \frac{2\lambda_H^2+1}{4\lambda_H^2} \Pi_L(-\lambda_H) \right)$. Then $\frac{d\mu_H^g}{d\rho} = \frac{1}{\Delta} \left(\frac{d}{d\rho} \Pi_L(0) - \frac{2\lambda_H^2+1}{4\lambda_H^2} \frac{d}{d\rho} \Pi_L(-\lambda_H) \right)$. Since $\frac{2\lambda_H^2+1}{4\lambda_H^2} < 1$, $\frac{d}{d\rho} \Pi_L(0) > \frac{d}{d\rho} \Pi_L(-\lambda_H)$, and $\frac{d}{d\rho} \Pi_L(0) > 0$, we have $\frac{d\mu_H^g}{d\rho} > 0$. ■

Proof of Proposition 7: (i) When $v_L \rightarrow c$, $\phi(z_L) \rightarrow 0$ and $\Phi(z_L) \rightarrow 0$. Then $\frac{d}{d\rho} \Pi_L(0) \rightarrow 0$, $\frac{d}{d\rho} \Pi_L(-\lambda_H) \rightarrow 0$, and thus $\frac{d}{d\rho} \Pi^g(\hat{x}^*) \rightarrow -\Delta \mu_H \frac{d}{d\rho} \hat{\xi}(\hat{x}^*)$. From Lemma 4(ii), $\frac{d}{d\rho} \hat{\xi}(\hat{x}^*) \leq 0$ at $\rho = 0$, and $\frac{d}{d\rho} \hat{\xi}(\hat{x}^*) \geq 0$ at $\rho = 1$.

(ii) From $A(\hat{x}) = \frac{1}{\Pi_L(-\lambda_H)} \left(\Pi_L(0) - \Delta \mu_H \Phi \left(\frac{\lambda_L + \rho \hat{x}}{\sqrt{1-\rho^2}} + 2z_L \right) \right)$, we have

$$\begin{aligned}\frac{d}{d\mu_H} A(\hat{x}) &= -\frac{\Delta}{\Pi_L(-\lambda_H)} \Phi \left(\frac{\lambda_L + \rho \hat{x}}{\sqrt{1-\rho^2}} + 2z_L \right) < 0, \\ \frac{d}{d\rho} A(\hat{x}) &= \frac{1}{\Pi_L(-\lambda_H)} \left(v_L \phi(z_L) \sigma_L \frac{\rho}{\sqrt{1-\rho^2}} - \Delta \mu_H \phi \left(\frac{\lambda_L + \rho \hat{x}}{\sqrt{1-\rho^2}} + 2z_L \right) \left(\frac{\hat{x}}{\sqrt{1-\rho^2}} + \rho \frac{\lambda_L + \rho \hat{x}}{(1-\rho^2)^{\frac{3}{2}}} \right) \right. \\ &\quad \left. - A(\hat{x}) \left(v_L \phi(z_L) \sigma_L \frac{\rho}{\sqrt{1-\rho^2}} - (v_L - c) \sigma_L \lambda_H \right) \right).\end{aligned}$$

When $\rho = 0$, $A(\hat{x}) = 1 - \frac{\Delta\mu_H}{\Pi_L(-\lambda_H)}\Phi(\lambda_L + 2z_L)$ is independent of \hat{x} . Then $\Pi^g(\hat{x}^*) = \Pi^p$ implies $\hat{x}^* = \lambda_H$, which means $A(\lambda_H) = B(\lambda_H) = \frac{2\lambda_H^2+1}{4\lambda_H^2}$ at $\mu_H = \hat{\mu}_H^g$. With $\rho = 0$, we have

$$\begin{aligned}\frac{d}{d\rho}A(\lambda_H) &= \lambda_H \left(-\frac{\Delta\mu_H}{\Pi_L(-\lambda_H)}\phi(\lambda_L + 2z_L)\lambda_H + A(\lambda_H)\frac{(v_L - c)\sigma_L}{\Pi_L(-\lambda_H)} \right) \\ &= \lambda_H \left(-\frac{\Delta\mu_H}{\Pi_L(-\lambda_H)}\phi(\lambda_L + 2z_L)\lambda_H + A(\lambda_H)\frac{\Phi(z_L)}{\Phi(z_L)\lambda_L - \phi(z_L)} \right),\end{aligned}$$

