

Supply Base Design for Supplier Competition and Investment of Effort under Cost and Demand Uncertainties

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Supplier competition and investment of effort are important concerns in supply base design. They are affected not only by the number of suppliers, but also by their capacity profile. This paper studies supply base design in terms of the number and capacities of suppliers, considering both supplier competition and investment of cost-reduction effort. We analyze a buyer who invests in capacities for two available suppliers. Given the capacity investment, suppliers exert cost-reduction effort. The realized production cost of a supplier is uncertain and is private information. After the costs and demand are realized, an auction is conducted to determine the quantity sourced from and the price paid to each supplier. The buyer benefits from both symmetry and asymmetry between suppliers in capacity: A symmetric supply base is effective at inducing supplier competition, while an asymmetric supply base delivers great value from suppliers' cost-reduction efforts. As a result, we find that the buyer may invest in capacity for only one supplier (sole sourcing), equal capacities for both suppliers (symmetric dual sourcing), or positive but unequal capacities for both suppliers (asymmetric dual sourcing), even though the suppliers are ex ante identical. We show that asymmetric dual sourcing is possible only in the presence of demand uncertainty, and the policy arises for intermediate capacity costs. We also find that, if the cost of supplier effort is low, with increasing demand uncertainty the optimal supply base structure shifts from symmetric dual sourcing to sole sourcing or asymmetric dual sourcing, along with reduction of the total capacity. Finally, we consider different timing scenarios for purchasing price and capacity investment decisions, investigate when the buyer may choose to commit to purchasing prices before the realization of supplier costs, or to postpone capacity investment decisions until after the cost realization, and compare the choice of supply base design under each mechanism.

1. Introduction

Many companies have recognized the strategic importance of optimizing their supply base¹, as they increasingly rely on suppliers to create value, reduce cost, and improve products or services (Maurer et al., 2004; Carbone, 2000, 2004; Michaels, 2006). According to a 2004 report from the Hackett

¹Choi and Krause (2006) define a supply base as the group of suppliers that the buyer actively manages through contracts and purchasing of parts, materials, and services.

Group, 75 percent of all world-class companies engage in significant supplier rationalization reviews on an annual basis (Duffy, 2005). While a supply base can be defined on various dimensions, with its design concerned with different issues (Choi and Krause, 2006; Duffy, 2005), in this paper, we study supply base design along two fundamental dimensions—the number of suppliers and their production capacities, considering supplier competition and suppliers’ investment of cost-reduction effort.

Over the past decade, there has been a noticeable trend of OEMs (buyers) consolidating sourcing needs with fewer suppliers. World-class manufacturers in the automotive sector reduced their supply base typically by 50 percent and many moved to single-sourcing and one supplier per part in the mid 1990s (Asmus and Griffin, 1993). See Baldwin et al. (2001, Appendix B) for a number of other examples. A number of different reasons exist for supply base reduction, including the benefits of reducing transaction costs and leveraging the volume. One important reason is that, with fewer suppliers, the buyer is able to build deeper trust and foster closer supplier relationships (Duffy, 2005), thereby encouraging suppliers to dedicate more resources or effort to improve their performance (Dyer, 2000). For example, in 2005 Ford planned to overhaul its supply system by offering larger contracts to a smaller group of suppliers, in order to motivate suppliers to provide the best technology to Ford (McCracken, 2005). In 2006 Airbus had to trim its supply base, “in hoping to forge durable relationships so contractors can make long-term investment plans to improve the quality of their products and increase efficiency.” (Michaels, 2006) Motivating suppliers to invest effort to reduce the cost or improve the product is important: Supplier innovation has fueled many product improvement projects in which the function of parts or subassemblies is maintained or improved but the cost is reduced (Ellram and Choi, 2000), and has been a major source for leading automotive companies to improve the size and functionality of their products while reducing the costs of manufacturing (Nelson et al., 1998, 2001).

While buyers have realized the advantages of supply base reduction, a smaller number of suppliers is not necessarily better for them. A small supply base adversely affects the benefits of a competitive marketplace (Duffy, 2005). When fewer suppliers compete for a contract, the buyer has less bargaining power in negotiation with the suppliers and, as a result, the buyer has to bear a higher purchasing cost (McMillan, 1990). Supplier competition is especially important when it is difficult for the buyer to estimate a supplier’s cost, for example, when the supplier serves as a system integrator for a complex subsystem. Such subassembly systems are much harder to price and evaluate than simple parts (Maurer et al., 2004). In these cases, supplier competition helps the buyer to form reasonable expectations of supplier cost, thus discovering a fair price (McMillan, 1990; Duffy, 2005). For example, GM set up a system of worldwide competitive bidding of suppliers, saving an estimated \$4 billion a year in the prices it paid for parts between 1992 and 1994 (Fine et al., 1996). Apple sourced its iPhone touchscreen display from four suppliers, who bid each

time Apple released a factory order—“by pitting suppliers against each other, Apple maintains the upper hand and keeps its costs low.” (Carson, 2007).

Supplier competition not only provides the benefit of competitive pricing, but also allows flexibility in supplier selection when supplier capabilities are subject to unpredictable changes. A supplier may experience performance slips due to managerial problems, or cost surges due to fluctuating exchange rates, energy and material costs, transportation prices or spikes of lower-tier supplier costs. Such variations of supplier costs, called procurement risks by Chopra and Sodhi (2004), cause unanticipated changes in the buyer’s acquisition costs. The uncertainty of supplier capability may also be caused by the uncertain outcome of innovations (Choi and Krause, 2006). If a buyer sources from multiple competing suppliers, then she is able to respond to such supplier capability changes by adjusting her business allocation among suppliers (Duffy, 2005). For example, both Toyota and Cisco maintain more suppliers than they need in their supplier networks, so that they can “keep the resulting higher costs in check by monitoring and benchmarking suppliers against each other” (Chopra and Sodhi, 2004). Nike developed a worldwide subcontracting system that allowed the company to quickly relocate production and disassociate itself with factories that failed to meet standards of performance or where price changes rendered an uncompetitive product (Donaghu and Barff, 1990).

Besides the number of suppliers, their production capacity is also an important dimension of a supply base. Certainly, the capacity of suppliers has a major impact on the buyer’s capability to meet uncertain demand. In addition, the capacity profile of suppliers influences supplier competition and the incentive of suppliers to invest effort in improving their performance.

In this paper we analyze the supply base design problem in which a buyer invests in the capacities of suppliers. Buyers often make direct investment of assets for a supplier, as a part of supplier development: Toyota invested in production capacity and technology for its suppliers as a way to enhance its relationship with them and lessen management risks as its sales grew. Recently, it invested in the equipment for its new brake system company, ADVICS, and for its new power steering company, Fabes (CAPS, 2005). Hyundai Motors also assisted its suppliers in expanding capacity to improve their delivery capability, as a stage of supplier development (Hahn et al., 1989). It is common in the automotive industry that an OEM invests in specific assets of a supplier, including facilities in close proximity to the buyer’s sites, physical assets such as customized machinery, tools, and equipment, or personnel located at the supplier’s site (Dyer, 2000, 1997)². Outside the automotive industry, Nike’s first-tier suppliers, in charge of final assembly of footwear, also invested in the machinery required to manufacture components specific to Nike, such as outsoles, midsoles, and themal plastic heel counters (Donaghu and Barff, 1990).

We consider the supply base development problem in which the suppliers invest effort to improve

²Even though in some cases the investment may be made by a supplier, the cost is often reimbursed by the OEM (Visteon, 2003).

their production efficiency, with the buyer investing in supplier capacity to improve their delivery capability. While a buyer can also invest in resources, such as engineering support or training programs, to help improve a supplier's efficiency, in this paper we focus on the effort initiated by a supplier, since the improvement pertains to the activities performed by the supplier, and thus requires great commitment on the part of the supplier (Handfield et al., 2000). While cost reduction is typically not possible without a supplier's initiative or commitment, capacity improvement may be achieved with investment made primarily by the buyer.

Our goal is to develop an understanding of supply base design in terms of the number and capacities of suppliers, taking into consideration both supplier competition and investment of cost-reduction effort. In order to focus on the impact of the buyer's decision on supply base performance, we assume the buyer faces two ex ante identical suppliers. The buyer first decides capacity investment for each supplier (a supplier is not included in the supply base if he is invested in zero capacity), facing uncertain future demand. Then suppliers invest cost-reduction effort. Due to uncertainty of the effort outcome as well as the change of a supplier's internal and external environments, the realized cost of each supplier is uncertain. In addition, the cost is a supplier's private information, for the difficulty of cost assessment from outside. Finally, facing information asymmetry, the buyer determines the quantity sourced from and price paid to each supplier via an auction, after the demand and supplier costs are realized.

We find that the buyer benefits from both asymmetry and symmetry between suppliers in their capacities. An asymmetric supply base delivers great value from suppliers' cost-reduction effort, while a symmetric supply base is effective at inducing supplier competition. In the end result, the buyer may maximize supplier asymmetry by investing in capacity for only one supplier (sole sourcing), maximize supplier symmetry by investing in equal capacities for the two suppliers (symmetric dual sourcing), or reach an intermediate case by investing in positive but unequal capacities for both suppliers (asymmetric dual sourcing), even though suppliers are identical ex ante. Among these three structures, asymmetric dual sourcing is possible only if demand is uncertain; otherwise, without demand uncertainty, the buyer should choose between sole sourcing and symmetric dual sourcing. We find that symmetric dual sourcing is favored with low capacity costs, sole sourcing favored with high capacity costs, and asymmetric dual sourcing is located in between, preferred with intermediate capacity costs. In addition, the favorability of symmetric dual sourcing (sole sourcing) increases (decreases) as the cost of supplier effort or the uncertainty of supplier production cost increases. Finally, we find that, if the cost of supplier effort is low, increasing demand uncertainty shifts the optimal supply base structure from symmetric dual sourcing to sole sourcing or asymmetric dual sourcing, meanwhile reducing total capacity investment.

In our main model, the buyer invests in (commits to) supplier capacity *before* the suppliers make their investment of cost-reduction effort, with the purchasing prices negotiated *afterwards*.

This is appropriate when cost-reduction is a continuous process that can outlast the period of capacity investment, and the parties are not able to write a court-enforceable contract when facing uncertainties of future cost and demand. We extend our analysis to the ex ante price commitment mechanism, in which the buyer commits to prices (along with the capacities) before suppliers invest their effort, and to the ex post capacity investment mechanism, in which the buyer leaves the capacities to be decided (along with the prices) after supplier cost realization. We compare these two mechanisms to our base mechanism. We find that ex ante price commitment makes sole sourcing more favorable, and results in a lower profit for the buyer unless the supplier cost uncertainty is very small, whereas ex post capacity investment considers dual sourcing more favorably, and improves the buyer's profit if the cost of effort, cost uncertainty, capacity cost or demand uncertainty is large.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. We formulate and analyze the model in Sections 3 and 4, respectively. In Section 5 we characterize the optimal supply base structure. Sections 6 and 7 examine the ex ante price commitment and ex post capacity investment mechanisms and compare them to the main model. We conclude in Section 9.

2. Literature review

We study the impact of supply base design on suppliers' investment of cost-reduction effort. In our model, supplier effort is non-contractible, and is determined before the supplier receives a contract (though we consider an extension with pre-specified contracts). This is different from the papers in supply contract (see Cachon, 2003; Chen, 2003, for a review) or procurement (e.g., Cachon and Zhang, 2006; Kim et al., 2007; Li and Scheller-Wolf, 2009) literature that have supplier investment (directly or indirectly) regulated with pre-specified contracts.

Some papers consider supplier investment without a pre-specified contract. Van Mieghem (1999) analyzes a subcontracting game and compares the two situations when the unit price is specified before and after the subcontractor makes his investment. Bernstein and Kök (2009) examine the dynamics of supplier investment in cost reduction initiatives in a multi-period setting. They compare cost-contingent contracts and target-price contracts, which are analogous to ex post price negotiations and ex ante price commitments. Taylor and Plambeck (2007) study repeated interactions between a buyer and a supplier, with the supplier investing in capacity without a formal contract. All these papers assume a single supplier (of each component). We consider the influence of supplier competition on the investment decision of suppliers.

When suppliers make investment in anticipation of future competition, the investment decision depends on the procurement mechanism used to allocate demand and negotiate contracts with competing suppliers. Li et al. (2006) analyze the equilibrium of investments made by suppliers and the optimal auction mechanism designed by the buyer. Cachon and Zhang (2007) examine

the buyer's choice over several demand allocation rules, considering its impact on two competing suppliers in their capacity investment decisions. Using second-price auctions, Li and Gupta (2007) analyze and compare the situations in which the buyer can and cannot pre-commit to the auction reserve price before suppliers make their investments. Also based on second-price auctions, Dasgupta (1990) finds that suppliers invest less than the socially optimal level when the buyer cannot pre-commit to a reserve price. In all these papers, the supply base is given, and the focus is on the relationship between supplier investments and the auction/allocation mechanism. We differ by considering supply base design as an endogenous decision. This decision influences supplier investments and shapes the follow-up auction mechanism designed by the buyer. Thus, the supply base design can be considered as a strategic decision, with the auction mechanism as a tactical decision, both influencing supplier investments, but on different levels.

All the above mentioned papers consider investment made only by suppliers. Some other papers focus on the investment made only by the buyer. Wang et al. (2008) analyze the decision of a buyer investing in resources to improve the reliability of two suppliers, considering the option of dual sourcing. Li and Debo (2009a,b) study a buyer that invests in (transferrable or untransferrable) capacity for a supplier, which influences the future supplier competition and alternative sourcing opportunities. None of these papers is concerned with investment of resources from suppliers. Unlike these papers, we consider investments made by both the buyer and suppliers, studying the impact of the buyer's investment on the incentive of suppliers.

Some papers analyze the situation when both the buyer and supplier commit resources. Iyer et al. (2005) examine a supply chain in which both the buyer and supplier allocate resources that jointly determine the supplier's production cost. Zhu et al. (2007) investigate the quality improvement problem in which both the buyer and supplier can exert effort to improve the quality of the final product. In these two papers, the buyer's investment is observable (or contractible) before the supplier commits his resource, as is the case in our base model. If the investment of neither party is observable or contractible, then a double moral hazard arises (Corbett et al., 2005; Plambeck and Taylor, 2006). All these papers with two-sided investment—whether observable or not—assume a single supplier. We consider supplier competition under the influence of the buyer.

In our paper, the supply base is an endogenous decision of the buyer. An endogenous supply base is considered by Riordan (1996), who analyzes a buyer who qualifies two suppliers *ex ante*, and awards production *ex post*. In his model, a supplier invests effort *after* receiving a contract. Thus, supplier effort is regulated by the contract, and is not an essential concern that drives supply base design. Wan and Beil (2008) analyze supply base configuration for procurement auctions, trading off the value of supplier competition in reducing purchasing costs and the benefit of supplier diversification in mitigating supplier cost shocks. They focus on the impact of the buyer's bargaining power, without considering suppliers' investment of effort. Agrawal and Nahmias (1997) analyze

the optimal number of suppliers when the yield of the product delivered from each supplier is random. They do not consider strategic suppliers.

To summarize, the existing literature ignores either the buyer's investment of resources for suppliers or suppliers' improvement effort, and/or assumes an exogenous supply base. We contribute to the literature by studying supply base design as an endogenous decision, considering both supplier competition and supplier investment of effort.

3. Model

A downstream firm (the "buyer") sources an essential input to her final product from external suppliers. Each unit of the final product requires a unit of the input, and is sold at price r . The demand of the product, X , is uncertain, and is drawn from a probability distribution $G(\cdot)$ (density $g(\cdot)$) with mean μ_d and on the support $[\underline{d}, \bar{d}]$. We let $X = \mu_d + \sigma_d X_0$, where X_0 is a random variable with zero mean, and σ_d characterizes the demand uncertainty.

There are two upstream firms, Supplier 1 and Supplier 2, that the buyer may use as her suppliers. The buyer invests in the production capacity for a supplier. Let the capacity invested for Supplier i be Q_i , $i = 1, 2$. Denote by $\mathbf{Q} \doteq (Q_1, Q_2)$ the suppliers' capacity profile, and $\bar{Q} \doteq Q_1 + Q_2$ the total capacity of suppliers. Each unit capacity costs the buyer k . Besides the capacity and purchasing costs, all other costs of the buyer are normalized to zero. Define $S(\mathbf{Q}) \doteq \mathbb{E}[\min(X, \bar{Q})]$, $X \sim G(\cdot)$, as the expected demand covered with supplier capacity \mathbf{Q} .

Observing the capacity profile \mathbf{Q} , each supplier i invests effort e_i to reduce his production cost. The effort, such as the activities to improve the production process, quality control, or design of tools, is unobservable by the other supplier or by the buyer, and incurs costs independent of the capacity of a supplier. Denote by $\mathbf{e} \doteq (e_1, e_2)$ the suppliers' effort profile. The cost of supplier effort e is characterized by an increasing convex function $\varphi(e)$, $\varphi'(e) > 0$, $\varphi''(e) > 0$. The realization of a supplier's production cost is uncertain, due to uncertainty of the outcome of cost-reduction effort and variations in the supplier's internal and external environments; the realized production cost γ_i of Supplier i is a random cost c_i reduced by his effort e_i : $\gamma_i = c_i - e_i$. The random cost $c_i \in [\underline{c}, \bar{c}]$ is drawn independently from a probability distribution $F_i(\cdot)$ (density $f_i(\cdot)$), and is called the supplier's *type*. Denote by $\boldsymbol{\gamma} \doteq (\gamma_1, \gamma_2)$ the profile of suppliers' final costs. In line with the mechanism design literature, we assume that $F_i(\cdot)$ is log-concave, i.e., $\frac{F_i(\cdot)}{f_i(\cdot)}$ is increasing. This condition is satisfied by a wide variety of commonly used distributions, including uniform and normal, and with some restrictions including beta, gamma and Weibull (Rosling, 2002; Bagnoli and Bergstrom, 2005). The realized production cost γ_i is a supplier's private information, although the distribution $F_i(\cdot)$ is common knowledge. In order to focus on the impact of the buyer's investment on the supply base performance, we assume that the suppliers are ex ante identical, i.e., $F(\cdot) \doteq F_1(\cdot) = F_2(\cdot)$.

After the demand and supplier costs are realized, the buyer conducts an auction between the

suppliers to determine the quantity sourced from and the price paid to each supplier. The buyer designs the auction to maximize her profit. According to the revelation principle, we restrict the auction to a direct auction mechanism, without loss of generality. Under this mechanism, the buyer offers each supplier i a menu of contracts parameterized by the suppliers' cost profile γ : $(q_i(\gamma), t_i(\gamma))$, where q_i is the quantity and t_i the payment for Supplier i . Suppliers report their costs given the contract menus. Based on the reported cost profile $\hat{\gamma} \doteq (\hat{\gamma}_1, \hat{\gamma}_2)$, each supplier i is awarded the contract $(q_i(\hat{\gamma}), t_i(\hat{\gamma}))$.

Since the buyer does not observe the suppliers' efforts when determining the auction mechanism (contract menus), the efforts and auction mechanism are determined simultaneously: In an equilibrium, each supplier's effort is his best response to the other supplier's effort and the buyer's auction mechanism, and the buyer's auction mechanism is her best response to the suppliers' effort profile.

The sequence of events, along with the major parameters, is summarized in Figure 1.

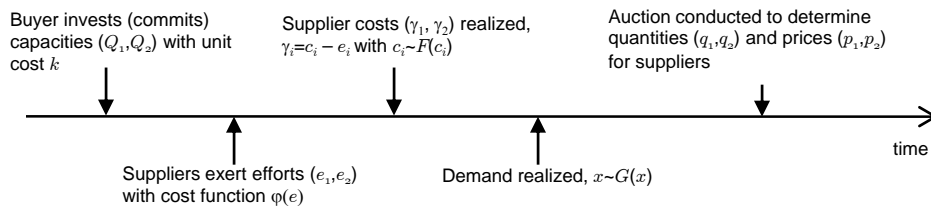


Figure 1: *Sequence of events*

Discussion is in order about our model of the sequence. We assume that the firms negotiate the contracts ex post after the suppliers' investment of effort and the realization of their costs. This assumption is based on the consideration that writing a contract ex ante with suppliers' costs uncertain can be difficult for the firms: First, it is costly to write a complete contract with the terms contingent on every possible state; and second, firms may not be able to credibly commit to a non-contingent contract, as it is likely ex post inefficient. For these reasons, the contracts are negotiated after the uncertainties are resolved. In Section 6, we consider an extension of the mechanism in which the buyer commits to the purchasing prices *before* suppliers invest cost-reduction effort.

