

Joint Pricing and Inventory/Production Decisions in a Two-Stage Dual-Channel Supply Chain System

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Abstract

The Internet is becoming increasingly important as a sales channel. Thus, most large retail firms have adopted a multi-channel strategy that includes both web-based channels and pre-existing off-line channels. Since competition exists between these two channels, for a supply chain with a dual-channel retailer, pricing in one channel will affect the demand in the other channel. This subsequently affects the retailer's replenishment (ordering) decisions, which have an impact on the producer's inventory/production plans and wholesale price decisions. It is clear then that pricing decisions and inventory/production decisions are interacting in each member of the supply chain and among the members in the chain as well. Thus, competition should play an important role when considering pricing strategies and inventory/production decision in a supply chain with dual-channel retailers. However, it is usually ignored. In this paper, we consider joint pricing and inventory/production decision problems for members in a monopoly two-stage dual-channel retailer supply chain. We analyze the problems by incorporating intra-product line price interaction (competition) in the EOQ model. We show that a unique equilibrium exists under certain realistic conditions. We also provide numerical results that offer insights for pricing strategies for the dual-channel retailer supply chain and for product design for different channels.

Our main findings are summarized as follows. Price pooling effect is found between online store and off-line store as the degree of substitution increases. The same observation holds for the wholesale prices. This provides managerial implications on pricing for different product category in online store and off-line store. The EOQ of the off-line store decreases with the degree of substitution; the EOQ of online store, on the other hand, increases with the degree substitution. This provide some guidance on inventory replenishment policies when intra-channel competition exists. It is also interesting to see that the total retail profit and the supplier's profit can either increase or decrease with the degree of substitution, depending on the production costs and self-sensitivity of demands.

1 Introduction

With the increasing popularity of the Internet, U.S. online retail sales are expected to rise 11% to \$156 billion in 2009 even in a down economy, according to projections from Forrester Research (Reuters, 2009). Since Internet is becoming increasingly important as a sales channel,

it is not surprising that web-based channels are fast being integrated into the channel strategy of traditional off-line retailers. Most large retail firms have adopted a multi-channel strategy that includes both web-based channels and traditional brick-and-mortar channels. For example, Chapters Inc. has both on-line and off-line book stores. Despite its advantages, such as reaching more potential customers, a retailer's dual-channel system introduces new management concerns. Logistics systems, for example, usually need to be redesigned when a traditional retailer becomes a dual-channel retailer. If the frequency of Internet orders for a dual-channel retailer becomes large, order fulfilment from its existing infrastructure is not desirable anymore. de Koster (2003) suggests that an Internet delivery distribution center with specially designed warehouses is established to help a dual-channel retailer obtain economies of scale. In addition, two channels serving common market segments compete. Thus, pricing and inventory decisions for the channels under competition are important issues that the management team has to deal with. In this paper, we consider a two-level supply chain with a dual-channel retailer and model the joint pricing and inventory/production decisions (from now on, joint decisions) at both the supplier and the retailer level.

There is a considerable amount of literature that describes advances in research and management practices in the area of joint decisions in a single firm. The literature deals with the decisions at the interface between marketing and manufacturing, specifically the simultaneous determination of pricing and inventory/production decisions. This line of research has spanned over 50 years with over 100 published articles. A broad review on this topic can be found in Eliashberg and Steinberg (1993) and Yano and Gilbert (2002). Research on joint decisions has drawn a great deal of attention because joint decisions can improve a firm's profitability over traditional sequential decisions. An example in Kunreuther and Richard (1971) indicates that profits can be increased by 12.5% if pricing and inventory/production decisions are made jointly rather than sequentially.

The idea of joint decisions can be extended to multi-firm supply chains. The main objective of supply chain management is often to maximize chain-wide profits, while satisfying service requirements and focusing on how to coordinate supply chain members in an efficient manner. Reviews on supply chain coordination can be found in Cachon (2003) and Chan et al. (2004). It is intuitive to think that supply chain members could further leverage chain-wide profits if their pricing and inventory/production decisions are made jointly.

There is indeed an emerging body of literature that deals with joint decisions in multi-firm supply chain settings. The first papers were published over 20 years ago, and there is a growing interest in the area. Most joint decisions research focuses on a single product through a supply chain (see, for example, Weng, 1995). However, when the retailer in a supply chain has dual channels that serve common market segments, the products sold in one channel can be seen as substitutes of the products sold in the other channel; thus creating channel competition. Real-world pricing and inventory/production decisions are not made in isolation for each channel in this type of supply chain. In a dual-channel retailer supply chain, pricing of the product in one channel will affect the demand in the other channel. This subsequently affects the retailer's replenishment (ordering) decisions, which have an impact on the supplier's inventory/production plans and wholesale price decisions. Thus, not only do each member's pricing decisions interact with its inventory/production decisions but also the supplier's decisions interact with the retailer's decisions.

We believe that it is important to further investigate joint decision problems in a dual-channel retailer supply chain. Given that research on joint decisions in a supply chain lies at the interface between marketing and operations management, we also believe that this line of research will foster collaboration between and contribute to these two functional areas of business.