and with $v_L \rightarrow c$, $\frac{\Phi(z_L)}{\Phi(z_L)\lambda_L - \phi(z_L)} \rightarrow 0$. Thus $\frac{d}{d\rho}A(\lambda_H) < 0$. Since $\frac{d}{d\mu_H}A(\hat{x}) < 0$, it follows that $\frac{d\hat{\mu}_H^g}{d\rho} < 0$.

When $\rho = 1$, $\frac{\lambda_L + \rho\hat{x}}{\sqrt{1-\rho^2}} + 2z_L > \lambda_H$ and thus $A(\hat{x}) = \frac{1}{\Pi_L(-\lambda_H)}(\Pi_L(0) - \Delta\mu_H)$ is independent of \hat{x} . Then $\Pi^g(\hat{x}^*) = \Pi^p$ implies $\hat{x}^* = \lambda_H$, which again means $A(\lambda_H) = B(\lambda_H) = \frac{2\lambda_H^2+1}{4\lambda_H^2}$ for $\mu_H = \hat{\mu}_H^g$. With $\rho = 1$,

$$\frac{d}{d\rho}A(\lambda_H) = \frac{1}{\Pi_L(-\lambda_H)} \left(v_L\phi(z_L)\sigma_L\frac{\rho}{\sqrt{1-\rho^2}} \left(1 - \frac{2\lambda_H^2+1}{4\lambda_H^2} \right) + \frac{2\lambda_H^2+1}{4\lambda_H^2}(v_L - c)\sigma_L\lambda_H \right)$$

is positive. Since $\frac{d}{d\mu_H}A(\hat{x}) < 0$, it follows that $\frac{d\hat{\mu}_H^g}{d\rho} > 0$. ■

Proof of Proposition 8: In the full-price guarantee mechanism, the price will be reduced if and only if $\Delta x_H < \Pi_L\left(\frac{x_H - \mu_H}{\sigma_H}\right)$, or

$$\Delta(\mu_H + \sigma_H x) < (v_L - c)(\mu_L + \rho\sigma_L x) - v_L\phi(z_L)\sigma_L\sqrt{1-\rho^2}.$$

Case 1: If $\Delta\sigma_H - (v_L - c)\rho\sigma_L > 0$, then this is equivalent to

$$x < \hat{x}^{bg} \equiv \frac{\Pi_L(0) - \Delta\mu_H}{\Delta\sigma_H - (v_L - c)\rho\sigma_L}.$$

Then by substituting \hat{x}^{bg} in

$$\begin{aligned}\Pi^{bg} &= (v_H - c)\mu_H + \mathbb{E}[\Pi_L(x) - \Delta(\mu_H + \sigma_H x) | x < \hat{x}^{bg}] \Phi(\hat{x}^{bg}) \\ &= (v_H - c - \Delta\Phi(\hat{x}^{bg}))\mu_H + \mathbb{E}[\Pi_L(0) + ((v_L - c)\rho\sigma_L - \Delta\sigma_H)x | x < \hat{x}^{bg}] \Phi(\hat{x}^{bg}) \\ &= (v_H - c)\mu_H + \Phi(\hat{x}^{bg})(\Pi_L(0) - \Delta\mu_H) - \phi(\hat{x}^{bg})((v_L - c)\rho\sigma_L - \Delta\sigma_H),\end{aligned}$$

we have

$$\begin{aligned}\frac{d}{d\rho}\Pi^{bg} &= \Phi(\hat{x}^{bg})\frac{d}{d\rho}\Pi_L(0) - \phi(\hat{x}^{bg})(v_L - c)\sigma_L \\ &= \Phi(\hat{x}^{bg})v_L\phi(z_L)\sigma_L\frac{\rho}{\sqrt{1-\rho^2}} - \phi(\hat{x}^{bg})(v_L - c)\sigma_L \\ &= v_L\sigma_L\phi(z_L)\phi(\hat{x}^{bg})\left(\frac{\Phi(\hat{x}^{bg})}{\phi(\hat{x}^{bg})}\frac{\rho}{\sqrt{1-\rho^2}} - \frac{\Phi(z_L)}{\phi(z_L)}\right).\end{aligned}$$

Since \hat{x}^{bg} is increasing in ρ (because $\Pi_L(0)$ is increasing in ρ), Π^{bg} is quasi-convex in ρ .