Considering that cost reduction is usually a continuous process that lasts beyond the time of capacity investment, we assume that the buyer invests in (commits to) supplier capacity before the suppliers exert cost-reduction effort. In some situations, the investments of the buyer and suppliers may occur in parallel, instead of following the sequence as we have assumed. In this case, our model still applies if the buyer's capacity commitment is contractible³, similar to that in Iyer

³Since the buyer's investment of capacity is usually realized in physical or financial forms, it is reasonable to assume that the buyer's investment is along a well-defined dimension and thus is contractible. This assumption is

et al. (2005). In Section 7, we extend our consideration to the situation where the buyer invests in supplier capacity *after* the suppliers' investment of effort (and the realization of supplier costs).

4. Analysis

The analysis is performed in three steps. We first examine, in §4.1, for given capacity profile \mathbf{Q} and supplier effort profile \mathbf{e} , the auction mechanism design as the buyer's best-response strategy. Then, in §4.2, for given \mathbf{Q} , we derive the equilibrium of suppliers' efforts based on the buyer's best-response auction mechanism. Finally, in §4.3, we analyze the buyer's optimal capacity investment for the suppliers, given the equilibrium of the auction mechanism and suppliers' efforts as a function of the capacity profile.

4.1 Best-response auction mechanism for given capacity profile \mathbf{Q} and effort profile \mathbf{e}

Given a supplier's type c_i and effort e_i , we define $J_i(c_i, e_i) \doteq c_i - e_i + \frac{F_i(c_i)}{f_i(c_i)}$ as the supplier's virtual cost, which can be interpreted as the true cost inflated by information rent (Myerson, 1981). Since $F_i(\cdot)$ is log-concave, $J_i(c_i, e_i)$ is increasing in c_i and decreasing in e_i . We assume that the buyer's revenue is large enough to cover the capacity cost and highest possible virtual cost of a supplier: $r > k + J_i(\bar{c}_i, 0)$ for $i = 1, 2$. This assumption implies that it is always profitable for the buyer to trade with a supplier, even if the supplier has the lowest possible efficiency. Based on this assumption, no supplier cutoff needs to be considered.

Let $-i$ denote the other supplier as opposed to Supplier i . It can be shown that in the optimal auction mechanism, the suppliers are compared by their virtual costs (Myerson, 1981). The one with the lower virtual cost wins the competition by supplying as much of the buyer's demand as possible subject to his capacity constraint. The supplier with the higher virtual cost loses, and receives only the demand overflow beyond the winner's capacity. Recall $S(Q) \doteq \mathbb{E}[\min(X, Q)]$ and $\bar{Q} \doteq Q_1 + Q_2$. Based on this demand allocation rule, the expected volume provided by Supplier i as a winner is $S(Q_i)$, and as a loser is $S(\bar{Q}) - S(Q_{-i})$. We now define:

$$A(\mathbf{Q}) \doteq 2S(Q_1) - S(\bar{Q}) \quad \text{and} \quad B(\mathbf{Q}) \doteq S(Q_2) + S(Q_1) - S(\bar{Q}).$$

$A(\mathbf{Q})$ is the expected quantity difference between Supplier 1 as the winner, $S(Q_1)$, and Supplier 2 as the loser, $S(\bar{Q}) - S(Q_1)$. $B(\mathbf{Q})$ is the expected quantity difference for Supplier i between when he is the winner, $S(Q_i)$, and when he is the loser, $S(\bar{Q}) - S(Q_{-i})$. Lemma 1 is intuitive and useful for later analysis:

in contrast to supplier investment of efforts, which typically involves activities such as personnel training, R&D, management and logistics improvement, etc., that are not easy to measure.

Lemma 1 For given total capacity \bar{Q} , $A(\mathbf{Q})$ increases with $Q_1 - Q_2$, maximized with $Q_1 = \bar{Q}$ and $Q_2 = 0$, and $B(\mathbf{Q})$ decreases with $|Q_1 - Q_2|$, maximized with $Q_1 = Q_2 = \frac{\bar{Q}}{2}$.

Lemma 2 characterizes the firms' profits from the buyer's best-response auction design.

Lemma 2 In an optimal auction mechanism for given \mathbf{Q} and \mathbf{e} ,

i) the expected profit of Supplier i with cost realization γ_i , excluding the cost of effort, is

$$u_i(\gamma_i, \mathbf{Q}, \mathbf{e}) = \begin{cases} \int_{\underline{c}_i - e_i}^{\bar{c}_i - e_i} \bar{q}_i(\rho, \mathbf{Q}, \mathbf{e}) d\rho + (\underline{c}_i - e_i - \gamma_i) \bar{q}_i(\underline{c}_i - e_i, \mathbf{Q}, \mathbf{e}) & \text{if } \gamma_i < \underline{c}_i - e_i \\ \int_{\gamma_i}^{\bar{c}_i - e_i} \bar{q}_i(\rho, \mathbf{Q}, \mathbf{e}) d\rho & \text{if } \gamma_i \in [\underline{c}_i - e_i, \bar{c}_i - e_i] \\ 0 & \text{else } \gamma_i > \bar{c}_i - e_i. \end{cases} \quad (1)$$

where for $\rho \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$,

$$\begin{aligned} \bar{q}_i(\rho, \mathbf{Q}, \mathbf{e}) &\doteq S(Q_i) \Pr(J_i(\rho + e_i, e_i) \leq J_{-i}(c_{-i}, e_{-i})) \\ &\quad + (S(\bar{Q}) - S(Q_{-i})) \Pr(J_i(\rho + e_i, e_i) > J_{-i}(c_{-i}, e_{-i})). \end{aligned}$$

ii) the expected profit of the buyer, excluding the cost of capacity investment, is:

$$\begin{aligned} U(\mathbf{Q}, \mathbf{e}) &= (r - \mathbb{E}[J_1(c_1, e_1) + J_2(c_2, e_2)]/2)S(\bar{Q}) - \mathbb{E}[J_1(c_1, e_1) - J_2(c_2, e_2)]A(\mathbf{Q})/2 \quad (2) \\ &\quad + \mathbb{E}[J_1(c_1, e_1) - J_2(c_2, e_2) | J_1(c_1, e_1) > J_2(c_2, e_2)] \Pr(J_1(c_1, e_1) > J_2(c_2, e_2))B(\mathbf{Q}). \end{aligned}$$

In Lemma 2 i), $\bar{q}_i(\rho, \mathbf{Q}, \mathbf{e})$ is the expected quantity to be provided by Supplier i when his cost realization is $\rho \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$. It is easily shown that a supplier's profit $u_i(\gamma_i, \mathbf{Q}, \mathbf{e})$ is decreasing in his cost realization γ_i . This provides incentive for a supplier to exert effort to (stochastically) reduce his cost. This will be analyzed in Section 4.2. As shown in Lemma 2 ii), the buyer's expected profit U takes into consideration suppliers' virtual costs as their ex ante unit costs. The composition of U can be interpreted as follows: The first line of Equation (2) characterizes the profit that the buyer would achieve if Supplier 1 would always be treated as the winner. The first term is the profit achieved if the suppliers have equal chances to win. The second term is the difference made by changing the winner from Supplier 2 to Supplier 1 for half of the cases: The quantity allocated to Supplier 1 is increased by $A(\mathbf{Q})$, and the (ex ante) unit cost for this part of quantity is changed by $\mathbb{E}[J_1(\gamma_1, e_1) - J_2(\gamma_2, e_2)]$. However, the allocation with Supplier 1 always as the winner ignores supplier competition: If Supplier 2 has a lower virtual cost, the buyer is better off by choosing Supplier 2, but not Supplier 1, as the winner. Such a change increases Supplier 2's expected volume by $B(\mathbf{Q})$, and improves the buyer's unit profit by $\mathbb{E}[J_1(\gamma_1, e_1) - J_2(\gamma_2, e_2) | J_1(\gamma_1, e_1) > J_2(\gamma_2, e_2)]$, with the total benefit of supplier competition characterized in the second line of (2).

4.2 Equilibrium efforts for given supplier capacity profile \mathbf{Q}

Given Lemma 2, if Supplier $-i$ invests effort e_{-i} and the auction mechanism is designed in response to supplier efforts $\mathbf{e}=(e_1, e_2)$, then Supplier i 's expected profit as a function of his own effort \hat{e}_i ,

taking into consideration the cost of effort, is $\hat{\Pi}_i(\hat{e}_i, \mathbf{Q}, \mathbf{e}) \doteq \mathbb{E}_{c_i} [u_i(c_i - \hat{e}_i, \mathbf{Q}, \mathbf{e})] - \varphi(\hat{e}_i)$. The supplier efforts e_1 and e_2 constitute an equilibrium for given \mathbf{Q} if $\hat{\Pi}_i(\hat{e}_i, \mathbf{Q}, \mathbf{e})$ is maximized at $\hat{e}_i = e_i$ for both $i = 1, 2$.

Since suppliers are ex ante identical, without loss of generality we impose $Q_1 \geq Q_2$, i.e., Supplier 1 is equipped with greater capacity than Supplier 2. Then based on Lemma 1, for given total capacity, $A(\mathbf{Q})$ increases while $B(\mathbf{Q})$ decreases with the capacity difference $Q_1 - Q_2$. For tractability of analysis, hereafter we assume that a supplier's type c_i follows a uniform distribution on $[\underline{c}, \bar{c}]$ with $\Delta \doteq \bar{c} - \underline{c}$. Furthermore, we assume a quadratic form of the cost function of supplier effort: $\varphi(e) = \frac{a}{2}e^2$, where $a > 0$ is a constant.

Given that the auction mechanism and supplier effort are determined simultaneously, a pure strategy equilibrium of such decisions may not exist. To see this, let the supplier type be a constant c (i.e., $\Delta = 0$). If a supplier invests effort e in a pure-strategy equilibrium, the buyer will always propose a price $p = c - e$, which leaves the supplier a total profit $-\varphi(e)$. Expecting such a strategy on the part of the buyer, however, the supplier will not invest any effort, resulting in $e = 0$. Then for the buyer, the best-response strategy is to offer a price $p = c$. However, expecting such a price, the supplier will invest positive effort e to maximize his total profit $e - \varphi(e)$. Thus, a pure strategy equilibrium does not exist with $\Delta = 0$. To ensure the existence of a pure strategy equilibrium, we assume that the supplier cost uncertainty Δ is sufficiently high: $\Delta \geq \frac{\mu_d}{a}$ (except in §6.2 where we analyze a mixed strategy equilibrium with $\Delta = 0$).

Assumption 1 $\Delta \geq \frac{\mu_d}{a}$.

Under these assumptions, the suppliers' equilibrium efforts are characterized in Lemma 3.

Lemma 3 *i) Given the supplier capacity profile \mathbf{Q} , $e_1(\mathbf{Q}) = \frac{S(\bar{Q})}{2a} + \Delta\eta(\mathbf{Q})$ and $e_2(\mathbf{Q}) = \frac{S(\bar{Q})}{2a} - \Delta\eta(\mathbf{Q})$ constitute the unique equilibrium of supplier efforts (for an auction mechanism in the best response to (e_1, e_2)), where $\eta(\mathbf{Q}) \in \left[0, \frac{S(Q_1)}{2a\Delta}\right]$ is uniquely defined by*

$$A(\mathbf{Q}) - (1 - \eta)^2 B(\mathbf{Q}) = 2a\Delta\eta. \quad (3)$$

ii) $e_i(\mathbf{Q})$ increases with Q_i and decreases with Q_{-i} .

Lemma 3 i) shows that the larger-capacity supplier, Supplier 1, always invests more effort than the smaller supplier, Supplier 2, and the average effort (unit cost reduction) of suppliers, $\frac{e_1 + e_2}{2}$, depends only on the total capacity, but not on the capacity allocation of suppliers. However, the difference of supplier efforts, $e_1 - e_2 = 2\Delta\eta$, depends on the capacity allocation according to Equation (3). Lemma 3 ii) suggests that a supplier is motivated to invest more effort when his own capacity increases or the opponent's capacity decreases. This implies that, for given total capacity,

a larger capacity difference causes a larger effort difference between suppliers. As we shall see in §4.3, not only the average effort, but also the difference of effort, affect the buyer's profit.

It is interesting to note that our result of positive supplier effort is based on the assumption that the effort is unobservable. In fact, if supplier effort is observable, then a supplier will *not* invest any effort: For any effort of the supplier, the buyer would deduct the cost reduction achieved by the supplier from the price offer, and the supplier is unable to derive any benefit from his cost reduction. Thus, a supplier will have no incentive to invest effort if such effort is observable.

4.3 Optimal capacity investment

The buyer's overall expected profit as a function of the capacity investment profile \mathbf{Q} is $\Pi(\mathbf{Q}) = U(\mathbf{Q}, \mathbf{e}(\mathbf{Q})) - k\bar{Q}$, where $\mathbf{e}(\mathbf{Q})$ is the equilibrium of supplier efforts as characterized in Lemma 3, and $U(\mathbf{Q}, \mathbf{e})$ is the buyer's profit from the auction, defined in Lemma 2. $\Pi(\mathbf{Q})$ can be rewritten as in Proposition 1.

Proposition 1 *For given capacity investment profile \mathbf{Q} , the buyer's total profit is equal to*

$$\Pi(\mathbf{Q}) = \underbrace{\left(r - \bar{c} + \frac{S(\bar{Q})}{2a}\right) S(\bar{Q}) - k\bar{Q}}_{1) \text{ profit if suppliers were combined}} + \underbrace{\Delta \frac{(1 - \eta(\mathbf{Q}))^3}{3} B(\mathbf{Q})}_{2) \text{ benefit of supplier symmetry}} + \underbrace{\Delta \eta(\mathbf{Q}) A(\mathbf{Q})}_{3) \text{ benefit of supplier asymmetry}}, \quad (4)$$

where $\eta(\mathbf{Q})$ is defined by (3).

The buyer's total profit can be decomposed into three parts. The first part depends only on the total capacity but not on capacity allocation between suppliers. This part can be regarded as the profit that the buyer would receive if the two suppliers were combined as one, keeping the same total capacity \bar{Q} and (average) cost reduction effort $\frac{S(\bar{Q})}{2a}$. With a single supplier, the buyer would pay a price equal to the cost of the least efficient type, $\bar{c} - \frac{S(\bar{Q})}{2a}$, due to information asymmetry. The second part of the profit is related to the benefit gained from supplier competition. For given total capacity investment, a smaller capacity difference between the suppliers (resulting in greater $B(\mathbf{Q})$ and lower $\eta(\mathbf{Q})$) intensifies supplier competition, reducing information rent as well as improving cost-uncertainty mitigation. We refer to this part as the *benefit of supplier symmetry*. The third part is related to supplier effort. For a given total capacity, enlarging the capacity difference between suppliers (resulting in greater $A(\mathbf{Q})$ and higher $\eta(\mathbf{Q})$) leads the larger supplier (Supplier 1) to increase while the smaller supplier (Supplier 2) to decrease their efforts, without affecting the average effort. Such effort changes benefit the buyer, since Supplier 1 provides more quantity on expectation than Supplier 2 (Supplier 1 has a larger capacity, and also is more likely to win the auction for his greater effort). We call this part of profit the *benefit of supplier asymmetry*.

If the buyer invests in equal capacities for the suppliers, then $\eta = 0$ and there is no benefit of supplier asymmetry. If the buyer invests in only one supplier with $Q_2 = 0$, then $B = 0$, and the

benefit of supplier symmetry does not exist. The buyer's profit and suppliers' effort profile in these two cases are characterized in Corollary 2. The expressions will be useful in later discussions.

Corollary 2 *i) With $Q_1 = Q_2 = Q$, each supplier invests effort $\frac{S(2Q)}{2a}$, and the buyer's profit is*

$$\Pi(Q, Q) = \left(r - \bar{c} + \frac{S(2Q)}{2a} \right) S(2Q) - 2kQ + \underbrace{\frac{\Delta}{3} (2S(Q) - S(2Q))}_{\text{benefit of supplier symmetry}}. \quad (5)$$

ii) With $Q_1 = Q$ and $Q_2 = 0$, Supplier 1 invests effort $\frac{S(Q)}{a}$ and Supplier 2 invests zero, and the buyer's profit is

$$\Pi(Q, 0) = \left(r - \bar{c} + \frac{S(Q)}{2a} \right) S(Q) - kQ + \underbrace{\frac{S(Q)^2}{2a}}_{\text{benefit of supplier asymmetry}}. \quad (6)$$

5. Optimal supply base structure

The optimal supplier capacity profile (Q_1^*, Q_2^*) maximizes the buyer's profit as characterized in Proposition 1. The solutions can be classified into three types: *Sole sourcing* (S), in which only one supplier receives positive capacity investment ($Q_1^* > 0$ and $Q_2^* = 0$), *symmetric Dual sourcing* (sD), in which the two suppliers receive positive and equal capacity investments ($Q_1^* = Q_2^* > 0$), and *asymmetric Dual sourcing* (aD), in which the two suppliers receive positive but unequal capacity investments ($Q_1^* > Q_2^* > 0$). Note that the S structure is the most asymmetric and sD the most symmetric of the three structures. The aD structure is an intermediate choice between S and sD.

In this section, we analyze when the buyer should choose which supply base structure and how the total capacity investment varies with the choice. Specifically, we investigate the impact of the capacity cost k , cost of effort a , supplier cost uncertainty Δ and demand uncertainty σ_d on the buyer's decision. In order to develop an understanding of the basic economic tradeoffs, we first analyze the benchmark cases in §5.1 for a very large, or k equal to zero. Then we focus on the impact of k , a and Δ in §5.2, and the impact of σ_d in §5.3.

5.1 Benchmark analysis

If the cost of effort a is so high that a supplier will never invest any effort, it is intuitive that the buyer should always adopt *symmetric dual sourcing* to take advantage of the lower price that can be obtained from competing suppliers. This is formalized in Proposition 3:

Proposition 3 *When the cost of effort, a , is infinite, symmetric dual sourcing (sD) is optimal.*

If the cost of effort a is low, it may be attractive for a supplier to invest some effort. In this case, the buyer may be better off creating some asymmetry between the suppliers via capacity

investment, in order to provide stronger incentives to the larger supplier (Supplier 1) to exert cost-reduction effort. Proposition 4 indicates that, when capacity cost is zero, the buyer should select either the most symmetric (sD) or the most asymmetric (S) supply base structure. Specifically, S is favored against sD if supplier cost uncertainty $\Delta = \bar{c} - \underline{c}$ or the cost of effort a is low. With Δ low, the value of supplier competition is small. With a low, supplier investment of effort is very sensitive to the capacity profile. Both effects lead to the benefit of supplier asymmetry (see Equation 6) dominating that of supplier symmetry (see Equation 5). Thus, with the supply base design influencing supplier effort, even if capacity is free, the buyer may invest in capacity for only one supplier. Recall that \bar{d} is the highest demand realization.

Proposition 4 *When the capacity cost k is zero,*

- i) if $a\Delta < \frac{3\mu_d}{2}$, then sole sourcing (S) is optimal, $(Q_1^*, Q_2^*) = (\bar{d}, 0)$,*
- ii) otherwise if $a\Delta \geq \frac{3\mu_d}{2}$, symmetric dual sourcing (sD) is optimal, $(Q_1^*, Q_2^*) = (\bar{d}, \bar{d})$.*

Notice that sD has double of the total capacity investment of S when $k = 0$: In order to stimulate supplier competition, sD requires that sufficient capacity redundancy be built into the supply base so that the quantity to be sourced from the inferior supplier will be kept low. (Even with a strictly positive capacity cost, it can be shown that, when sD generates more profits than S, the former requires more capacity investment than the latter.) When $k = 0$, the capacity investment cost is not a concern in the choice of the supply base structure. However, if k is significant, then sD is disadvantageous because it requires a larger capacity. As we shall see in §5.2, not only will such a capacity cost concern influence the choice between S and sD, but may also lead to the intermediate choice of aD.