We note that in a supply chain network with demand substitution, joint decisions can potentially become complicated. The interaction between demand for different products in addition to other interactions such as resource sharing, makes the problem difficult because substitution prevents the separate analysis of each product. Furthermore, the large number of decision variables makes the problem difficult to solve. A two-stage supply chain with one supplier and a dual-channel retailer has seven decision variables: Two retail prices (different prices in different channels), two wholesale prices (if product configurations for the two channels are different), two replenishment decisions for the retailer, and the product replenishment decision for the supplier.

Although many types of contracts, such as buy-back and discount contracts, can be employed by the members in decentralized chains, we limit our attention to wholesale price contracts because this type of contract is one of the most popular in practice. We ask the following key questions:

1. Does a pure strategy equilibrium exist for the two-player game? How do the supplier and the dual-channel retailer in a supply chain make their pricing and inventory/production

decisions in equilibrium?

2. How do the cross-price effect between the products in a dual-channel retailer supply chain and other system parameters affect the supplier's and the retailer's decisions and profits?

We perform our investigation in the following two-stage supply chain setting: The supplier in the chain purchases or manufactures the product according to the Economic Order Quantity (EOQ) model and sells the product through a retailer with an on-line and an off-line retail store. The retailer makes decisions for the inventory replenishment and retail prices for the two stores, while the supplier makes its own wholesale price and production or purchase decisions. Price-sensitive EOQ joint decision supply chain models are proposed to determine and understand the interactions between pricing and production/inventory decisions across products at both the supplier and the retailer levels. The models are formulated as Stackelberg games in which the supplier, as a leader, has the economic power and managerial ability to take into account the reactions of the downstream retailer.

We assume a general linear demand function in which demand for each retailer channel is a linear function of retail prices in both channels. The parameters of the demand functions for the online store and the off-line stores may take different values. Thus, the two channels are not symmetric in our model. We first analytically prove that a unique pure strategy Nash equilibrium exists without assuming any specific relationship between the parameters for two stores in our mathematical analysis. Then, we use numerical experiments to show the impact of the substitution of the stores and products, i.e., the cross-price effect, on the supplier's and the retailer's decisions in the two channels. In the numerical experiments, we mainly assume that customers are more sensitive to price changes in the online store than the off-line store, and are equally sensitive to price difference between two channels. The analysis results in several insights. We find a price convergence effect between the online and the off-line stores as the degree of substitution between the stores and the products increases. The same observation holds for wholesale prices. This finding provides managerial implications on pricing for different product categories in online and off-line stores. For products with strong cross-price effect such as CDs and books, the prices in the two channels should be close. The opposite pricing strategies should be applied to products with lower cross-price effects.

The EOQ of the off-line store decreases with the degree of substitution; the EOQ of the

online store, on the other hand, increases with the degree of substitution. This result provides some guidance on inventory replenishment policies when intra-channel competition exists. It is also interesting to see that the retailer's and the supplier's profits either increase or decrease with the degree of substitution depending on the production costs and self-price sensitivities of demands. Whether the firms can benefit by considering the cross-price effect depends on the production costs and self-price sensitivity of demands as well.

The rest of the paper is organized as follows: Section 2 reviews related literature. Section 3 discusses our assumptions and presents our models. Section 4 analyze the models. Section 5 presents numerical examples. Finally, Section 6 offers concluding comments. Mathematical proofs that are not in the main body can be found in the Appendix.

2 Literature Review

The popularity of online shopping has forced traditional brick-and-mortar companies to redesign their distribution channels (Kopczak, 2001). Many manufacturers have added direct internet-enabled distribution channels to distribute their products in addition to their traditional retail stores. Netessine and Rudi (2006) and Chiang and Monahan (2005) investigate inventory management strategies in such a supply chain setting. At the same time, we can also see that many traditional retail stores have added an online channel to their existing traditional brick-and-mortar channels. However, companies should be cautious when deciding to have multiple channels because of the possible competition among the channels. King et al. (2004) examine the conditions under which traditional retailers should incorporate online channels. In this paper, we assume that the supply chain is designed and in place to accommodate both web-based channels and pre-existing off-line channels for a given retailer. We investigate the operational decisions under this structure.

Research on joint decisions in supply chains combines research on supply chain management and research on joint decision problems in single firms. Research shows that joint decisions can often improve chain-wide performance by incorporating pricing into the system, which is ignored in most supply chain management research. Most models in research on joint decisions are formulated to solve pricing and inventory/production decisions for each member in a two-stage supply chain with a single supplier and a single buyer. A few papers, such as Bernstein and Federgruen (2003), Chen et al. (2001), and Weng and Zeng (2001), consider single-supplier

and multi-buyer supply chains. Multiple competing retailers selling the same product, such as in Bernstein and Federgruen (2005), can be seen as equivalent to selling multiple substitutable products. Competition among retailers has the same effect as competition between substitutable products, which is what we consider in the retailer’s two channels. However, we consider the retailer’s interest in both channels as a whole. Mechanisms to achieve coordination among members without commercial integration, which is the focus of many papers, are out of the scope of this paper.