Case 2: If $\Delta\sigma_H - (v_L - c)\rho\sigma_L < 0$, then $\hat{x}^{bg} < -\lambda_H$ and price reduction will always occur. The seller's profit is then

$$\begin{aligned}\Pi^{bg} &= (v_H - c)\mu_H + \mathbb{E}[\Pi_L(x) - \Delta(\mu_H + \sigma_H x)] \\ &= (v_L - c)\mu_H + \Pi_L(0),\end{aligned}$$

which is increasing in ρ . ■

Proof of Proposition 9: From Equation (17),

$$\begin{aligned}\frac{d}{d\mu_H}\Pi^{fg}(\hat{x}^{fg}) &= v_L + \eta(\hat{x}^{fg})(1 - \Phi(\hat{x}^{fg})) - c + \frac{d\Pi^{fg}(\hat{x}^{fg})}{d\hat{x}^{fg}} \frac{d\hat{x}^{fg}}{d\mu_H} \\ &= v_L - c + \frac{\Pi_L(\hat{x}^{fg})}{\mu_H + \sigma_H \hat{x}^{fg}} (1 - \Phi(\hat{x}^{fg})) + \frac{d\Pi^{fg}(\hat{x}^{fg})}{d\hat{x}^{fg}} \frac{d\hat{x}^{fg}}{d\mu_H},\end{aligned}$$

where \hat{x}^{fg} satisfies $\frac{\Pi_L(\hat{x}^{fg})}{\Delta\mu_H(1+\lambda_H^{-1}\hat{x}^{fg})} = \frac{1-\hat{\xi}(\hat{x}^{fg})}{1-\Phi(\hat{x}^{fg})}$. Since $\frac{\Pi_L(\hat{x}^{fg})}{\Delta\mu_H(1+\lambda_H^{-1}\hat{x}^{fg})}$ is decreasing in μ_H , \hat{x}^{fg} is decreasing in μ_H . It can be shown that

$$\frac{d\Pi^{fg}(\hat{x})}{d\hat{x}} = \frac{d}{d\hat{x}} \left(\frac{\Pi_L(\hat{x})}{\mu_H + \sigma_H \hat{x}} \right) (\mu_H - \mathbb{E}[\mu_H + \sigma_H x | x < \hat{x}] \Phi(\hat{x}))$$

is negative because $\frac{d}{d\hat{x}} \left(\frac{\Pi_L(\hat{x})}{\mu_H + \sigma_H \hat{x}} \right) = -\frac{\sigma_H}{(\mu_H + \sigma_H \hat{x})^2} \Pi_L(-\lambda_H) < 0$. Thus

$$\begin{aligned}\frac{d}{d\mu_H}\Pi^{fg}(\hat{x}^{fg}) &> v_L - c + \frac{\Pi_L(\hat{x}^{fg})}{\mu_H + \sigma_H \hat{x}^{fg}} (1 - \Phi(\hat{x}^{fg})) \\ &= v_L - c + \Delta(1 - \hat{\xi}(\hat{x}^{fg})).\end{aligned}$$

Since $\frac{d}{d\mu_H}\Pi^p = v_H - c - \Delta\xi = v_L - c + \Delta(1 - \xi)$ and $\xi \geq \hat{\xi}(\hat{x}^{fg})$, we have $\frac{d}{d\mu_H}\Pi^p \leq \frac{d}{d\mu_H}\Pi^{fg}(\hat{x}^{fg})$. ■