5.2 Impact of k , a , and Δ

As we have discussed, sD requires more capacity investment than S as it exploits supplier competition. Thus, it is intuitive that a higher capacity cost will negatively affect sD more than S, making S increasingly favored over sD. In addition, Proposition 5 shows that a new structure, aD, can arise for intermediate capacity costs.

Proposition 5 *When demand uncertainty σ_d is positive and very small, there exist k_1, k_2 and \hat{a} such that $k_1 < k_2$ if and only if $a > \hat{a}$, and*

- i) when $k \leq k_1$, symmetric dual sourcing (sD) is optimal, with $Q_1^* = Q_2^*$, and both Q_1^* and Q_2^* approaching μ_d as σ_d reduces to zero;*
- ii) when $k \in (k_1, \max(k_1, k_2))$, asymmetric dual sourcing (aD) is optimal, with Q_1^* approaching μ_d and Q_2^* approaching zero as σ_d reduces to zero;*
- iii) when $k \geq \max(k_1, k_2)$, sole sourcing (S) is optimal, with $Q_2^* = 0$, and Q_1^* approaching μ_d as σ_d reduces to zero.*

Proposition 5 i), ii) and iii) characterize the conditions of the capacity cost k for the buyer to adopt sD, aD, and S structures. Note the condition in ii) for aD, $k_1 < k < \max(k_1, k_2)$, exists only if supplier effort is costly: $a > \hat{a}$, resulting in $k_1 < k_2$. With a high, the benefit of supplier asymmetry inducing supplier effort is relatively small. In this case, aD arises for intermediate capacity costs ($k_1 < k < k_2$), while sD and S are optimal when the capacity costs are low ($k \leq k_1$) and high ($k \geq k_2$). We have explained that increasing k improves the attractiveness of S over sD. For intermediate k , aD is optimal because the capacity cost is too high to justify sD, but still low enough to make it beneficial for the buyer to create some supplier redundancy in order to enjoy the benefit of supplier competition. With aD, the buyer relies on the larger supplier for cost reduction, and uses the smaller supplier to maintain supplier competition, achieving a tradeoff between the benefits of supplier asymmetry and symmetry. Thus, *even though suppliers are ex ante identical, the optimal solution involving both suppliers may not be symmetric.*

Note further from Proposition 5 ii) that, when demand uncertainty is zero, aD has $Q_2^* = 0$, reducing to S. Thus, the buyer should consider aD as an option only when facing uncertain demand; otherwise, either sD or S is optimal in an environment without demand uncertainty. Our results are new and complement Riordan (1996). Assuming unit demand and supplier cost-reduction effort occurring after contracting, Riordan (1996) shows that the buyer should fully qualify either a single supplier or both suppliers, resulting in our single sourcing or symmetric dual sourcing structures. In that model, sole sourcing is driven only by capacity cost concerns, but not by the incentive of supplier effort. Considering the influence of supply base design on suppliers' investment of cost-reduction effort, we show that sole sourcing can be optimal even if capacity is free, and that demand uncertainty introduces asymmetric dual sourcing.

If the cost of effort a is sufficiently low ($a \leq \hat{a}$, resulting in $k_1 \geq k_2$), the benefit of supplier asymmetry will be so substantial that aD should be replaced with S. In this case, as shown in Proposition 5 i) and iii), the supply base structure considers only between sD and S: sD is preferred for $k \leq k_1$, while S is preferred for $k > k_1$.

The two threshold capacity costs, k_1 and k_2 , define the boundaries between sD and aD/S, and between aD and S. It can be shown that both k_1 and k_2 increase in a and Δ . This suggests that sD (S) becomes more (less) favorable as a or Δ increases. This is consistent with the insight revealed in Proposition 4 based on $k = 0$.

Proposition 5 is based on the demand uncertainty σ_d being very small. Through numerical experiments, we find that the insights from Proposition 5 about the impacts of k , a and Δ on the supply base structure hold in general situations when σ_d can be large. Figure 4 demonstrates, as an example, the supply base structure with varying k and a . We see that sD is preferred when k is low and a is large and, vice versa S preferred for k high and a small. When a is sufficiently high, aD is optimal for intermediate k values. The impact of Δ is similar to that of a .

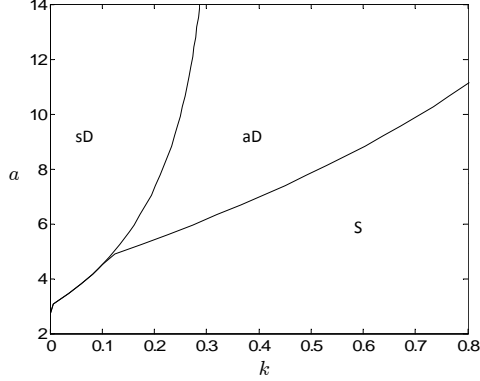


Figure 2: *The optimal supply base structure based on k and a . $r = 5$, $\underline{c} = 2$, $\bar{c} = 3$, demand following a uniform distribution on $[1.5, 2.5]$.*

5.3 Impact of σ_d

In this subsection, we discuss how demand uncertainty influences supply base design. We first focus on the case in which the demand uncertainty is very small:

Proposition 6 *With the demand uncertainty σ_d remaining small, increasing σ_d from zero may cause the following changes of the supply base structure:*

- i) When a is sufficiently small, the structure shifts from sD to S.*
- ii) When a is sufficiently large, the structure shifts from S to aD, or from aD to sD.*

When the cost of effort a is small, based on Proposition 5 the supply base structure considers only sD and S. Proposition 6 i) suggests that, if the cost of effort a is sufficiently small, then increasing demand uncertainty leads to the supply base structure shifting from sD to S. Since the total capacity investment is close to $2\mu_d$ in sD and close to μ_d in S when demand uncertainty is very small (see Proposition 5), the total capacity is reduced abruptly with the structure shifting from sD to S. This is somewhat counter-intuitive as one may think that a larger supply base would be desirable for a buyer facing more volatile demand, since it helps to mitigate demand uncertainty. This result can be explained by the influence of demand uncertainty on the service level (expected satisfied demand), supplier competition and supplier effort in each structure. We show that increasing demand uncertainty negatively affects sD more than S when a is small:

The buyer's profit from sD is characterized in Equation (5). Since the total capacity $2Q$ is close to $2\mu_d$ when demand uncertainty is very small, almost all demand will be satisfied, $S(2Q) \approx \mu_d$. Thus, neither the total expected demand satisfied by suppliers, $S(2Q)$, nor the effort of each supplier, $\frac{S(2Q)}{2a}$, change significantly with demand uncertainty σ_d . However, increasing σ_d reduces the benefit of the supplier competition generated in sD: As σ_d increases, the expected volume covered by the winning supplier, $S(Q)$, decreases, and meanwhile there will be more demand

overflow, $S(2Q) - S(Q)$, that is satisfied by the losing supplier ($S(Q)$ is much more sensitive to σ_d than $S(2Q)$). As the winner has a lower cost than the loser, higher σ_d increases the quantity allocated to the inefficient supplier and decreases the quantity allocated to the efficient supplier, reducing the benefit of supplier competition.

The buyer's profit from S is characterized in Equation (6). There is no supplier competition in S. Increasing demand uncertainty reduces the expected quantity satisfied by the supplier, $S(Q)$. In addition, it leads to the supplier investing less effort, $\frac{S(Q)}{2a}$. However, when a is low, a large capacity Q will be invested for the supplier, making $S(Q)$ insensitive to demand uncertainty. The negative impact of demand uncertainty in sD on both the service level and supplier effort is then negligible.

Therefore, if the cost of effort a is small, the negative effect of increasing demand uncertainty under S is quite small, while its effect under sD is still significant (for its influence on supplier competition). As a result, when demand uncertainty increases, the optimal supply base structure shifts from sD to S.

If a is high, the negative effects of demand uncertainty in S will become significant. This causes the supply base structure to shift in the opposite direction, from S to sD, as σ_d increases. However, since large a introduces aD as a structure between S and sD for $k \in (k_1, k_2)$ (see Proposition 5 ii), the structure will shift from S to aD (if k is close to k_2), or from aD to sD (if k is close to k_1), as stated in Proposition 6 ii).

Proposition 6 i) and ii) together suggest that demand uncertainty σ_d may drive the supply base design in opposite directions depending on the cost of effort: Increasing demand uncertainty (when it is low) drives the supply base away from sD and towards S when a is small, but towards sD and away from S when a is large. In other words, facing greater demand uncertainty, the buyer is more inclined toward sole sourcing (S) if a is small and toward dual sourcing (aD/sD) if a is large, while she prefers symmetric suppliers (sD) if a is large and asymmetric suppliers (S/aD) if a is small. The impact of σ_d on the supply base design is illustrated in Figure 3. In the space of capacity cost and effort cost ($k-a$), the left plot draws the boundary between symmetric (sD) and asymmetric (aD or S) sourcing, and the right plot draws the boundary between sole (S) and dual (sD or aD) sourcing. As σ_d increases, the boundaries in both plots shift to the left when a is low, and to the right when a is high.

Proposition 6 is based on the case when demand uncertainty is very small. We investigate numerically the situation when demand uncertainty can be large. When a is large, while increasing demand uncertainty up to a certain level improves the preference for sD rather than for S/aD, we find that the direction can reverse if σ_d increases further. This is illustrated in Figure 4. It shows the optimal supply base structure with varying k and σ_d . For σ_d less than about 4, as σ_d increases the structure shifts from S/aD to sD. This is consistent with Proposition 6 for the case when σ_d is

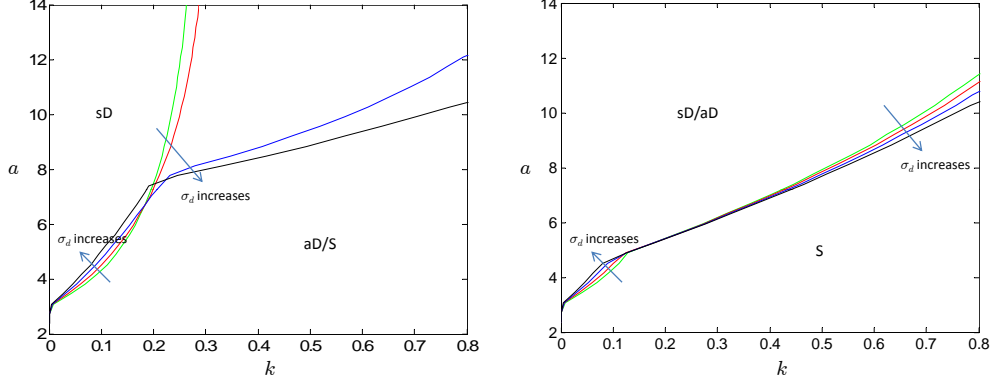


Figure 3: *The boundary of the region for sD (left plot) and for S (right plot). $\underline{c} = 2$, $\bar{c} = 3$, $r = 5$, and demand following a uniform distribution on $[2 - \sigma_d/2, 2 + \sigma_d/2]$. Along the direction of the arrows, $\sigma_d = 0.1, 1, 2, 3$ for the curves.*

small. However, as σ_d increases above 4, the structure shifts in the opposite direction—the area of k values for sD shrinks, while that for aD/S expands.

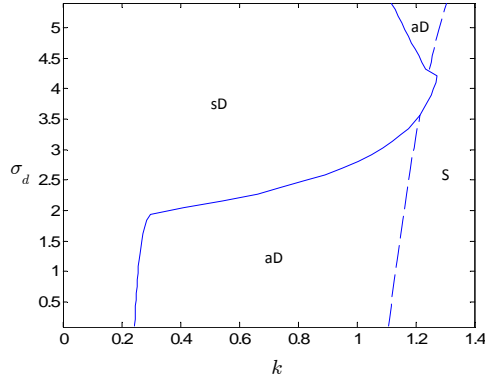


Figure 4: *The optimal supply base structure. $\underline{c} = 2$, $\bar{c} = 3$, $r = 6$, $a = 15$, and demand following a uniform distribution on $[2.7 - \sigma_d/2, 2.7 + \sigma_d/2]$*

Thus, when a is large (as for Figure 4), both very low and high demand uncertainty may favor supplier asymmetry (S or aD). The reason for sD losing its advantage under high σ_d is that both the service level and supplier effort in sD become susceptible to the influence of demand uncertainty: Recall that, when σ_d is small, the total and average expected quantities provided by the suppliers in sD are robust to the change of demand uncertainty, thanks to the protection provided by the redundant capacity. In the result, increasing demand uncertainty affects neither the service level nor the supplier effort. However, if σ_d is high, even the capacity in sD may not be sufficient to cover all demand, which results in the expected satisfied demand $S(2Q)$ decreasing in demand uncertainty. The decreasing quantity also leads to a decrease in the investment of effort made by

a supplier, $\frac{S(2Q)}{2a}$. As a result, when σ_d is high, not only supplier competition, but also the service level and supplier effort will be negatively affected by increasing demand uncertainty, thus reducing the relative advantage of sD.

We summarize the influence of a , Δ , k , and σ_d on the supply base structure in Table 1. Increasing cost of effort ($a \uparrow$), increasing supplier cost uncertainty ($\Delta \uparrow$), and decreasing capacity cost ($k \downarrow$) all cause the supply base structure to shift from S to sD, directly or indirectly through aD. However, increasing demand uncertainty ($\sigma_d \uparrow$) may drive the supply base design in different directions depending on a : If a is small, increasing σ_d causes the supply base structure to shift away from sD toward aD or S. If a is large, the opposite is true, although further increases in σ_d may reverse the direction.

Changing factor	Impact on supply base structure
$a \uparrow, \Delta \uparrow, k \downarrow$	S→aD→sD
$\sigma_d \uparrow$	sD→aD/S (a small) aD/S→sD(→aD/S) (a large)

Table 1: *The influence of a , Δ , k and σ_d on the supply base structure*

We discuss the managerial implications of Table 1. The cost of effort can be interpreted as the maturity of production technology: The cost to improve a mature technology is usually high. Our results suggest that, when sourcing items produced with a mature technology, inducing fierce supplier competition through supply base design (i.e., investing in equal or close capacities for both suppliers with an sD or aD supply base structure) is beneficial for the buyer. The supplier cost uncertainty measures the complexity of the product: The more subcomponents included in the product, the more uncertain is the total production cost. Our results suggest that supplier competition is also preferred when the buyer sources complex products. The capacity cost is related to the level of customization of the product: A highly customized product requires more specific capacity, which is difficult to redeploy for other buyers. For such products, the capacity cost is high, thus sole sourcing or a highly asymmetric supply base is more profitable for the buyer. Finally, demand uncertainty depends on the product life cycle: Demand uncertainty is usually high for a new product, but low for a mature product. Our results suggest that the impact of demand uncertainty varies with the maturity of production technology (a): For production based on an innovative technology (a small), the buyer should adopt a single supplier or a highly asymmetric supply base for a new product, then increase the level of supplier competition by shifting to symmetric dual sourcing as the product matures; however, the supply base may evolve in the opposite direction for production based on a mature technology (a large).

6. Ex ante price commitment mechanism

In our base model, we have considered an ex post price negotiation mechanism in which the purchasing prices are determined *after* suppliers exert cost-reduction efforts. In this section, we consider an alternative mechanism, whereby the buyer commits to the purchasing price (and capacity) *before* a supplier invests his effort. We call it the ex ante price commitment mechanism. In this mechanism, the contract price is specified with uncertainty of a supplier's production cost. Because it is difficult for the parties to write a court-enforceable complete contract with the terms contingent on every possible state of the outcome (Grossman and Hart, 1986; Hart and Moore, 1988), we focus on non-contingent contracts, with the terms independent of the realization of suppliers' costs. A non-contingent contract is likely ex post inefficient—the committed price may be lower than a supplier's cost. In view of such a possibility, we allow a supplier to quit his contract after he observes his cost. With the effort cost sunk at this time point, a supplier will quit if the contract price is lower than his cost.⁴ (Allowing suppliers to quit brings a fair comparison of the ex ante price commitment mechanism to our base mechanism, which allows a supplier not to participate in the auction after he observes his cost.) We differentiate the notations for the price commitment mechanism with a superscript “*P*”.

In this ex ante price commitment mechanism, the buyer first decides the capacity investments $\mathbf{Q} \doteq (Q_1, Q_2)$ and unit purchasing prices $\mathbf{p} \doteq (p_1, p_2)$ offered to each supplier. Receiving the capacity and price commitments, the suppliers invest cost reduction efforts $\mathbf{e} \doteq (e_1, e_2)$. Then the demand x and suppliers' production costs $\boldsymbol{\gamma} \doteq (\gamma_1, \gamma_2)$ are realized. A supplier i will quit if $\gamma_i > p_i$. Finally, the buyer decides the quantity sourced from each remaining supplier. It is obvious that the buyer will source as much as possible from the lower-price supplier, with only the demand overflow sourced from the higher-price supplier, subject to their capacity constraints.

Without loss of generality, we assume $p_1 \leq p_2$. Thus, if Supplier 1 stays, i.e., $c_1 \leq p_1 + e_1$, he always receives an expected quantity $q_1(\mathbf{Q}, p_2, e_2) = S(Q_1)$, regardless of whether Supplier 2 stays. If Supplier 2 stays, i.e., $c_2 \leq p_2 + e_2$, he receives an expected quantity $S(Q_2)$ if Supplier 1 quits, and $S(Q_1 + Q_2) - S(Q_1)$ if Supplier 1 stays. Thus the total expected quantity of Supplier 2 if he stays is equal to

$$q_2(\mathbf{Q}, p_1, e_1) = (S(Q_1 + Q_2) - S(Q_1)) \Pr(c_1 \leq p_1 + e_1) + S(Q_2) (1 - \Pr(c_1 \leq p_1 + e_1)).$$

Then for given \mathbf{Q} , \mathbf{p} , and \mathbf{e} , the expected profit of Supplier i is:

$$\Pi_i^P(\mathbf{Q}, \mathbf{p}, \mathbf{e}) = \mathbb{E}[p_i - c_i + e_i | c_i \leq p_i + e_i] \Pr(c_i \leq p_i + e_i) q_i(\mathbf{Q}, p_{-i}, e_{-i}) - \varphi(e_i).$$

⁴The supplier quitting is different from bankruptcy (Swinney and Netessine, 2009). A supplier will not quit as long as the *future* cash flow is positive, i.e., the production cost is lower than the price, even if the total cash flow, after taking into consideration the cost of effort, may be negative. A supplier without access to other financial resources will bankrupt if the total cash flow is negative. We assume a supplier has access to financial resources so that he will not bankrupt even if his total cash flow is negative (a supplier's expected total cash flow will always be positive, as the supplier decides his effort to maximize the expected profit).

For given \mathbf{Q} and \mathbf{p} , $\mathbf{e}^P \doteq (e_1^P, e_2^P)$ constitutes an equilibrium of supplier efforts if e_i^P maximizes $\Pi_i^P(\mathbf{Q}, \mathbf{p}, \mathbf{e})$ on e_i for $e_{-i} = e_{-i}^P$, $i = 1, 2$. The equilibrium of efforts \mathbf{e}^P is characterized in Lemma 4. It can be shown that e_i^P increases in Q_i and p_i , $i = 1, 2$, and e_2^P decreases in Q_1 and p_1 . Thus, in the price commitment mechanism, both the price and capacity decisions influence supplier efforts. This is different from the ex post price negotiation mechanism in our base model, in which only capacity investment influences the suppliers' efforts.

Lemma 4 *In the ex ante price commitment mechanism, for given \mathbf{Q} and \mathbf{p} , the suppliers' efforts are:*

$$e_1^P(\mathbf{Q}, \mathbf{p}) = \begin{cases} \frac{S(Q_1)}{a} & \text{if } p_1 \geq \bar{c} - \frac{S(Q_1)}{a} \\ (p_1 - \underline{c}) \frac{S(Q_1)}{a\Delta - S(Q_1)} & \text{if } \underline{c} \leq p_1 < \bar{c} - \frac{S(Q_1)}{a} \\ 0 & \text{else } p_1 < \underline{c} \end{cases},$$

$$e_2^P(\mathbf{Q}, \mathbf{p}) = \begin{cases} \frac{q_2(\mathbf{Q}, \mathbf{p}, e_1^P(\mathbf{Q}, \mathbf{p}))}{a} & \text{if } p_2 \geq \bar{c} - \frac{q_2(\mathbf{Q}, \mathbf{p}, e_1^P(\mathbf{Q}, \mathbf{p}))}{a} \\ (p_2 - \underline{c}) \frac{q_2(\mathbf{Q}, \mathbf{p}, e_1^P(\mathbf{Q}, \mathbf{p}))}{a\Delta - q_2(\mathbf{Q}, \mathbf{p}, e_1^P(\mathbf{Q}, \mathbf{p}))} & \text{if } \underline{c} \leq p_2 < \bar{c} - \frac{q_2(\mathbf{Q}, \mathbf{p}, e_1^P(\mathbf{Q}, \mathbf{p}))}{a} \\ 0 & \text{else } p_2 < \underline{c} \end{cases}.$$

Based on $\mathbf{e}^P(\mathbf{Q}, \mathbf{p})$, the buyer designs \mathbf{Q} and \mathbf{p} to maximize her profit:

$$\Pi^P(\mathbf{Q}, \mathbf{p}) = -k(Q_1 + Q_2) + \sum_{i=1}^2 \Pr(c_i \leq p_i + e_i^P(\mathbf{Q}, \mathbf{p})) (r - p_i) q_i(\mathbf{Q}, p_{-i}, e_{-i}^P(\mathbf{Q}, \mathbf{p})). \quad (7)$$

Note that the price p_i committed to Supplier i influences both the supplier's probability to stay, $\Pr(c_i \leq p_i + e_i^P(\mathbf{Q}, \mathbf{p}))$, and the buyer's profit margin from the supplier, $r - p_i$, if the supplier stays. Thus, the buyer's price decision trades off the margin and the probability that the supplier stays. Recall that we assume $p_1 \leq p_2$. It can be shown that, in the optimal solution, the buyer should make the higher price, p_2 , high enough to ensure that Supplier 2 will never quit (i.e., $\Pr(c_2 \leq p_2 + e_2) = 1$), but may design the lower-price, p_1 , to allow positive probability that Supplier 1 will quit. Thus, the buyer maintains a supply base consisting of a supplier that provides a low profit margin and will always stay, and a supplier that provides a high profit margin and may quit.

Lemma 5 *In the optimal price commitment mechanism, the capacity and price commitments, \mathbf{Q} and \mathbf{p} , satisfy $\Pr(c_2 \leq p_2 + e_2^P(\mathbf{Q}, \mathbf{p})) = 1$ with $p_2 = \bar{c} - \frac{q_2(\mathbf{Q}, p_1, e_1^P(\mathbf{Q}, \mathbf{p}))}{a}$.*

6.1 Optimal supply base structure

As the suppliers differ in terms of risk of quitting and profit margin provided to the buyer, we find in our study that the buyer will not invest equal capacities in the suppliers. That is, symmetric dual sourcing will never be optimal, even if the cost of effort, a , is very large. Proposition 7 analyzes the supply base design when demand uncertainty is very small. Let (Q_1^{P*}, Q_2^{P*}) be the optimal supplier capacity profile.

Proposition 7 *When demand uncertainty σ_d is positive and very small, there exist k_1^P , k_2^P and \hat{a}^P such that $k_1^P < k_2^P$ if and only if $a > \hat{a}^P$, and*

i) when $k \leq k_1^P$, asymmetric dual sourcing (aD) is optimal, with $Q_2^{P} > Q_1^{P*} > 0$, and both Q_1^{P*} and Q_2^{P*} approaching μ_d as σ_d reduces to zero;*

ii) when $k \in (k_1^P, \max(k_1^P, k_2^P))$, asymmetric dual sourcing (aD) is optimal, with $Q_2^{P} > Q_1^{P*} > 0$, and Q_1^{P*} approaching zero and Q_2^{P*} approaching μ_d as σ_d reduces to zero;*

iii) when $k \geq \max(k_1^P, k_2^P)$, sole sourcing (S) is optimal, with $Q_1^{P} = 0$, and Q_2^{P*} approaching μ_d as σ_d reduces to zero.*

Proposition 7 exhibits a similar structure to Proposition 5, except that, even when k is very small ($k \leq k_1^P$), aD is preferred over sD in the ex ante price commitment mechanism, while sD is optimal when k is sufficiently low in the ex post negotiation mechanism. Thus, for k in both ranges of small ($k \leq k_1^P$) and intermediate ($k_1^P < k \leq k_2^P$, existing for $a > \hat{a}^P$) values, aD is optimal under ex ante price commitment. However, the aD structures in the two capacity-cost ranges exhibit different features: When demand becomes deterministic, the aD for the lower range of k reduces to sD, while that for the higher range of k reduces to S (as in the ex post negotiation mechanism). Thus, we may consider the aD structure for the lower range of k qualitatively close to sD, while that for the higher range of k close to S.

Despite the fact that the buyer will never invest equal capacities in the two suppliers in the price commitment mechanism, we find that the parameters influence the supply base design in a similar way as in the ex post negotiation mechanism: The preference for dual sourcing improves while the preference for sole sourcing declines as the capacity cost k decreases, the cost of effort a increases, or supplier cost uncertainty Δ increases. In addition, under increasing demand uncertainty σ_d , the supply base structure shifts from (asymmetric) dual sourcing to sole sourcing, along with reduction of the total capacity, when a is low, and the opposite is true when a is high.

In summary, the price commitment mechanism exhibits major features that are qualitatively similar to those of the ex post price negotiation mechanism, except that symmetric dual sourcing does not arise in this mechanism, and is replaced with asymmetric dual sourcing where the two supplier capacities are different, but are relatively close.

6.2 Ex ante vs. ex post price decision

If both ex post price negotiation and ex ante price commitment are feasible, which mechanism should the buyer choose, and how does the buyer's choice of mechanism influence the supply base structure? In this section, we compare the two mechanisms in the buyer's profit and the preferred supply base structure. Proposition 8 presents the results for the situation when the capacity cost k or demand uncertainty σ_d is very small.

Proposition 8 *When the capacity cost k or demand uncertainty σ_d is very small,*

i) if sole sourcing is optimal with ex post price negotiation, then it is optimal with ex ante price commitment;

ii) ex ante price commitment generates less profits for the buyer than ex post price negotiation.

Proposition 8 i) indicates that, in the ex ante price commitment mechanism, sole sourcing is considered more favorably than in the ex post price negotiation mechanism. This is because the buyer benefits more from supplier competition with ex post price negotiation than with ex ante price commitment. Recall that, in the ex post negotiation mechanism, supplier competition brings two positive effects: 1) It mitigates supplier cost uncertainty, as the buyer is able to select the lower-cost supplier; and 2) it reduces the information rent, as the winning supplier must reduce the price in the face of the pressure of competition. In the price commitment mechanism, the prices are pre-specified and independent of the realized costs. Thus, supplier competition no longer provides the benefit of reducing information rent, although it still allows the buyer to hedge against supplier-cost uncertainty by providing an alternative sourcing opportunity in case of a supplier quits. Thus, the buyer benefits less from supplier competition and favors more sole sourcing in the ex ante price commitment mechanism, than in the ex post negotiation mechanism.

Proposition 8 ii) concludes that the buyer is better off with ex post price negotiating than with ex ante price commitment. We have explained that ex ante price commitment delivers less value from supplier competition. Without supplier competition, in sole sourcing, it can be shown that ex ante price commitment becomes equivalent to ex post negotiation (the buyer offers a price based on the least efficient supplier type \bar{c} in both mechanisms in order to ensure supplier participation). As a result, ex post negotiation is always (weakly) better for the buyer than ex ante price commitment.

Our conclusion of ex post price negotiation dominating ex ante price commitment is established by Assumption 1: Supplier cost uncertainty is sufficiently high, $\Delta \geq \frac{\mu_d}{a}$. This assumption guarantees the existence of a pure strategy equilibrium of supplier efforts and the auction mechanism. It is worth noting that the conclusion may reverse if Assumption 1 is violated, with Δ lower than $\frac{\mu_d}{a}$. This can be shown analytically for the extreme case without supplier cost uncertainty, $\Delta = 0$ (the analysis for general $\Delta < \frac{\mu_d}{a}$ is difficult). We focus on sole sourcing for both mechanisms, since the results with $\Delta \geq \frac{\mu_d}{a}$ have shown that the two mechanisms both favor sole sourcing, which leads to equal profits, when Δ is sufficiently small.

Without supplier cost uncertainty, the supplier type is a constant c . In sole sourcing, if the buyer leaves the price to be negotiated after the supplier effort is invested, the auction mechanism reduces to the buyer proposing a price p to the supplier, who may accept or reject the price based on his cost, $c - e$. Thus, an equilibrium of the price p and supplier effort e needs to be analyzed. Recall from the discussion before Assumption 1 in §4.2 that a pure-strategy equilibrium does not exist in this case. A mixed-strategy equilibrium is presented in Lemma 6.

Lemma 6 *When $\Delta = 0$, in the ex post price negotiation mechanism under sole sourcing, a price p following a uniform distribution on the support $\left[c - \frac{S(Q)}{a}, c\right]$ and supplier effort $e \geq 0$ drawn from a cumulative distribution $\Pr(e \leq x) = 1 - \frac{r-c}{r-c+x}$ constitute a mixed-strategy equilibrium. Based on this equilibrium, the buyer's expected profit is*

$$\tilde{\Pi}(Q) \doteq (r - c) S(Q) - kQ. \quad (8)$$

We see from Equation (8) that the buyer does not capture the supplier's cost reduction benefit in the ex post negotiation mechanism, even though the supplier may have exerted positive effort. This is not the case in the ex ante price commitment mechanism, as is shown in Lemma 7. In this mechanism, the buyer, with her first-mover advantage on pricing, is able to request a price based on her expectation of the supplier's effort.

Lemma 7 *When $\Delta = 0$, in the ex ante price commitment mechanism under sole sourcing, the buyer proposes a price $p = c - \frac{S(Q)}{a}$, and the supplier invests effort $e = \frac{S(Q)}{a}$. The buyer's profit is*

$$\tilde{\Pi}^P(Q) \doteq \left(r - c + \frac{S(Q)}{2a}\right) S(Q) - kQ.$$

It is clear that $\tilde{\Pi}^P(Q) \geq \tilde{\Pi}(Q)$ for any Q . Thus, without supplier cost uncertainty, ex ante price commitment is better than ex post price negotiation for the buyer (under sole sourcing).

Proposition 9 *When $\Delta = 0$, ex ante price commitment is preferred to ex post price negotiation under sole sourcing.*

We have shown that ex post price negotiation is better than price commitment if $\Delta > \frac{\mu_d}{a}$ (they are equal and both favor sole sourcing when Δ is sufficiently close to $\frac{\mu_d}{a}$), and the opposite is true if $\Delta = 0$ (under sole sourcing). Therefore, we may conclude that decreasing supplier cost uncertainty improves the favorability of ex ante price commitment over ex post negotiation. Based on a different setting, Bernstein and K ok (2009) compare target-price contracts and cost-contingent contracts, which are similar to our ex ante price commitment and ex post price negotiation mechanisms. They assume deterministic supplier costs with no supplier competition. They show that target-price contracts lead to higher investment levels by a supplier and greater profits of the buyer than cost-contingent contracts. Our result with $\Delta = 0$ reinforces their conclusion. However, we show that such a result is reversed in the presence of supplier competition under sufficiently large cost uncertainty.

7. Ex post capacity investment

We have assumed in our base model that the supplier capacity is invested or committed by the buyer before suppliers exert their cost reduction efforts. What if the buyer can postpone capacity

investment until *after* the suppliers exert cost-reduction effort and have their costs realized (but before the demand realizes)? In this section, we consider such an ex post capacity investment mechanism. In this mechanism, the buyer first decides to invite a single supplier or two suppliers to join the supply base; we call the former option *sole sourcing* and the latter *dual sourcing*. Then, the suppliers in the supply base invest cost-reduction effort. After suppliers' costs are realized, an auction is conducted to determine the capacity investment and purchasing price for each supplier. Finally, the demand is realized, and the contracts are executed. We differentiate the notation for this mechanism by the superscript “*C*”.

With ex post capacity investment, the buyer is able to decide the capacity investment based on suppliers' cost realizations. Even in a dual sourcing policy, *only* the supplier with a lower (virtual) cost will receive capacity investment and will produce for the buyer. This is unlike the dual sourcing mechanism in the base model with ex ante capacity investment—in that mechanism, the buyer invests in capacities for *both* suppliers, and the capacity investment is independent of suppliers' costs. Proposition 10 characterizes the suppliers' efforts and buyer's profits under sole and dual sourcing with ex post capacity investment. We define $Q^*(t) \doteq G^{-1}\left(1 - \frac{k}{r-t}\right)$ as the solution to $\max_Q ((r-t)S(Q) - kQ)$; it is the capacity investment for a single supplier with unit cost t .

Proposition 10 *i) With a single supplier, the supplier's effort $e^{C,S}$ is the unique solution to $e = \frac{1}{a}\mathbb{E}_c[S(Q^*(J(c,e)))]$ on e . The total expected profit of the buyer is*

$$\Pi^{C,S} = \mathbb{E}_c [(r - J(c, e^{C,S})) S(Q^*(J(c, e^{C,S}))) - kQ^*(J(c, e^{C,S}))]. \quad (9)$$

ii) With two suppliers, each supplier invests effort $e^{C,D}$ that is the unique solution to $e = \frac{1}{2a}\mathbb{E}_{c_{(1)}}[S(Q^(J(c_{(1)}, e)))]$ on e , where $c_{(1)} = \min(c_1, c_2)$ is the lower statistic of the two suppliers' types. The total expected profit of the buyer is*

$$\Pi^{C,D} = \mathbb{E}_{c_{(1)}} [(r - J(c_{(1)}, e^{C,D})) S(Q^*(J(c_{(1)}, e^{C,D}))) - kQ^*(J(c_{(1)}, e^{C,D}))]. \quad (10)$$

Proposition 10 i) shows the outcome of sole sourcing. The supplier makes investment of effort e_S^C based on his expectation of the sourcing quantity $\mathbb{E}_c[S(Q^*(J(c,e)))]$; the more quantity expected, the more effort invested. As shown in Equation (9), the buyer's profit takes into consideration the supplier's virtual cost $J(c, e^{C,S})$ as the ex ante unit cost. Proposition 10 ii) shows the outcome of dual sourcing. In this mechanism, since a supplier has only a 50 percent chance to be the winner (in the equilibrium), his total expected quantity is equal to a half of the expected quantity he will receive as the winner ($\mathbb{E}[S(Q^*(J(c_{(1)}, e)))]$). The buyer's profit, as characterized in Equation (10), is similar to that from sole sourcing, except that the supplier effort is different and the supplier type is based on the lower statistic $c_{(1)}$, the smaller of the two suppliers' types.

Compared to sole sourcing, dual sourcing allows the buyer to take advantage of supplier competition, investing only in the lower-cost supplier and negotiating a competitive price. However, the supplier effort in dual sourcing, $e^{C,D}$, may be smaller than that in sole sourcing, $e^{C,S}$. Thus, the buyer's choice between the two strategies trades off supplier effort and supplier-competition benefits.

7.1 Supply base decision

In this subsection, we analyze when the buyer should invite only one supplier (sole sourcing) vs. both suppliers (dual sourcing) to join her supply base. We focus only on the number of suppliers and not on capacity investment, since the latter is not a decision for supply base design. Through analytical and numerical study, we find that dual sourcing is preferred over sole sourcing for high values of the cost of effort a or supplier cost uncertainty Δ . In particular, if a is too high for a supplier to invest in any effort, dual sourcing is always better than sole sourcing. Thus, the impact of a and Δ on supply base design is similar to that in our base model with ex ante capacity investment. However, the capacity cost k and demand uncertainty σ_d influence supply base design differently than in the base mechanism: As shown in Section 5 for the base model with ex ante capacity investment, increasing k always drives toward S, and increasing σ_d may influence the choice between S and sD in either direction depending on a . In contrast, we find that, in the ex post capacity investment mechanism, increasing k or σ_d makes dual sourcing, but not sole sourcing, more favorable. Proposition 11 shows the result based on small k or σ_d .

Proposition 11 *In the ex post capacity investment mechanism, when the capacity cost k or demand uncertainty σ_d is small, increasing k or σ_d can only shift the optimal supply base structure from sole sourcing to dual sourcing, but not in the opposite direction.*

Through numerical experiments, we find that the result of Proposition 11 can be extended to the situations with general values of k and σ_d . Figure 5 shows representative results, demonstrating the regions of k and σ_d values that lead to sole and dual sourcing for different a values. For each value of a as marked, the area left of and below (right of and above) the corresponding curve is where sole (dual) sourcing is optimal.

The impact of k and σ_d on supply base design can be explained by their influence on supplier effort and supplier competition. First, increasing k or σ_d reduces supplier effort: As k or σ_d increases, the expected volume provided by the winning supplier decreases. This leads a supplier to reduce his investment of cost-reduction effort. Such a negative effect on supplier effort is more pronounced for sole sourcing than for dual sourcing, since the volume decrease is shared by two suppliers in the dual sourcing policy, but is shouldered by the only supplier in the sole sourcing policy. Second, increasing k or σ_d enhances the benefit of supplier competition: Increasing k or

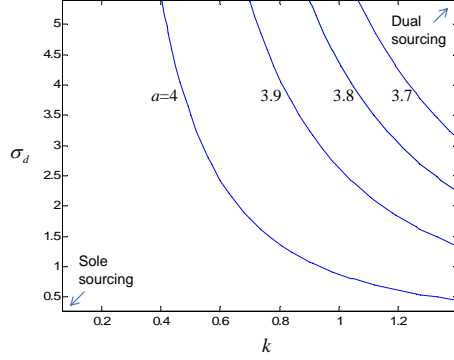


Figure 5: Profit indifference curves between sole sourcing and dual sourcing with ex post capacity investment. The area to the left (right) of each curve is where sole (dual) sourcing is better. $\underline{c} = 2$, $\bar{c} = 3$, $r = 6$, and demand following a uniform distribution on $[2.7 - \sigma_d/2, 2.7 + \sigma_d/2]$

σ_d makes the capacity investment decision $G^{-1}\left(1 - \frac{k}{r - J(c,e)}\right)$ more sensitive to the supplier cost c (for given effort e), which allows the buyer to better discriminate supplier types with responsive capacity investment, thus enlarging the profit advantage of sourcing from a lower-type supplier. This effect enhances the benefit of supplier competition, as it allows the buyer to discover a lower-type supplier. Thus, as k or σ_d increases, both effects on supplier effort and supplier competition make dual sourcing increasingly favorable over sole sourcing.

To summarize the above findings, in the ex post capacity investment mechanism, dual sourcing becomes more favorable as supplier cost uncertainty, cost of effort, demand uncertainty, or capacity cost increases.

7.2 Ex ante vs. ex post capacity investment

In this subsection, we analyze when the buyer is better off delaying capacity investment and the change on supply base design caused by the delay. Compared to ex ante capacity investment in our base model, ex post investment allows the buyer to make capacity decisions based on the realization of suppliers' costs. Meanwhile, however, the buyer's influence on supplier effort through ex ante capacity investment is lost (see Lemma 3 for the relationship between supplier effort and capacity investment). The selection of ex post capacity investment against ex ante investment thus depends on the tradeoff between the advantage of cost-contingent capacity investment and the disadvantage of losing influence on supplier effort.

Proposition 12 compares ex ante and ex post capacity investment for the situation in which the capacity cost k or demand uncertainty σ_d is very small. In this situation, the supply base design with ex ante capacity investment focuses on the sD and S structures; aD is either non-optimal (if k very small, see Proposition 4) or is close to S (if σ_d very small, see Proposition 5).

Proposition 12 *When the capacity cost k or demand uncertainty σ_d is very small,*

i) if symmetric dual sourcing is optimal with ex ante capacity investment, then dual sourcing is optimal with ex post capacity investment;

ii) if $a\Delta > \frac{3\mu_d}{2}$, then ex post capacity investment generates more profits for the buyer than ex ante capacity investment; otherwise, the opposite is true.

Proposition 12 i) shows that (symmetric) dual sourcing is considered more favorably in the ex post than in the ex ante capacity investment mechanism. This is because, with ex ante capacity investment, (symmetric) dual sourcing requires substantial redundant capacity, costing much more in capacity than sole sourcing. With ex post investment, such a capacity-cost concern does not exist for dual sourcing—the buyer will invest in the capacity for only one supplier even if both suppliers join her supply base; this improves the relative advantage of dual sourcing over sole sourcing.

Proposition 12 ii) shows that ex ante capacity investment is better than ex post capacity investment for the buyer if the cost of effort, a , is low or supplier cost uncertainty, Δ , is small; otherwise, the latter is superior. We explain the impact of a and Δ on the choice between the two mechanisms: With a small, a supplier’s effort is highly sensitive to the ex ante capacity investment decision, suggesting high value of capacity commitment in inducing supplier effort. With Δ low, the benefit of cost-contingent capacity investment is small. Thus, small a or low Δ makes ex ante capacity investment preferred to ex post investment.

Proposition 12 is based on the case when k or σ_d is very small. In this case, the ex post capacity investment decision is not very sensitive to the realization of supplier costs (note that the capacity investment will be the maximum demand \bar{d} if $k = 0$, or be the mean of demand μ_d if $\sigma_d = 0$). As k or σ_d increases, the capacity decision will be more sensitive to the cost realization, improving the benefit of cost-contingent capacity investment. Indeed, through numerical experiments we find that, even though ex ante investment is better than ex post investment when k or σ_d is low (for a or Δ sufficiently small), the preference is reversed when k or σ_d is sufficiently high. Figure 5 demonstrates the regions of k and σ_d values that lead to the choice of ex ante or ex post capacity investment for different a values. The area left of and below (right of and above) each curve is where ex ante (ex post) investment is better. We find that the insights from Proposition 12 continue to hold in the situations where k and σ_d can be large: Dual sourcing is considered more favorably with ex post capacity investment than with ex ante investment, and increasing a or Δ makes ex post investment more favorable compared to ex ante investment.

8. Summary of the three mechanisms

We have analyzed and compared our base mechanism with the ex ante price commitment mechanism and the ex post capacity investment mechanism. The three mechanisms differ in the time of the

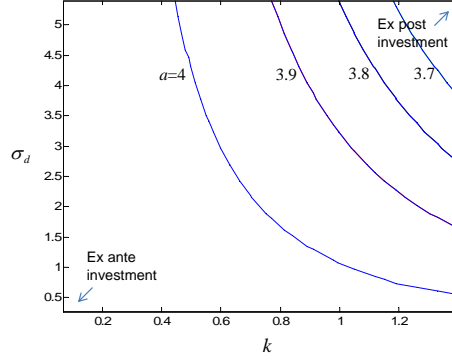


Figure 6: Profit indifference curves between ex post and ex ante capacity investment. The area to the left (right) of each curve is where ex ante (ex post) capacity investment is better. $\underline{c} = 2$, $\bar{c} = 3$, $r = 6$, and demand following a uniform distribution on $[2.7 - \sigma_d/2, 2.7 + \sigma_d/2]$

price and capacity decisions: In the base mechanism, the capacity is decided before supplier cost realization and the price is negotiated afterwards, while in the ex ante price commitment mechanism, the price decision (along with the capacity decision) is made *before* supplier cost realization, and in the ex post capacity investment mechanism, the capacity decision (along with the price decision) is delayed until *after* supplier cost realization. Compared to the base mechanism, the ex ante price commitment mechanism endows the buyer with greater influence on suppliers' investment of effort, but the buyer loses the benefit of supplier price competition. The ex post capacity investment mechanism enhances the benefit of supplier competition with cost-contingent capacity decisions, but it weakens suppliers' incentive to invest cost-reduction effort.

In each mechanism, the supply base design has to trade off the benefit of supplier competition and the suppliers' incentive to invest effort. Because of the different features of the three mechanisms, their preferences for involving suppliers in competition by supply base design are also different. For ex ante capacity investment (the base mechanism and the ex ante price commitment mechanism), we say that greater supplier competition be designed in the supply base if the suppliers' capacity profile is more balanced: A supply base with equal capacity is more competitive than one with unequal capacity between suppliers, and a supply base with two suppliers is more competitive than one with a single supplier. For ex post capacity investment, we say that a supply base joined by both suppliers be more competitive than one consisting of a single supplier. Table 2 summarizes the influence of the cost of effort a , supplier cost uncertainty Δ , capacity cost k , and demand uncertainty σ_d on supply base design in each mechanism. The mark + (-) in each cell means that greater (less) supplier competition is designed in the supply base with the increase of the corresponding factor. In all three mechanisms, increasing a or Δ calls for greater supplier competition. Increasing k leads to less supplier competition with ex ante capacity investment (in the base mechanism and the ex ante price commitment mechanism), but calls for greater supplier

competition with ex post capacity investment. While greater σ_d can change supplier competition in either way (depending on a) with ex ante capacity investment, it only increases supplier competition with ex post capacity investment.

Changing factor	Impact on supply base design for supplier competition		
	ex ante price commitment	base mechanism	ex post capacity investment
$a \uparrow, \Delta \uparrow$	+	+	+
$k \uparrow$	-	-	+
$\sigma_d \uparrow$	+/-	+/-	+

Table 2: *Summary of the impact of parameters on supply base design in the ex ante price commitment mechanism, the base mechanism, and the ex post capacity investment mechanism*

The choice among the three mechanisms depends on the environment. We find that ex ante price commitment is preferred when the supplier cost uncertainty Δ is very low, while ex post capacity investment is favored when Δ is high. The supply base design varies with the buyer's choice among the three mechanisms. Compared to the base mechanism, the ex ante price commitment mechanism considers sole sourcing more favorably, while the ex post capacity investment mechanism considers dual sourcing more favorably. In other words, the ex ante price commitment mechanism, the base mechanism, and the ex post capacity investment mechanism are ordered by increasing supplier competition in the supply base. Therefore, as supplier cost uncertainty increases, the buyer changes the time of price decision and capacity decision, shifting from ex ante price commitment to the base mechanism, and then to ex post capacity investment, and, meanwhile, she increases the competitiveness of the supply base. Figure 7 demonstrates conceptually the choice among the three mechanisms based on supplier cost uncertainty (horizontal axis), and the change of supply base design (vertical axis).

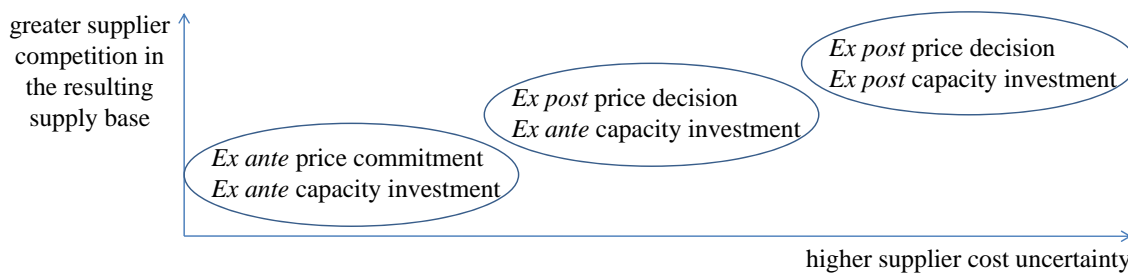


Figure 7: *Choice of the ex ante price commitment mechanism, the base mechanism, and the ex post capacity investment mechanism based on supplier cost uncertainty Δ , and the change of supply base design along the mechanism choice*

9. Conclusion

A buying firm relying on suppliers for critical production activities must make strategic decisions concerning supply base design. A buyer has two levers to reduce supply-related costs: (1) involving suppliers in competition, which reduces information asymmetry and hedges against supplier cost uncertainty, and (2) motivating suppliers to invest cost-reduction effort. The design of the supply base, in terms of the number of suppliers and capacity investment for each, influences both levers. While a sole-supplier base (sole sourcing) provides great incentive for a supplier to invest effort, a dual-supplier base with equal-capacity suppliers (symmetric dual sourcing) maximizes supplier competition.

We find that a buyer without demand uncertainty should choose only between sole sourcing and symmetric dual sourcing. With demand uncertainty, however, the buyer may trade off the benefits of supplier competition and effort investment by creating an unbalanced supply base—equipping both suppliers, but with unequal capacities. In other words, even if suppliers are *ex ante* identical, asymmetric dual sourcing may be optimal. We find that asymmetric dual sourcing is preferred for intermediate capacity cost, with low and high capacity costs favoring symmetric dual sourcing and sole sourcing, respectively. We also find that higher costs of supplier effort and greater production cost uncertainty enhance the preference for symmetric dual sourcing, while reducing that for sole sourcing. In addition, we find that increasing demand uncertainty may prompt the buyer to switch from symmetric dual sourcing to sole sourcing or to asymmetric dual sourcing, while at the same time reducing total capacity investment. This occurs when the cost of supplier effort is low. Our model ignores the economy-of-scale in a supplier’s production cost and the buyer’s fixed cost of managing a supplier. Considering these costs would drive the decision towards sole sourcing, without changing the qualitative insights about the influence of supply base design on supplier competition and investment of effort.

Our results deliver valuable managerial guidance for supply base design: A buyer should engage suppliers in close competition (by including more suppliers in the supply base and investing similar capacities in the suppliers) when sourcing inputs that require a relatively low level of customization. For these items, the capacity may be redeployed for other buyers, implying that the capacity cost is low. Supplier competition is also preferred when the buyer sources complex products that include many subcomponents. The production cost for such items usually features greater uncertainty, and is more difficult to estimate for the buyer, than the cost of simple parts. In addition, supplier competition is favored if the production of the sourced item uses a mature technology or a standard process. In such conditions, it is difficult to achieve further efficiency improvement. Finally, supply base design may change over the product life cycle. A buyer may start with a sole supplier for a new product when the demand is highly uncertain, and then increase the level of supplier competition

by shifting to dual sourcing as demand uncertainty decreases with product maturation. However, the path of the supply base structure may reverse if the production is based on a mature technology or a standard process.

Our results suggest that a buyer facing uncertain demand should not only focus on the total capacity and the total number of suppliers needed in the supply base, but also should carefully decide how the capacity should be allocated between suppliers—some unbalance in capacity between suppliers may be created intentionally. Furthermore, when designing the supply base capacity, the buyer should not only be concerned with the capability to meet uncertain demand, but also should consider the capacity redundancy needed to stimulate supplier competition. The capacity serving the first purpose may be referred to as the *operational capacity*, and that for the latter purpose the *strategic capacity*. While greater demand uncertainty typically calls for a greater buffer of the operational capacity, it dampens supplier competition, lowering the value of the strategic capacity. Thus, the total capacity, along with the number of suppliers, may decrease with demand uncertainty.

In our base model, we assume that capacities are invested before suppliers exert efforts and have their costs realized, and prices are negotiated later. We extend our analysis to the ex ante price commitment mechanism and ex post capacity investment mechanism. We find that ex ante price commitment may be used with a sole-supplier base, when the cost uncertainty is very small, and ex post capacity investment may be adopted with a dual-supplier base, when the cost uncertainty is relatively large. Thus, the sequence of supplier development activities and timing of price decision should be considered jointly with supply base design.

Our results are consistent with empirical observations. Japanese auto-makers typically form close relationships with their suppliers by sharing knowledge and providing engineering support (Dyer, 2000). A close supplier relationship reduces supplier cost uncertainty, as it allows the buyer to gain information about a supplier's cost structure and keep a supplier's performance variation under control. It also reduces a supplier's cost of effort to improve his performance, with the support he receives from the buyer. Under such deep supplier relationships, Japanese auto-makers often source from a single supplier (or a small number of suppliers) and set target prices (similar to our ex ante price commitment) for suppliers (McMillan, 1990; Liker and Choi, 2004). Compared to their Japanese peers, U.S. auto-makers do not work as closely with their suppliers. They are more inclined to engage suppliers in competition by forming a larger supply base, and often renegotiate prices based on suppliers' achieved costs (similar to our ex post price negotiation). (Liker and Choi, 2004; Fine et al., 1996)

We hope this research leads to future work on sourcing considering buyer-supplier collaboration along with supplier competition, both important strategies of supply base development (Krause, 1997). On one hand, under the trend of building deeper supplier relationships in practice, the idea of closer buyer-supplier collaboration, in which both parties invest resources to improve performance,

has been increasingly embraced by practitioners and receives growing attention from researchers. On the other hand, the value of supplier competition has long been recognized in the practice and research of procurement. While supplier competition has been generally considered as an adversarial force for collaboration, it provides some benefits and is not necessarily exclusive to collaboration. How can or should these two strategies effectively combined? When may one replace the other? Answers to these questions depend on the nature of collaboration and competition. We consider in this paper the collaboration scenario in which a buyer and supplier(s) invest complementary resources that improve delivery capability and reduce cost, respectively, and suppliers compete on costs. Other types of collaboration and competition can occur. For example, the buyer and supplier(s) may invest substitutable resources to improve product quality or delivery performance, and suppliers may compete on cost, quality, or delivery. Further study is required to develop an understanding of these different types of collaboration and competition.

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Appendix

Proof of Lemma 1:

For given \bar{Q} , $\frac{d}{dQ_1}A(\mathbf{Q}) = 2(1 - G(Q_1)) > 0$. Thus $A(\mathbf{Q})$ increases with $Q_1 - Q_2$.

Define $\delta = Q_1 - \frac{\bar{Q}}{2}$. Then $\frac{d}{d\delta}B(\mathbf{Q}) = G\left(\frac{\bar{Q}}{2} - \delta\right) - G\left(\frac{\bar{Q}}{2} + \delta\right)$. $\frac{d}{d\delta}B(\mathbf{Q}) \leq 0$ if $\delta \geq 0$. Thus $B(\mathbf{Q})$ decreases with $|Q_1 - Q_2|$.

Proof of Lemma 2:

We omit \mathbf{Q} and \mathbf{e} in the parameters for simplicity of exposition. We first prove ii): Based on Myerson (1981), we restrict the consideration to direct auction mechanisms that are incentive compatible (IC) and individual rational (IR). For any $\hat{\gamma}_i \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$, define $\bar{q}_i(\hat{\gamma}_i) \doteq \mathbb{E}_{\gamma_{-i}}[q_i(\hat{\gamma}_i, \gamma_{-i})]$ and $\bar{t}_i(\hat{\gamma}_i) \doteq \mathbb{E}_{\gamma_{-i}}[t_i(\hat{\gamma}_i, \gamma_{-i})]$ as a supplier's expected quantity and payment when he reports cost $\hat{\gamma}_i$ and the other supplier reports truthfully. Then by reporting $\hat{\gamma}_i$, a supplier receives profit $u_i(\hat{\gamma}_i) \doteq \bar{t}_i(\hat{\gamma}_i) - \gamma_i \bar{q}_i(\hat{\gamma}_i)$. In an IC mechanism, $u_i(\gamma_i) = \max_{\hat{\gamma}_i} u_i(\hat{\gamma}_i)$. The IR constraint implies $u_i(\gamma_i) \geq 0$ for all $\gamma_i \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$. From Myerson (1981), the IC and IR constraints hold if and only if (1) $\bar{q}_i(\gamma_i)$ is a decreasing function of γ_i , and (2) $u_i(\gamma_i) = \underline{u}_i + \int_{\gamma_i}^{\bar{c}_i - e_i} \bar{q}_i(\rho) d\rho$ for any constant $\underline{u}_i \geq 0$. We analyze the optimal mechanism based on the condition (2), assuming the condition (1) is satisfied. Then we show that the condition (1) is satisfied in the result.

Based on the condition (2), the ex ante profit of a supplier, $U_i = \mathbb{E}[u_i(\gamma_i)]$, can be written as $U_i = \underline{u}_i + \mathbb{E}_{c_i} \left[\bar{q}_i(c_i - e_i) \frac{F_i(c_i)}{f_i(c_i)} \right]$ by changing the order of integrations. The expected profit of the buyer is equal to the total profit of the supply chain minus the suppliers' profits:

$$\begin{aligned} U &= \mathbb{E}_{\gamma} [r \min(x, q_1(\gamma) + q_2(\gamma)) - \gamma_1 q_1(\gamma) - \gamma_2 q_2(\gamma)] - U_1 - U_2 \\ &= \mathbb{E}_{\gamma} [r \min(x, q_1(\gamma) + q_2(\gamma)) - J_1(c_1, e_1) q_1(\gamma) - J_2(c_2, e_2) q_2(\gamma)] - \sum_{i=1}^2 \underline{u}_i. \end{aligned}$$

Since $r \geq J_i(c_i, e_i)$ for any $c_i \in [\underline{c}_i, \bar{c}_i]$, the buyer should satisfy as much demand as possible, $q_1(\gamma) + q_2(\gamma) = \min(x, \bar{Q})$. In addition, the buyer should source as much of the quantity as possible from whichever supplier that has a lower $J_i(c_i, e_i)$. Thus, the allocation rule is: $q_i = \min(x, Q_i)$ and $q_{-i} = \min(Q_{-i}, (x - Q_i)^+)$ if $J_i(c_i, e_i) \leq J_{-i}(c_{-i}, e_{-i})$. Since $J_i(c_i, e_i) = \gamma_i + \frac{F(\gamma_i + e_i)}{f(\gamma_i + e_i)}$, this implies that $\bar{q}_i(\gamma_i)$ is a decreasing function; the condition (1) is satisfied. To maximize U , \underline{u}_i is set to 0. Thus, the buyer's profit is equal to

$$\begin{aligned} U &= rS(\bar{Q}) - \mathbb{E} [J_1(c_1, e_1) S(Q_1) + J_2(c_2, e_2) (S(\bar{Q}) - S(Q_1)) | J_1(c_1, e_1) \leq J_2(c_2, e_2)] \\ &\quad \Pr(J_1(c_1, e_1) \leq J_2(c_2, e_2)) \\ &\quad - \mathbb{E} [J_1(c_1, e_1) (S(\bar{Q}) - S(Q_2)) + J_2(c_2, e_2) S(Q_2) | J_1(c_1, e_1) > J_2(c_2, e_2)] \\ &\quad \Pr(J_1(c_1, e_1) > J_2(c_2, e_2)), \end{aligned}$$

which can be transformed to Equation (2).

To prove i): Based on the optimal mechanism, Supplier i 's expected quantity is $S(Q_i)$ if $J_i(c_i, e_i) \leq J_{-i}(c_{-i}, e_{-i})$, and otherwise is $S(\bar{Q}) - S(Q_{-i})$ if $J_i(c_i, e_i) > J_{-i}(c_{-i}, e_{-i})$. Thus the total expected quantity of Supplier i is

$$\bar{q}_i(c_i - e_i) = S(Q_i) \Pr(J_i(c_i, e_i) \leq J_{-i}(c_{-i}, e_{-i})) + (S(\bar{Q}) - S(Q_{-i})) \Pr(J_i(c_i, e_i) > J_{-i}(c_{-i}, e_{-i})).$$

We have shown that a supplier's expected profit is $u_i(\gamma_i) = \int_{\gamma_i}^{\bar{\gamma}_i} \bar{q}_i(\rho) d\rho$ for any $\gamma_i \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$. Now we analyze for supplier costs outside of $[\underline{c}_i - e_i, \bar{c}_i - e_i]$. Note that a supplier has to choose a contract from the menu design for Supplier i 's cost on $[\underline{c}_i - e_i, \bar{c}_i - e_i]$, $i = 1, 2$. In other words, a supplier has to report a cost within that range, even if his cost is outside that range.

If $\gamma_i < \underline{c}_i - e_i$, then Supplier i should report cost $\underline{c}_i - e_i$: By reporting $\hat{\gamma}_i \in [\underline{c}_i - e_i, \bar{c}_i + e_i]$, Supplier i achieves a profit $\hat{u}_i(\hat{\gamma}_i|\gamma_i) = u_i(\hat{\gamma}_i) + (\hat{\gamma}_i - \gamma_i) \bar{q}_i(\hat{\gamma}_i)$. $\hat{u}_i(\hat{\gamma}_i|\gamma_i)$ is maximized at $\hat{\gamma}_i = \underline{c}_i - e_i$ because $\frac{\partial}{\partial \hat{\gamma}_i} \hat{u}_i(\hat{\gamma}_i|\gamma_i) = (\hat{\gamma}_i - \gamma_i) \bar{q}'_i(\hat{\gamma}_i) \leq 0$. This means Supplier i 's optimal profit is $\int_{\underline{c}_i - e_i}^{\bar{c}_i - e_i} \bar{q}_i(\rho) d\rho + (\underline{c}_i - e_i - \gamma_i) \bar{q}_i(\underline{c}_i - e_i)$.

If $\gamma_i > \bar{c}_i - e_i$, then Supplier i should quit, achieving zero profit. With $\frac{\partial}{\partial \hat{\gamma}_i} \hat{u}_i(\hat{\gamma}_i|\gamma_i) = (\hat{\gamma}_i - \gamma_i) \bar{q}'_i(\hat{\gamma}_i) \geq 0$ for any $\hat{\gamma}_i \in [\underline{c}_i - e_i, \bar{c}_i - e_i]$, and $\hat{u}_i(\bar{c}_i - e_i|\gamma_i) = (\bar{c}_i - e_i - \gamma_i) \bar{q}_i(\bar{c}_i - e_i) < 0$, Supplier i would achieve a negative profit if he chooses any contract.

Proof of Lemma 3:

We omit \mathbf{Q} and \mathbf{e} in the parameters for simplicity of exposition. i) By investing effort \hat{e}_i , Supplier i 's expected profit is $\mathbb{E}[u_i(c_i - \hat{e}_i)] - \varphi(\hat{e}_i)$. Based on Equation (1), $\frac{d^2}{d\hat{e}_i^2} \mathbb{E}[u_i(c_i - \hat{e}_i)]$ is positive, increasing in \hat{e}_i for $\hat{e}_i < e_i$ and decreasing in \hat{e}_i for $\hat{e}_i \geq e_i$. The maximum of $\frac{d}{d\hat{e}_i} \mathbb{E}[u_i(c_i - \hat{e}_i)]$ is equal to $\frac{1}{\Delta} \bar{q}_i(\underline{c} - e_i)$, achieved when \hat{e}_i approaches e_i from the left. With $\varphi(e) = \frac{a}{2} e^2$ for $a \geq \frac{\mu a}{\Delta}$, $\frac{d}{d\hat{e}_i} \mathbb{E}[u_i(c_i - \hat{e}_i)]$ single crosses $\varphi'(\hat{e}_i)$ from above at some $\hat{e}_i \geq 0$. Then e_i as an equilibrium effort is the unique solution to $\varphi'(e_i) = \frac{d}{d\hat{e}_i} \mathbb{E}[u_i(c_i - \hat{e}_i)]$, i.e., $ae_i = \frac{1}{\Delta} u_i(\underline{c} - e_i)$.

Define $d_e \doteq e_1 - e_2$. With identical uniform cost distributions for the two suppliers, $J_1(\gamma_1, e_1) \leq J_2(\gamma_2, e_2)$ corresponds to $c_1 - c_2 \leq \frac{d_e}{2}$. Then

$$\bar{q}_1(c_1 - e_1) = \begin{cases} S(Q_1) & \text{if } c_1 \leq \underline{c} + \frac{d_e}{2} \\ S(Q_1) + (S(\bar{Q}) - S(Q_2) - S(Q_1)) \frac{1}{\Delta} (c_1 - \underline{c} - \frac{d_e}{2}) & \text{else } c_1 > \underline{c} + \frac{d_e}{2} \end{cases},$$

$$\bar{q}_2(c_2 - e_2) = \begin{cases} S(\bar{Q}) - S(Q_1) - (S(\bar{Q}) - S(Q_2) - S(Q_1)) \frac{1}{\Delta} (\bar{c} - c_2 - \frac{d_e}{2}) & \text{if } c_2 \leq \bar{c} - \frac{d_e}{2} \\ S(\bar{Q}) - S(Q_1) & \text{else } c_2 > \bar{c} - \frac{d_e}{2} \end{cases}.$$

Based on $u_i(\underline{c} - e_i) = \int_{\underline{c}}^{\bar{c}} \bar{q}_i(c_i - e_i) dc_i$, it follows that the equilibrium efforts (e_1, e_2) are defined by

$$\begin{cases} ae_1 = \frac{1}{\Delta} u_1(\underline{c} - e_1) = S(Q_1) + (S(\bar{Q}) - S(Q_2) - S(Q_1)) \frac{1}{2} \left(1 - \frac{d_e}{2\Delta}\right)^2, \\ ae_2 = \frac{1}{\Delta} u_2(\underline{c} - e_2) = (S(\bar{Q}) - S(Q_1)) - (S(Q_1 + Q_2) - S(Q_2) - S(Q_1)) \frac{1}{2} \left(1 - \frac{d_e}{2\Delta}\right)^2. \end{cases}$$

Summing up the two equations, we have $e_1 + e_2 = \frac{S(\bar{Q})}{a}$, and thus $e_1 = \frac{S(\bar{Q})}{2a} + \frac{d_e}{2}$, $e_2 = \frac{S(\bar{Q})}{2a} - \frac{d_e}{2}$. Define $\eta \doteq \frac{d_e}{2\Delta}$. Subtracting the above two equations, we have $2a\Delta\eta = A(\mathbf{Q}) - (1 - \eta)^2 B(\mathbf{Q})$.

The RHS is an increasing concave function of $\eta \in [0, 1]$. When $\eta = 0$, the RHS is $S(Q_1) - S(Q_2)$, greater than the LHS. When $\eta = \frac{S(Q_1)}{2a\Delta}$, the LHS is $S(Q_1)$, greater than the RHS. Thus a unique solution of $\eta \in \left[0, \frac{S(Q_1)}{2a\Delta}\right]$ exists.

ii) From Equation (3),

$$\frac{\partial \eta}{\partial Q_1} = \frac{\left(2 - (1 - \eta)^2\right) (1 - G(Q_1)) - \left(1 - (1 - \eta)^2\right) (1 - G(\bar{Q}))}{2a\Delta - 2(S(Q_2) + S(Q_1) - S(\bar{Q})) (1 - \eta)} \geq \frac{1 - G(\bar{Q})}{2a\Delta},$$

$$\frac{\partial \eta}{\partial Q_2} = -\frac{1 - G(\bar{Q}) + (G(\bar{Q}) - G(Q_2)) (1 - \eta)^2}{2a\Delta - 2(S(Q_2) + S(Q_1) - S(\bar{Q})) (1 - \eta)} \leq -\frac{1 - G(\bar{Q})}{2a\Delta}.$$

Thus $\frac{\partial e_1}{\partial Q_1} = \frac{\bar{G}(\bar{Q})}{2a} + \Delta \frac{\partial \eta}{\partial Q_1} \geq \frac{\bar{G}(\bar{Q})}{a} \geq 0$ and $\frac{\partial e_1}{\partial Q_2} = \frac{\bar{G}(\bar{Q})}{2a} + \Delta \frac{\partial \eta}{\partial Q_2} \leq 0$. Similarly, $\frac{\partial e_2}{\partial Q_1} = \frac{\bar{G}(\bar{Q})}{2a} - \Delta \frac{\partial \eta}{\partial Q_1} \leq 0$, and $\frac{\partial e_2}{\partial Q_2} = \frac{\bar{G}(\bar{Q})}{2a} - \Delta \frac{\partial \eta}{\partial Q_2} \geq \frac{\bar{G}(\bar{Q})}{a} \geq 0$. Therefore, $e_i(\mathbf{Q})$ increases with Q_i and decreases with Q_{-i} .

Proof of Proposition 1:

With a uniform cost distribution, $J_i(c_i, e_i) = 2c_i - \underline{c} - e_i$. In addition, with identical cost distributions, $J_1(c_1, e_1) \leq J_2(c_2, e_2)$ corresponds to $c_1 - c_2 \leq \Delta\eta$. Then with $e_1 = \frac{S(\bar{Q})}{2a} + \Delta\eta$ and $e_2 = \frac{S(\bar{Q})}{2a} - \Delta\eta$, Equation (4) follows $\Pi(\mathbf{Q}) = U(\mathbf{Q}, \mathbf{e}(\mathbf{Q})) - k\bar{Q}$ based on $U(\mathbf{Q}, \mathbf{e})$ as characterized in Equation (2).

Proof of Proposition 3:

If $a \rightarrow \infty$, then based on Lemma 3, $\eta(\mathbf{Q}) = 0$ and $e_1 = e_2 = 0$. Then the buyer's profit reduces to $\Pi(\mathbf{Q}) = (r - \bar{c})S(\bar{Q}) - k\bar{Q} + \frac{\Delta}{3}B(\mathbf{Q})$. For given \bar{Q} , the optimal solution maximizes $B(\mathbf{Q})$, which results in symmetric dual sourcing according to Lemma 1.

Proof of Proposition 4:

Based on Equation (4),

$$\begin{aligned} \frac{\partial \Pi(\mathbf{Q})}{\partial Q_1} &= -k + \left(r - \bar{c} + \frac{S(\bar{Q})}{2a}\right) (1 - G(\bar{Q})) + \Delta \frac{(1 - \eta(\mathbf{Q}))^3}{3} (G(\bar{Q}) - G(Q_1)) \\ &\quad + \Delta \eta(\mathbf{Q}) (1 + G(\bar{Q}) - 2G(Q_1)) + 2a\Delta^2 \eta(\mathbf{Q}) \frac{\partial \eta(\mathbf{Q})}{\partial Q_1}. \end{aligned}$$

Note $\frac{\partial \eta}{\partial Q_1} > 0$. If $k = 0$, then $\frac{\partial \Pi(\mathbf{Q})}{\partial Q_1} > 0$ for any given Q_2 . Thus $Q_1 = \bar{d}$. It follows that $A(\mathbf{Q}) = \mu_d$ and $B(\mathbf{Q}) = S(Q_2)$, and thus $\mu_d - (1 - \eta)^2 S(Q_2) = 2a\Delta\eta$, i.e., $S(Q_2) = \frac{\mu_d - 2a\Delta\eta}{(1 - \eta)^2}$. Then $\Pi(\mathbf{Q})$ can be transformed as a function of η :

$$\hat{\Pi}(\eta) = \left(r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{3}\right) \mu_d + \frac{2\Delta}{3} \eta (\mu_d - a\Delta) + \frac{2a\Delta^2}{3} \eta^2.$$

$\hat{\Pi}(\eta)$ is a convex function of η . $\eta = 0$ corresponds to dual sourcing with $Q_1 = Q_2 = \bar{d}$. $\eta = \frac{\mu_d}{2a\Delta}$ corresponds to sole sourcing with $Q_1 = \bar{d}$ and $Q_2 = 0$. Since $\hat{\Pi}(0) = \left(r - \bar{c} + \frac{\mu_d}{2a}\right) \mu_d + \frac{\Delta}{3} \mu_d$ and $\hat{\Pi}\left(\frac{\mu_d}{2a\Delta}\right) = \left(r - \bar{c} + \frac{\mu_d}{a}\right) \mu_d$, $\hat{\Pi}\left(\frac{\mu_d}{2a\Delta}\right) \geq \hat{\Pi}(0)$ if and only if $\frac{\Delta}{3} < \frac{\mu_d}{2a}$, i.e., $a\Delta < \frac{3}{2}\mu_d$.

To prove Proposition 5, we first prove the following lemma:

Lemma 8 *When demand is certain and equal to μ_d ,*

- i) if $k \leq k_1$, then symmetric dual sourcing is optimal, $Q_1^* = Q_2^* = \mu_d$,*
- ii) else if $k > k_1$, then sole sourcing is optimal, $Q_1^* = \mu_d$ and $Q_2^* = 0$.*

With certain demand, in the optimal solution we have $Q_i \in [0, \mu_d]$, $i = 1, 2$, and $Q_1 + Q_2 \geq \mu_d$. Then $A(\mathbf{Q}) = 2Q_1 - \mu_d$, $B(\mathbf{Q}) = Q_1 + Q_2 - \mu_d$, and the buyer's profit reduces to

$$\widehat{\Pi}(\eta) = \left(r - \bar{c} + \frac{\mu_d}{2a} - \frac{\Delta}{3} (2\eta + 1) (\eta - 1)^2 \right) \mu_d + 2a\Delta^2\eta^2 + \left(\frac{\Delta}{3} (2\eta + 1) (\eta - 1)^2 - k \right) \bar{Q},$$

where $\bar{Q} = Q_1 + Q_2$ with (Q_1, Q_2) satisfying Equation (3) while minimizing $Q_1 + Q_2$. Below we show that $\widehat{\Pi}(\eta)$ is a quasi-convex function of η . Note the maximum of η is $\bar{\eta} \doteq \frac{\mu_d}{2a\Delta} < \frac{1}{2}$.

Case 1: η is small enough so that $\frac{\Delta}{3} (2\eta + 1) (\eta - 1)^2 > k$

For given η , we have $Q_1 = \mu_d$ and $Q_2 = \frac{\mu_d - 2a\Delta\eta}{(1-\eta)^2}$ to maximize the buyer's profit. Particularly, if $\eta = 0$, then $Q_1 = Q_2 = \mu_d$ (symmetric dual sourcing). If $\eta = \bar{\eta}$, then $Q_1 = \mu_d$ and $Q_2 = 0$ (sole sourcing). The buyer's profit reduces to

$$\widehat{\Pi}(\eta) = \left(r - \bar{c} + \frac{\mu_d}{2a} - k \right) \mu_d + 2a\Delta^2\eta^2 + \left(\frac{\Delta}{3} (2\eta + 1) (1 - \eta)^2 - k \right) \frac{\mu_d - 2a\Delta\eta}{(1 - \eta)^2}.$$

With $\mu_d < a\Delta$, it can be shown that $\widehat{\Pi}$ is a convex function of η .

Case 2: η is large enough so that $\frac{\Delta}{3} (2\eta + 1) (\eta - 1)^2 \leq k$

For given η , we have $Q_1 + Q_2 = \mu_d$ and $Q_1 - Q_2 = 2a\Delta\eta$ to maximize the buyer's profit. Then $\widehat{\Pi}(\eta) = \left(r - \bar{c} + \frac{\mu_d}{2a} - k \right) \mu_d + 2a\Delta^2\eta^2$ is increasing convex in η . Again in this range, $\eta = \bar{\eta}$ corresponds to $Q_1 = \mu_d$ and $Q_2 = 0$ (sole sourcing).

Thus $\widehat{\Pi}(\eta)$ is quasi-convex over the entire range of $\eta \in [0, \bar{\eta}]$. It is easily shown that $\widehat{\Pi}(0) < \widehat{\Pi}(\bar{\eta})$ if and only if $k > k_1 \doteq \frac{\Delta}{3} - \frac{\mu_d}{2a}$. ■

Proof of Proposition 5:

From Lemma 8, when $\sigma_d = 0$, the buyer's profit as a function of η , $\widehat{\Pi}(\eta)$, is quasi-convex, and sD ($\eta = 0$) and S ($\eta = \frac{S(Q_1)}{2a\Delta}$) are locally optimal.

First we prove that sD remains to be locally optimal if σ_d is positive and infinitely small: When $\eta = 0$, $Q_1 = Q_2 = Q^* \doteq G^{-1} \left(1 - \frac{k}{\Delta/3} \right)$. Then for η very small, $Q_1 + Q_2 > \bar{d}$, $S(Q_1 + Q_2) = \mu_d$, and the buyer's profit is

$$\widehat{\Pi}(\eta) = \left(r - \bar{c} + \frac{\mu_d}{2a} \right) \mu_d - k(Q_1 + Q_2) + \Delta \frac{(1 - \eta)^3}{3} (S(Q_1) + S(Q_2) - \mu_d) + \Delta\eta(2S(Q_1) - \mu_d),$$

subject to Equation (3). Note when $\eta = 0$, we have $\frac{\partial \widehat{\Pi}}{\partial \eta} = 2a\Delta^2\eta = 0$, and $\frac{\partial \widehat{\Pi}}{\partial Q_1} = \frac{\partial \widehat{\Pi}}{\partial Q_2} = \frac{\Delta}{3} (1 - G(Q^*)) - k = 0$. Thus $\widehat{\Pi}'(0) = \frac{\partial \widehat{\Pi}}{\partial \eta} + \frac{\partial \widehat{\Pi}}{\partial Q_1} \frac{\partial Q_1}{\partial \eta} + \frac{\partial \widehat{\Pi}}{\partial Q_2} \frac{\partial Q_2}{\partial \eta} = 0$. This suggests that $\eta = 0$ remains to be locally optimal.

Next we analyze when S is locally optimal: The optimal sole sourcing capacity Q_1 is determined by $Q_1 = G^{-1} \left(1 - \frac{k}{r - \bar{c} + 4\eta\Delta} \right)$ where $\eta = \frac{S(Q_1)}{2a\Delta} \approx \bar{\eta}$. S is locally optimal if $\frac{\partial \Pi(Q_1, 0)}{\partial Q_2} < 0$

at $Q_2 = 0$; otherwise aD is locally optimal. Note for $Q_2 = 0$, $\frac{\partial \Pi(Q_1, 0)}{\partial \eta} \Big|_{\eta = \frac{\mu_d}{2a\Delta}} = 2a\Delta^2\eta$, $\frac{\partial \eta}{\partial Q_2} = -\frac{1 - (1 - (1 - \eta)^2)G(Q_1)}{2a\Delta}$, and

$$\frac{\partial \Pi(Q_1, 0)}{\partial Q_2} \Big|_{\eta = \frac{\mu_d}{2a\Delta}} = (r - \bar{c} + \Delta\eta)(1 - G(Q_1)) - k + \Delta \frac{(1 - \eta)^3}{3} G(Q_1).$$

Thus

$$\begin{aligned} \frac{\partial \Pi(Q_1, 0)}{\partial Q_2} &= \frac{\partial \Pi(Q_1, 0)}{\partial Q_2} \Big|_{\eta = \frac{S(Q_1)}{2a\Delta}} + \frac{\partial \Pi(Q_1, 0)}{\partial \eta} \Big|_{\eta = \frac{S(Q_1)}{2a\Delta}} \frac{\partial \eta}{\partial Q_2} \\ &= (r - \bar{c})(1 - G(Q_1)) - k + \frac{\Delta(1 - \eta)^2(1 - 4\eta)}{3} G(Q_1) \\ &\approx \frac{\Delta}{3} \left((1 - \bar{\eta})^2(1 - 4\bar{\eta}) - k \frac{(1 - \bar{\eta})^2(1 - 4\bar{\eta}) + 12\bar{\eta}}{r - \bar{c} + 4\bar{\eta}\Delta} \right). \end{aligned}$$

Thus $\frac{\partial \Pi(Q_1, 0)}{\partial Q_2} < 0$ if and only if $k > k_2 \doteq \frac{(1 - \bar{\eta})^2(1 - 4\bar{\eta})}{12\bar{\eta} + (1 - \bar{\eta})^2(1 - 4\bar{\eta})} (r - \bar{c} + 4\Delta\bar{\eta})$.

From Lemma 8, if $k \leq k_1$, sD remains optimal. Otherwise if $k > k_1$, S or aD is optimal. Specifically, with $k > k_1$, S is optimal for $k > k_2$, and aD is optimal for $k \leq k_2$.

Finally, we analyze the condition for $0 < k_1 < k_2$. This is equivalent to $\frac{r - \bar{c}}{\Delta} > \frac{(1 - 3\bar{\eta})(12\bar{\eta} + (1 - \bar{\eta})^2(1 - 4\bar{\eta}))}{3(1 - \bar{\eta})^2(1 - 4\bar{\eta})} - 4\bar{\eta}$ for $\bar{\eta} \in [0, \frac{1}{4}]$. The RHS is an increasing function with the minimum equal to 1. Note $\frac{r - \bar{c}}{\Delta} > 1$. Thus, for given $\frac{r - \bar{c}}{\Delta}$, the inequality is satisfied if $\bar{\eta}$ is sufficiently small. This is equivalent to a sufficiently large.

To prove Proposition 6, we first prove the following lemma:

Lemma 9 *With $X = \mu_d + \sigma_d X_0$, define $S_{\sigma_d}(Q)$ as the derivative of $S(Q)$ with respect to σ_d . Then $\frac{S_{\sigma_d}(Q)}{1 - G(Q)}$ is negative and decreases in Q .*

For given Q , define $Q_0 \doteq \frac{Q - \mu_d}{\sigma_d}$ and $S_0(Q_0) \doteq \mathbb{E}[\min(X_0, Q_0)]$. Then $S(Q) = \mu_d - \sigma_d S_0\left(\frac{Q - \mu_d}{\sigma_d}\right)$, $S_{\sigma_d}(Q) = -S_0(Q_0) + Q_0(1 - G_0(Q_0))$, and $\frac{S_{\sigma_d}(Q)}{1 - G(Q)} = -\frac{S_0(Q_0)}{1 - G_0(Q_0)} + Q_0$. Based on $\frac{d}{dQ} \frac{S_{\sigma_d}(Q)}{1 - G(Q)} = -\frac{S_0(Q_0)g_0(Q_0)}{\sigma_d(1 - G_0(Q_0))^2} < 0$, and $\frac{S_{\sigma_d}(d)}{1 - G(d)} = 0$, $\frac{S_{\sigma_d}(Q)}{1 - G(Q)}$ is negative and decreases in Q . ■

Proof of Proposition 6:

We shall first analyze the derivative of the buyer's profit with respect to σ_d under different supply base structures. Then we compare these derivatives. We use superscripts sD , aD , and S in the notations to differentiate the sD, aD, and S structures, unless where the structure is clear.

1) Symmetric dual sourcing

Let Q be the capacity for each supplier. When demand uncertainty is very small,

$$\Pi^{sD}(Q) = \left(r - \bar{c} + \frac{\mu_d}{2a}\right) \mu_d - 2kQ + \frac{\Delta}{3} (2S(Q) - \mu_d).$$

Thus the capacity of each supplier Q^{sD} in dual sourcing satisfies $G(Q^{sD}) = 1 - \frac{k}{\Delta/3}$ if $k < \frac{\Delta}{3}$, otherwise $Q^{sD} = \underline{d}$. Then based on the envelop theorem,

$$\frac{d\Pi^{sD}}{d\sigma_d} = \frac{2\Delta}{3} S_{\sigma_d}(Q^{sD}) = 2k \frac{S_{\sigma_d}(Q^{sD})}{1 - G(Q^{sD})}.$$

2) Sole sourcing

In sole sourcing, $\eta = \frac{S(Q^S)}{2a\Delta} \approx \frac{\mu_d}{2a\Delta}$ and $B = 0$, $A = S(Q)$. Then the profit from sole sourcing is

$$\Pi^S(Q) = \left(r - \bar{c} + \frac{S(Q)}{a} \right) S(Q) - kQ,$$

and Q^S satisfies $G(Q^S) = 1 - \frac{k}{r - \bar{c} + 2S(Q^S)/a} \approx 1 - \frac{k}{r - \bar{c} + 2\mu_d/a}$. Then based on the envelop theorem,

$$\frac{d\Pi^S}{d\sigma_d} = \left(r - \bar{c} + \frac{2S(Q^S)}{a} \right) S_{\sigma_d}(Q^S) = k \frac{S_{\sigma_d}(Q^S)}{1 - G(Q^S)}.$$

3) Asymmetric dual sourcing

With demand uncertainty very small, we have $Q_2 \approx 0$, $S(Q_2) = Q_2$, and $\eta \approx \bar{\eta}$. Then

$$\begin{aligned} \Pi^{aD} &= \left(r - \bar{c} + \frac{S(Q_1 + Q_2)}{2a} \right) S(Q_1 + Q_2) - k(Q_1 + Q_2) \\ &\quad + \Delta \frac{(1 - \eta)^3}{3} (S(Q_1) + Q_2 - S(Q_1 + Q_2)) + \Delta \eta (2S(Q_1) - S(Q_1 + Q_2)), \end{aligned}$$

subject to Equation (3). From $\frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_i} = \frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_i} \Big|_{\eta} + \frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial \eta} \Big|_{\eta} \frac{\partial \eta}{\partial Q_i}$, it follows

$$\begin{aligned} \frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_1} &= -k + \left(r - \bar{c} - \frac{\Delta}{3} (1 - 4\eta) (1 - \eta)^2 \right) (1 - G(\bar{Q})) \\ &\quad + \left(\frac{\Delta}{3} (1 - 4\eta) (1 - \eta)^2 + 4\Delta\eta \right) (1 - G(Q_1)), \end{aligned}$$

$$\frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_2} = -k + \frac{\Delta}{3} (1 - 4\eta) (1 - \eta)^2 + \left(r - \bar{c} - \frac{\Delta}{3} (1 - 4\eta) (1 - \eta)^2 \right) (1 - G(\bar{Q})).$$

Since (Q_1, Q_2) is an interior solution, we have $\frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_1} = 0$ and $\frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial Q_2} = 0$, resulting in $G(Q_1 + Q_2) = 1 - \frac{k - \Delta(1 - 4\bar{\eta})(1 - \bar{\eta})^2/3}{r - \bar{c} - \Delta(1 - 4\bar{\eta})(1 - \bar{\eta})^2/3}$ and $G(Q_1) = 1 - \frac{(1 - 4\bar{\eta})(1 - \bar{\eta})^2}{(1 - 4\bar{\eta})(1 - \bar{\eta})^2 + 12\bar{\eta}}$. Then based on the envelop theorem,

$$\begin{aligned} \frac{d\Pi^{aD}}{d\sigma_d} &= \frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial \sigma_d} \Big|_{\eta} + \frac{\partial \Pi^{aD}(Q_1, Q_2)}{\partial \eta} \Big|_{\eta} \frac{\partial \eta}{\partial \sigma_d} \\ &= \left(k - \frac{\Delta}{3} (1 - 4\bar{\eta}) (1 - \bar{\eta})^2 \right) \frac{S_{\sigma_d}(Q_1 + Q_2)}{1 - G(Q_1 + Q_2)} + \frac{\Delta}{3} (1 - 4\bar{\eta}) (1 - \bar{\eta})^2 \frac{S_{\sigma_d}(Q_1)}{1 - G(Q_1)}. \end{aligned}$$

We now compare the derivatives for $k = \frac{\Delta}{3} - \frac{\mu_d}{2a} > 0$, when sD is equivalent to S (aD) for $\sigma_d = 0$.

i) When a is sufficiently small so that $\frac{\Delta}{3} \approx \frac{\mu_d}{2a}$, we have $\bar{\eta} \approx \frac{1}{3}$, and $k_2 < 0 \approx k_1$. Thus aD cannot be optimal, and the structure shifts between sD and S. In this case, $G(Q^S) < G(Q^{sD}) \approx 1$, and thus $Q^S < Q^{sD}$. Then based on Lemma 9, $\frac{S_{\sigma_d}(Q^S)}{1 - G(Q^S)} > \frac{S_{\sigma_d}(Q^{sD})}{1 - G(Q^{sD})}$. Therefore, $\frac{d\Pi^{sD}}{d\sigma_d} = 2k \frac{S_{\sigma_d}(Q^{sD})}{1 - G(Q^{sD})} < k \frac{S_{\sigma_d}(Q^{sD})}{1 - G(Q^{sD})} < k \frac{S_{\sigma_d}(Q^S)}{1 - G(Q^S)} = \frac{d\Pi^S}{d\sigma_d}$.

ii) When a is very large so that $\bar{\eta} \approx 0$, then $k \approx \frac{\Delta}{3}$, $k_1 < k_2$, and the structure sD has a boundary with aD. In this case, $Q^{sD} \approx \underline{d}$, and thus $\frac{d\Pi^{sD}}{d\sigma_d} = 2k \frac{S_{\sigma_d}(Q^{sD})}{1 - G(Q^{sD})} \approx 0$. In addition, $Q_1^{aD} \approx \bar{d}$, and $\frac{d\Pi^{aD}}{d\sigma_d} \approx k \frac{S_{\sigma_d}(Q_1^{aD})}{1 - G(Q_1^{aD})} < \frac{d\Pi^{sD}}{d\sigma_d}$.

The structure S also has a boundary with aD. Following the proof of Proposition 5, aD is better than S if $k < \frac{(1-\eta)^2(1-4\eta)}{12\eta+(1-\eta)^2(1-4\eta)}(r - \bar{c} + 4\Delta\eta)$, where $\eta = \frac{S(Q^S)}{2a\Delta}$. The RHS of the inequality is a decreasing function of $\eta \in [0, \frac{1}{4}]$ for $r - \bar{c} \geq \Delta$. Since $S(Q^S)$ decreases in σ_d , η decreases and thus the RHS of the inequality increases in σ_d . This means that S should shift to aD when σ_d increases.

Proof of Lemma 4:

For given \mathbf{Q} and \mathbf{p} , Supplier i 's expected profit is

$$\begin{aligned} \Pi_i^P &= \mathbb{E}[p_i - c_i + e_i | c_i \leq p_i + e_i] \Pr(c_i \leq p_i + e_i) q_i(\mathbf{Q}, p_{-i}, e_{-i}) - \varphi(e_i) \\ &= \begin{cases} (p_i - \mu_c + e_i) q_i(\mathbf{Q}, p_{-i}, e_{-i}) - \varphi(e_i) & \text{if } \bar{c} \leq p_i + e_i \\ \frac{1}{2\Delta} (p_i - \underline{c} + e_i)^2 q_i(\mathbf{Q}, p_{-i}, e_{-i}) - \varphi(e_i) & \text{if } \underline{c} \leq p_i + e_i < \bar{c} \\ -\varphi(e_i) & \text{else } p_i + e_i < \underline{c} \end{cases} . \end{aligned}$$

With $\varphi(e) = \frac{a}{2}e^2$, we have

$$\frac{\partial \Pi_i^P}{\partial e_i} = \begin{cases} q_i(\mathbf{Q}, p_{-i}, e_{-i}) - ae_i & \text{if } \bar{c} \leq p_i + e_i \\ \frac{1}{\Delta} ((p_i - \underline{c}) q_i(\mathbf{Q}, p_{-i}, e_{-i}) - (a\Delta - q_i(\mathbf{Q}, p_{-i}, e_{-i})) e_i) & \text{if } \underline{c} \leq p_i + e_i < \bar{c} \\ -ae_i & \text{else } p_i + e_i < \underline{c} \end{cases} .$$

With $p_1 \leq p_2$, the optimal effort of Supplier 1, e_1 , maximizes Π_1^P for $q_1(\mathbf{Q}, p_2, e_2) = S(Q_1)$. This leads to $e_1 = e_1^P(\mathbf{Q}, \mathbf{p})$. Then for given e_1 , we can calculate the optimal effort of Supplier 2, e_2 , that maximizes Π_2^P . This leads to $e_2 = e_2^P(\mathbf{Q}, \mathbf{p})$.

Proof of Lemma 5:

Based on the equilibrium of supplier efforts,

$$\Pr(c_2 \leq p_2 + e_2) = \begin{cases} 1 & \text{if } p_2 > \bar{c} - \frac{q_2(\mathbf{Q}, p_1, e_1)}{a} \\ \frac{a(p_2 - \underline{c})}{a\Delta - q_2(\mathbf{Q}, p_1, e_1)} & \text{if } \underline{c} \leq p_2 < \bar{c} - \frac{q_2(\mathbf{Q}, p_1, e_1)}{a} \\ 0 & \text{else } p_2 < \underline{c} \end{cases} .$$

Based on Equation (7), p_2 maximizes $\Pr(c_2 \leq p_2 + e_2)(r - p_2)$. Given $r \geq \bar{c} + \Delta$, this results in $p_2 = \bar{c} - \frac{q_2(\mathbf{Q}, p_1, e_1)}{a}$. Then $\Pr(c_2 \leq p_2 + e_2) = 1$.

In order to prove Proposition 7, we first prove the following lemma:

Lemma 10 *When demand is certain and equal to μ_d ,*

- i) if $k > \frac{\Delta}{4} - \frac{\mu_d}{2a}$, sole sourcing is optimal, $Q_i^{P*} = 0$ and $Q_{-i}^{P*} = \mu_d$ for $i = 1, 2$;*
- ii) else if $k \leq \frac{\Delta}{4} - \frac{\mu_d}{2a}$, symmetric dual sourcing is optimal, $Q_1^{P*} = Q_2^{P*} = \mu_d$.*

If demand is certain, we have $Q_1, Q_2 \in [0, \mu_d]$ and $Q_1 + Q_2 \geq \mu_d$ in the optimal solution. Then $S(Q_1) = Q_1$, $S(Q_2) = Q_2$, and $S(Q_1 + Q_2) = \mu_d$. Define $x \doteq \frac{a(p_1 - \underline{c})}{a\Delta - S(Q_1)} = \frac{a(p_1 - \underline{c})}{a\Delta - Q_1}$ as the probability of Supplier 1 not to quit. It follows that $p_1 = \underline{c} + x \left(\Delta - \frac{Q_1}{a} \right)$, $q_2(\mathbf{Q}, p_1, e_1) = Q_2 - (Q_1 + Q_2 - \mu_d)x$, and

$$\Pi^P = -k\bar{Q} + xQ_1 \left(r - \underline{c} - x \left(\Delta - \frac{Q_1}{a} \right) \right) + \left(r - \bar{c} + \frac{Q_2 - (\bar{Q} - \mu_d)x}{a} \right) (Q_2 - (\bar{Q} - \mu_d)x) .$$

Then for given x ,

$$\frac{\partial \Pi^P}{\partial Q_2} \Big|_x = -k + (1-x) \left(r - \bar{c} + \frac{2}{a} \left((1-x) Q_2 + x(\mu_d - Q_1) \right) \right)$$

is increasing in Q_2 , and

$$\frac{\partial \Pi^P}{\partial Q_1} \Big|_x = -k + x \left((1-x) \Delta + x \frac{2}{a} (2Q_1 + Q_2 - \mu_d) - \frac{2}{a} Q_2 \right)$$

is increasing in Q_1 . Thus for given x , the optimal solution is achieved with $Q_1 + Q_2 = \mu_d$ or $Q_1 = Q_2 = \mu_d$. Below we analyze and compare these two cases.

If $Q_1 + Q_2 = \mu_d$, then for given x ,

$$\Pi^P = -k\mu_d + xQ_1 \left(r - \underline{c} - x \left(\Delta - \frac{Q_1}{a} \right) \right) + \left(r - \bar{c} + \frac{Q_2}{a} \right) Q_2.$$

This leads to $Q_2 = \mu_d$ and $Q_1 = 0$, or $Q_1 = \mu_d$ and $Q_2 = 0$, both resulting in sole sourcing. The optimal profit (by selecting x) with S is $\Pi^{P,S} \doteq \mu_d \left(r - \bar{c} + \frac{\mu_d}{a} - k \right)$.

If $Q_1 = Q_2 = \mu_d$, then for given x , the buyer's expected profit is

$$\Pi^P = \mu_d \left(\left(\Delta - 2\frac{\mu_d}{a} \right) x(1-x) + r - \bar{c} + \frac{\mu_d}{a} - 2k \right).$$

To optimize Π^P , we have $x = \frac{1}{2}$, and thus the optimal profit with sD is $\Pi^{P,D} \doteq \mu_d \left(r - \bar{c} + \frac{\Delta}{4} + \frac{\mu_d}{2a} - 2k \right)$.

$\Pi^{P,D} \geq \Pi^{P,S}$ if and only if $\frac{\Delta}{4} - \frac{\mu_d}{2a} > k$. ■

Proof of Proposition 7:

From the proof of Lemma 10, when $\sigma_d = 0$, either (1) $Q_1^P = Q_2^P = \mu_d$, (2) $Q_1^P = \mu_d$, $Q_2^P = 0$, or (3) $Q_1^P = 0$, $Q_2^P = \mu_d$ is optimal. We now examine the three solutions when σ_d increases from zero by a small amount. Define $\xi(p_1, Q_1) \doteq \frac{a(p_1 - \underline{c})}{a\Delta - S(Q_1)}$.

(1) We have $S(Q_1 + Q_2) = \mu_d$, $S(Q_1) \approx S(Q_2) \approx \mu_d$, and

$$\frac{\partial \Pi^P}{\partial Q_1} \approx \xi(1 - G(Q_1)) \left(\Delta - 2\frac{\mu_d}{a} - \xi \left(\Delta - 4\frac{\mu_d}{a} \right) \right) - k,$$

$$\frac{\partial \Pi^P}{\partial Q_2} \approx -k + (1 - \xi) \left(r - \bar{c} + 2\frac{\mu_d}{a} (1 - \xi) \right) (1 - G(Q_2)).$$

With $\xi \approx \frac{1}{2}$, the optimal Q_1 and Q_2 are determined by $1 - G(Q_1) \approx \frac{4k}{\Delta}$ and $1 - G(Q_2) \approx \frac{2k}{r - \bar{c} + \frac{\mu_d}{a}}$. Therefore, $Q_1 \approx Q_2 \approx \mu_d$. Since $r - \bar{c} + \frac{\mu_d}{a} > \frac{\Delta}{2}$, we have $Q_2 > Q_1$.

(2) With $Q_2 \approx 0$ and $Q_1 \approx \mu_d$, we have $\frac{\partial \Pi^P}{\partial p_1} \approx \frac{aS(Q_1)(a\Delta - S(Q_1))}{(a\Delta - S(Q_1))^2} (r + \underline{c} - 2p_1)$. $\frac{\partial \Pi^P}{\partial p_1} \geq 0$ for any $p_1 \leq \bar{c}$. Therefore, in the optimal solution $p_1 = \bar{c} - \frac{S(Q_1)}{a}$, $\Pr(c_1 \leq p_1 + e_1) = 1$, and $g_2(Q, p_1, e_1) = S(Q_1 + Q_2) - S(Q_1)$. It follows that $\frac{\partial \Pi^P}{\partial Q_2} \approx -k + (r - \bar{c})(1 - G(Q_1 + Q_2))$, leading to $G(Q_1 + Q_2) = 1 - \frac{k}{r - \bar{c}}$. As a result,

$$\begin{aligned} \frac{\partial \Pi^P}{\partial Q_1} &\approx -k + \left(r - \bar{c} + \frac{2\mu_d}{a} \right) (1 - G(Q_1)) + (r - \bar{c}) \left(G(Q_1) - 1 + \frac{k}{r - \bar{c}} \right) \\ &= \frac{2\mu_d}{a} (1 - G(Q_1)) \geq 0 \end{aligned}$$

Thus in the optimal solution $Q_1 = G^{-1}\left(1 - \frac{k}{r-\bar{c}}\right)$ and $Q_2 = 0$, i.e., S remains to be optimal.

(3) With $Q_1 \approx 0$ and $Q_2 \approx \mu_d$, we have $q_2(\mathbf{Q}, p_1, e_1) \approx \mu_d$, $\xi(p_1, Q_1) = \frac{p_1 - \underline{c}}{\Delta}$, $\frac{\partial}{\partial p_1} \xi(p_1, Q_1) = \frac{1}{\Delta}$, $\frac{\partial}{\partial Q_1} \xi(p_1, Q_1) = \frac{p_1 - \underline{c}}{a\Delta^2}$, and $\frac{\partial \Pi^P}{\partial Q_2} \approx -k + (r - \bar{c} + 2\frac{\mu_d}{a})(1 - G(Q_2))$. Thus $G(Q_2) = 1 - \frac{k}{r - \bar{c} + 2\frac{\mu_d}{a}}$. It follows

$$\frac{\partial \Pi^P}{\partial Q_1} \approx -k + \left(-p_1 + \bar{c} - 2\frac{\mu_d}{a} + k\right) \xi(p_1, Q_1).$$

For given $Q_1 \approx 0$ and $Q_2 \approx \mu_d$, $\frac{\partial \Pi^P}{\partial p_1} \approx -2\frac{S(Q_1)}{\Delta} \left(p_1 - \mu_c + \frac{\mu_d}{a} - \frac{k}{2}\right)$. Since $p_2 = \bar{c} - \frac{S(Q_2)}{a} \approx \bar{c} - \frac{\mu_d}{a}$ has to be greater than p_1 , we have $p_1 = \min\left(\frac{\bar{c} + \underline{c}}{2} - \frac{\mu_d}{a} + \frac{k}{2}, \bar{c} - \frac{\mu_d}{a}\right)$.

If $k > \Delta$, then $p_1 = \bar{c} - \frac{\mu_d}{a}$. In this case, $\xi(p_1, Q_1) = \frac{a\Delta - \mu_d}{a\Delta}$, $\frac{\partial}{\partial Q_1} \xi(p_1, Q_1) = \frac{a\Delta - \mu_d}{(a\Delta)^2}$, and $\frac{\partial \Pi^P}{\partial Q_1} = -k + k\frac{a\Delta - \mu_d}{a\Delta} < 0$. Thus $Q_1 = 0$.

If $k \leq \Delta$, then $p_1 = \mu_c - \frac{\mu_d}{a} + \frac{k}{2}$. In this case, $\xi(p_1, Q_1) = \frac{1}{\Delta} \left(\frac{\Delta}{2} - \frac{\mu_d}{a} + \frac{k}{2}\right)$, and $\frac{\partial \Pi^P}{\partial Q_1} = -k + \frac{1}{4\Delta} \left(k + \Delta - \frac{2\mu_d}{a}\right)^2$. If $k < k_2^P \doteq \Delta + 2\frac{\mu_d}{a} - 2\sqrt{2\Delta\frac{\mu_d}{a}}$ (the RHS is less than Δ with $\frac{\mu_d}{a} < \Delta$), then $\frac{\partial \Pi^P}{\partial Q_1} > 0$ and thus $Q_1 > 0$. This means that aD but not S is locally optimal in this case. From Lemma 10, if in addition $k > k_1^P \doteq \frac{\Delta}{4} - \frac{\mu_d}{2a}$, then this solution is global optimal.

$k_2^P > k_1^P$ is equivalent to $\left(\sqrt{\Delta} - \sqrt{2\frac{\mu_d}{a}}\right) \left(3\sqrt{\Delta} - 5\sqrt{2\frac{\mu_d}{a}}\right) > 0$, i.e., $a > \frac{50\mu_d}{9\Delta}$.

In order to prove Proposition 8, we first prove the following lemma:

Lemma 11 *When the capacity cost k is zero,*

- i) if $a\Delta < 2\mu_d$, then $(Q_1^{P*}, Q_2^{P*}) = (\bar{d}, 0)$ with the buyer's profit $\Pi^P = (r - \bar{c} + \frac{\mu_d}{a}) \mu_d$;
- ii) otherwise if $a\Delta \geq 2\mu_d$, $(Q_1^{P*}, Q_2^{P*}) = (\bar{d}, \bar{d})$ with the buyer's profit $\Pi^P = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{4}) \mu_d$.

With $k = 0$, the optimal solution has $Q_1 + Q_2 \geq \mu_d$ and $S(Q_1 + Q_2) = \mu_d$. It follows that $q_2(\mathbf{Q}, p_1, e_1) = S(Q_2) - (S(Q_1) + S(Q_2) - \mu_d) \frac{a(p_1 - \underline{c})}{a\Delta - S(Q_1)}$, and

$$\Pi^P = \frac{a(p_1 - \underline{c})}{a\Delta - S(Q_1)} (r - p_1) S(Q_1) + \left(r - \bar{c} + \frac{q_2(\mathbf{Q}, p_1, e_1)}{a}\right) q_2(\mathbf{Q}, p_1, e_1).$$

$Q_2 = \bar{d}$ to optimize Π^P . Thus $q_2(\mathbf{Q}, p_1, e_1) = \mu_d - S(Q_1) \frac{a(p_1 - \underline{c})}{a\Delta - S(Q_1)}$, and for given Q_1 ,

$$\frac{\partial \Pi^P}{\partial p_1} = \frac{aS(Q_1)}{a\Delta - S(Q_1)} \left(\Delta - \frac{2\mu_d}{a} - 2(p_1 - \underline{c}) \frac{a\Delta - 2S(Q_1)}{a\Delta - S(Q_1)}\right).$$

1) If $\Delta - \frac{2\mu_d}{a} > 0$, then $a\Delta - 2S(Q_1) > 0$, and $\frac{\partial \Pi^P}{\partial p_1}$ is decreasing in p_1 . Since $\frac{\partial \Pi^P}{\partial p_1} \leq 0$ when $p_1 = \bar{c} - \frac{S(Q_1)}{a}$, the optimal p_1 is an interior solution that leads to $\frac{\partial \Pi^P}{\partial p_1} = 0$: $p_1 = \frac{(a\Delta - 2\mu_d)(a\Delta - S(Q_1))}{2a(a\Delta - 2S(Q_1))} + \underline{c}$. Then we have $\frac{d\Pi^P}{dQ_1} = \frac{\Delta(1-G(Q_1))(a\Delta - 2\mu_d)^2}{4(a\Delta - 2S(Q_1))^2} > 0$. Thus $Q_1 = \bar{d}$ and $p_1 = \mu_c - \frac{\mu_d}{2a}$. This means for $\Delta - \frac{2\mu_d}{a} > 0$, $(Q_1^{P*}, Q_2^{P*}) = (\bar{d}, \bar{d})$, with the buyer's profit $\Pi^{P*} = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{4}) \mu_d$.

2) If $\Delta - \frac{2\mu_d}{a} \leq 0$, then $p_1 = \underline{c}$ for $a\Delta - 2S(Q_1) > 0$. Next we focus on $a\Delta - 2S(Q_1) < 0$. In this case, Π^P is a quasi-convex function of p_1 , and thus $p_1 = \underline{c}$ or $p_1 = \bar{c} - \frac{S(Q_1)}{a}$. $p_1 = \underline{c}$ results in sole sourcing, with $\Pi^P = (r - \bar{c} + \frac{\mu_d}{a}) \mu_d$. When $p_1 = \bar{c} - \frac{S(Q_1)}{a}$,

$$\Pi^P = \left(r - \bar{c} + \frac{S(Q_1)}{a}\right) S(Q_1) + \left(r - \bar{c} + \frac{1}{a}(\mu_d - S(Q_1))\right) (\mu_d - S(Q_1)),$$

and $\frac{d\Pi^P}{dQ_1} = \frac{2}{a}(1 - G(Q_1))(2S(Q_1) - \mu_d)$. Since $\mu_d < a\Delta < 2S(Q_1)$, $\frac{d\Pi^P}{dQ_1} > 0$ and thus $Q_1 = \bar{d}$. In this case, $\Pi^P = (r - \bar{c} + \frac{\mu_d}{a})\mu_d$, and the buyer sole sources from Supplier 1 ($q_2(\mathbf{Q}, p_1, e_1) = 0$). This is equivalent to sole sourcing with $(Q_1^{P*}, Q_2^{P*}) = (\bar{d}, 0)$.

To summarize, when $\Delta - \frac{2\mu_d}{a} > 0$, the buyer should invest in capacity \bar{d} for both suppliers, resulting in profits $\Pi^P = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{4})\mu_d$, and when $\Delta - \frac{2\mu_d}{a} \leq 0$, the buyer should invest in capacity \bar{d} for only one supplier, resulting in profits $\Pi^P = (r - \bar{c} + \frac{\mu_d}{a})\mu_d$. ■

Proof of Proposition 8:

It can be proven that under sole sourcing, the buyer's profit is equal with ex ante price commitment and with ex post price negotiation: In the former mechanism, with only Supplier 2, the solution has $q_2(\mathbf{Q}, p_1, e_1^P(\mathbf{Q}, \mathbf{p})) = S(Q_2)$ and thus $p_2 = \bar{c} - \frac{S(Q_2)}{a}$. Then the buyer's profit is $-kQ_2 + (r - \bar{c} + \frac{S(Q_2)}{a})S(Q_2)$, which is same as the sole sourcing profit in the base model (see Corollary 2).

When k or σ_d is very small, from Lemmas 11 and 10, the supply base design focuses on the S and sD structures in both the ex post price negotiation and ex ante price commitment mechanisms (aD is close to either S or sD). Denote the buyer's profits by Π^S ($\Pi^{P,S}$) and Π^D ($\Pi^{P,D}$) under S and sD with ex post price negotiation (ex ante price commitment).

When $k = 0$, from Lemma 11, $\Pi^{P,D} = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{4})\mu_d$, and from Proposition 4, $\Pi^D = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{3})\mu_d$.

When $\sigma_d = 0$, from Proposition 7, $\Pi^{P,D} = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{4} - 2k)\mu_d$, and from Proposition 5, $\Pi^D = (r - \bar{c} + \frac{\mu_d}{2a} + \frac{\Delta}{3} - 2k)\mu_d$.

Note $\Pi^D - \Pi^{P,D} = \frac{\Delta}{12}\mu_d$ with either $k = 0$ or $\sigma_d = 0$. Thus when k or σ_d is very small, $\Pi^D > \Pi^{P,D}$. We already have $\Pi^{P,S} = \Pi^S$. Therefore, i) if $\Pi^S > \Pi^D$, then $\Pi^{P,S} > \Pi^{P,D}$, and ii) $\max(\Pi^S, \Pi^D) \geq \max(\Pi^{P,S}, \Pi^{P,D})$.

Proof of Lemma 6:

Based on the mixed-strategy equilibrium, for any price $p \leq c$, the buyer's profit is

$$\begin{aligned} \tilde{\Pi}(Q) &= (r - p) \Pr(c - e \leq p) S(Q) - kQ \\ &= (r - p) \frac{r - c}{r - c + (c - p)} S(Q) - kQ \\ &= (r - c) S(Q) - kQ. \end{aligned}$$

For any effort $e \geq 0$, the supplier's profit is

$$\tilde{\Pi}_i(e) = \frac{a}{S(Q)} \int_{c-e}^c (p - c + e) dp \cdot S(Q) - \frac{a}{2}e^2 = 0.$$

Proof of Lemma 7:

For given price p , the supplier's profit is $\max((p - c + e)S(Q) - \frac{a}{2}e^2, 0)$. The supplier will invest effort $\frac{S(Q)}{a}$ for any price p greater than $c - \frac{S(Q)}{2a}$; otherwise the effort is zero if the price is lower than $c - \frac{S(Q)}{2a}$. Thus the optimal price for the buyer is $c - \frac{S(Q)}{2a}$.

Proof of Proposition 9:

This directly follows $\tilde{\Pi}^P(Q) - \tilde{\Pi}(Q) = \frac{S^2(Q)}{2a}$.

Proof of Proposition 10:

We prove for the dual sourcing mechanism. The sole sourcing mechanism can be proved similarly with the number of suppliers equal to one.

The proof is similar as for Lemma 2, except that in the menu of contracts the buyer has to specify the capacity investment $Q_i(\gamma_i, \gamma_{-i})$, and the quantity allocation $q_i(\gamma_i, \gamma_{-i}, X) \leq Q_i(\gamma_i, \gamma_{-i})$ is random, depending on the future demand X . Then the expected quantity of Supplier i with type c_i will be $\bar{q}_i(c_i - e_i) = \mathbb{E}_{c_{-i}, X} [q_i(c_i - e_i, c_{-i} - e_{-i}, X)]$. Thus, the ex ante profit of a supplier can be written as $U_i = \mathbb{E}_{c_i} \left[\bar{q}_i(c_i - e_i) \frac{F_i(c_i)}{f_i(c_i)} \right]$, and the expected profit of the buyer is equal to

$$U = \mathbb{E} \left[r \min \left(\sum_{i=1,2} q_i(\gamma_1, \gamma_2, X), X \right) - \sum_{i=1,2} J(c_i, e_i) q_i(\gamma_1, \gamma_2, X) - k \sum_{i=1,2} Q_i(\gamma_1, \gamma_2) \right],$$

where $\gamma_i = c_i - e_i$. To maximize U , the buyer should invest in capacity only for the supplier with a lower virtual cost $J(c_i, e_i)$, and this supplier provide a quantity up to his capacity to satisfy the demand. Thus, the buyer's profit can be simplified as

$$U = \sum_{i=1,2} \mathbb{E} [(r - J(c_i, e_i)) S(Q_i(\gamma_1, \gamma_2)) - kQ_i(\gamma_1, \gamma_2) | J(c_i, e_i) \leq J(c_{-i}, e_{-i})] \Pr(J(c_i, e_i) \leq J(c_{-i}, e_{-i})).$$

Since the two suppliers are ex ante identical, we focus on the symmetric equilibrium of effort $e_1 = e_2 = e$. Then $J(c_i, e_i) \leq J(c_{-i}, e_{-i})$ corresponds to $c_i \leq c_{-i}$. Thus $Q_i(\gamma_1, \gamma_2) = Q^*(J(c_i, e))$, and

$$\begin{aligned} U &= \sum_{i=1,2} \mathbb{E} [(r - J(c_i, e)) S(Q^*(J(c_i, e))) - kQ^*(J(c_i, e)) | c_i \leq c_{-i}] \Pr(c_i \leq c_{-i}) \\ &= \mathbb{E}_{c_{(1)}} [(r - J(c_{(1)}, e)) S(Q^*(J(c_{(1)}, e))) - kQ^*(J(c_{(1)}, e))]. \end{aligned}$$

In this mechanism, by investing effort \hat{e} , a supplier's expected profit is $\mathbb{E}_{\gamma_i} [u_i(\gamma_i) | \hat{e}_i] - \varphi(\hat{e}_i)$, and the equilibrium effort e is defined by $ae = \frac{1}{\Delta} u_i(\underline{c} - e)$. Since $u_i(\underline{c} - e) = \int_{\underline{c}}^{\bar{c}} S(Q^*(J(c, e))) (1 - F(c)) dc = \frac{\Delta}{2} \int_{\underline{c}}^{\bar{c}} S(Q^*(J(c, e))) f_{(1)}(c) dc$, the equilibrium effort e is the solution to $ae = \frac{1}{2} \mathbb{E}_{c_{(1)}} [S(Q^*(J(c_{(1)}, e)))]$.

Proof of Proposition 11:

We first analyze the influence of σ_d . When sole sourcing and dual sourcing are equal with $\sigma_d = 0$, we have $\mathbb{E}_c [r - J(c, e^{C,S})] = \mathbb{E}_{c_{(1)}} [r - J(c_{(1)}, e^{C,D})]$. Since $f_{(1)}(c) = 2f(c)(1 - F(c))$, this is equivalent to $\mathbb{E}_c [\varphi^S(c)] = \mathbb{E}_c [\varphi^D(c)]$, where $\varphi^S(c) \doteq r - 2c + e^{C,S}$, $\varphi^D(c) \doteq (r - 2c + e^{C,D}) \frac{2(\bar{c}-c)}{\Delta}$. $\varphi^D(c) - \varphi^S(c)$ is a decreasing convex function of c . With $\mathbb{E}_c [\varphi^D(c) - \varphi^S(c)] = 0$, there exists $\hat{c} \in [\underline{c}, \bar{c}]$ such that $\varphi^D(c) - \varphi^S(c) > 0$ for $c < \hat{c}$, and the opposite for $c > \hat{c}$. For $\sigma_d > 0$, $S(Q^*(J(c, e^C_S)))$ is a decreasing function of c . Then $\mathbb{E}_c [(\varphi^D(c) - \varphi^S(c)) S(Q^*(J(c, e^{C,S})))] > \mathbb{E}_c [(\varphi^D(c) - \varphi^S(c)) S(Q^*(J(\hat{c}, e^{C,S})))] = 0$. Since $\Pi^{C,D} - \Pi^{C,S} > \mathbb{E}_c [(\varphi^D(c) - \varphi^S(c)) S(Q^*(J(c, e^{C,S})))]$, we have proved $\Pi^{C,D} - \Pi^{C,S} > 0$ when demand uncertainty is very small.

Next we analyze the influence of k . Note:

$$\frac{d}{dk}\Pi^{C,S} = \frac{1}{a} \frac{d}{dk} \mathbb{E}_c [S(Q^*(J(c, e^{C,S})))] \mathbb{E}_c [S(Q^*(J(c, e^{C,S})))] - \mathbb{E}_c [Q^*(J(c, e^{C,S}))]$$

$$\frac{d}{dk}\Pi^{C,D} = \frac{1}{2a} \frac{d}{dk} \mathbb{E}_c [S(Q^*(J(c_{(1)}, e^{C,D})))] \mathbb{E}_{c_{(1)}} [S(Q^*(J(c_{(1)}, e^{C,D})))] - \mathbb{E}_{c_{(1)}} [Q^*(J(c_{(1)}, e^{C,D}))].$$

When $k = 0$, $Q^*(J(c, e^{C,S})) = Q^*(J(c, e^{C,D})) = \bar{d}$, and $e^{C,S} = e^{C,D} = \frac{\mu_d}{2a}$. Since $\frac{d}{dk} \mathbb{E}_c [S(Q^*(J(c, e)))] < 0$, we have $\frac{d}{dk}\Pi^{C,S} < \frac{d}{dk}\Pi^{C,D}$ at $k = 0$.

Proof of Proposition 12:

When k or σ_d is very small, the supply base design with ex ante capacity investment focuses on the S and sD structures (aD is approximately S). Denote by Π^S ($\Pi^{C,S}$) and Π^D ($\Pi^{C,D}$) the buyer's profits under sole sourcing and dual sourcing (or sD) with ex ante capacity investment (ex post capacity investment).

When $k = 0$, $\Pi^{C,S} = \Pi^S = (r - \bar{c} + \frac{\mu_d}{a}) \mu_d$, and $\Pi^{C,D} = \Pi^D = (r - \bar{c} + \frac{\Delta}{3} + \frac{\mu_d}{2a}) \mu_d$. Thus, if $\Pi^D > \Pi^S$, then $\Pi^{C,D} > \Pi^{C,S}$. This proves i). When $a\Delta > \frac{3\mu_d}{2}$, $\Pi^D > \Pi^S$ and $\Pi^{C,D} > \Pi^{C,S}$. We compare Π^D and $\Pi^{C,D}$ for k very small (they are equal for $k = 0$). From the proof of Proposition 11, $\frac{d}{dk}\Pi^{C,D} \approx \frac{1}{2a} \frac{d}{dk} \mathbb{E}_c [S(Q^*(J(c_{(1)}, \frac{\mu_d}{2a})))] \mu_d - \bar{d} \approx -\bar{d}$. Since $\frac{d}{dk}\Pi^D \approx -2\bar{d}$, we have $\frac{d}{dk}\Pi^{C,D} > \frac{d}{dk}\Pi^D$. Thus $\Pi^{C,D} \geq \Pi^D$ when k is sufficiently small. This proves ii).

When $\sigma_d = 0$, $\Pi^{C,S} = \Pi^S = (r - \bar{c} + \frac{\mu_d}{a} - k) \mu_d$, and $\Pi^{C,D} = (r - \bar{c} + \frac{\Delta}{3} + \frac{\mu_d}{2a} - k) \mu_d$ while $\Pi^D = (r - \bar{c} + \frac{\Delta}{3} + \frac{\mu_d}{2a} - 2k) \mu_d$. Thus, if $\Pi^D > \Pi^S$, then $\Pi^{C,D} > \Pi^{C,S}$. This proves i). When $\frac{\Delta}{3} > \frac{\mu_d}{2a} + k$, we have $\Pi^{C,D} > \Pi^{C,S}$ and $\Pi^D > \Pi^S$, with $\Pi^{C,D} - \Pi^D = k\mu_d > 0$. When $\frac{\mu_d}{2a} < \frac{\Delta}{3} \leq \frac{\mu_d}{2a} + k$, we have $\Pi^{C,D} > \Pi^{C,S}$ and $\Pi^S > \Pi^D$, with $\Pi^{C,D} - \Pi^S = (\frac{\Delta}{3} - \frac{\mu_d}{2a}) \mu_d > 0$. Thus ex post investment is preferred to ex ante investment for $\frac{\mu_d}{2a} < \frac{\Delta}{3}$. This proves ii).