Another line of related work considers pricing strategies in different supply chain structures. Choi (1991), Choi (1996), and Lee and Staelin (1997) examine channel pricing strategies for substitutable products under various supply chain models. However, this line of work does not consider any member’s inventory/production decisions.

Next, we discuss the model assumptions and introduce the joint decision models.

3 Assumptions and Problem Formulation

We consider a two-stage supply chain with one supplier and one retailer who has two distribution channels. In practice, a retailer may have one online store and more than one retail outlet. In our stylized model, we consider one off-line outlet store that competes with the online store; thus, products in one channel are seen as substitutes for the products in the other channel. We refer to the online store as store 1 and to the off-line store as store 2. Each store carries the product manufactured or distributed by the supplier to meet the end market demand $\mathbf{D} = (D_1, D_2)$. The demand for the online store and the demand for the off-line store depend on the prices in the two channels. For many products, these two prices are different. For example, the book “The Snowball: Warren Buffett And The Business Of Life” is sold \$35 in stores, but it is sold \$28 online (www.bn.com, 2009). The supplier may charge different wholesale prices for orders from the online store and from the off-line store due to different product configurations or processing costs.

We assume that the off-line store has its own inventory to supply the demand of local customers. There are many possible ways for an online store to distribute its products to its customers (de Koster, 2003). We assume that the demand for the online store is satisfied by a warehouse, which is strategically located for convenient delivery to the online customers. Thus, we have two separate inventory storage locations fulfilling orders from the online channel and the

off-line channel, respectively. This assumption is realistic for many retail situations and enables us to focus on the impact of price interaction between products on inventories and performance. The orders from the two channels are processed and fulfilled separately by the supplier. In this paper, we use stores interchangeably with distribution centers regarding inventory management because we consider only two stores in the model and each store corresponds to a different distribution center.

We model demand as the following linear function of prices:

$$D_i = k_i - \alpha_i p_i + \theta_{ij}(p_j - p_i) \quad (1)$$

with $k_i > 0$, $\alpha_i > 0$, $i = 1, 2$ and $j = 3 - i$, where $\theta_{ij} \geq 0$ represents the competition (substitution or cross-price) effect between products in the online store (store 1) and the off-line store (store 2). This effect is caused by channel differentiation and product differentiation. The higher θ_{ij} , the more substitutable the product in store j is for the product in store i ; that is, competition is more severe between the two stores.

In addition, we have the following assumption about demand:

$$\alpha_i + \theta_{ij} > \theta_{ji} \quad (2)$$

for $i = 1, 2$ and $j = 3 - i$. This assumption states that an increase in the price in either store results in a decrease of total sales in the market. It also implies that a price change in store i has a greater effect on its own demand than on that of the other store. This is an intuitive condition and can be seen in research that adopts a linear demand function such as Bernstein and Federgruen (2003) and Choi (1991).

In our model, each retail store operates under a deterministic price-sensitive EOQ model. The retailer seeks the optimal retail prices $\mathbf{p} = (p_1, p_2)$ and the optimal order quantities $\mathbf{q} = (q_1, q_2)$ from the supplier for both retail stores. The total ordering cost of each product consists of a fixed cost per order s_i plus a cost per unit w_i . The unit inventory holding cost is h_i . All other assumptions follow those of the classic EOQ model except that demand is price-sensitive. The retailer's yearly profit under any given wholesale prices $\mathbf{w} = (w_1, w_2)$, which equals the gross revenue minus the ordering and the inventory holding costs of the two stores, is a function of \mathbf{p}

and \mathbf{q} :

$$\pi^r(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^2 (p_i - w_i)D_i(\mathbf{p}) - s_i D_i(\mathbf{p})/q_i - h_i q_i/2 \quad (3)$$

We assume that the supplier purchases the product from an outside supplier or manufacturers the product under a deterministic EOQ model and then sells the product to the retail stores at \mathbf{w} . If the demands from the two retail stores can be approximated as a constant demand rate $D_1 + D_2$, the supplier's economic order quantity can be modeled as $Q = \sqrt{2S(D_1 + D_2)/H}$, where S and H are the supplier's setup cost and unit holding cost, respectively. The approximation considers the integrated impact of the demands from the two stores on the supplier's inventory decisions. In addition, it is possible that the supplier provides the products to other retailers in addition to these two retail stores. The approximation is more accurate when the supplier supply a large number of retailers, and the demand from the two retail stores is approximately a fixed percentage of the total demand for the product from all retailers.

The resulting ordering and holding cost for the supplier is $\sqrt{2SH(D_1 + D_2)}$. We denote by c_i the unit cost of acquiring or manufacturing the product if the supplier has different product configurations or different order processing costs for the online store and the off-line store which result in different unit costs. The supplier's yearly profit for two stores is equal to the sum of yearly gross revenue from each store minus the ordering and holding costs.

$$\pi^s(\mathbf{p}, \mathbf{w}) = \sum_{i=1}^2 (w_i - c_i)D_i(\mathbf{p}) - \sqrt{2SH(D_1(\mathbf{p}) + D_2(\mathbf{p}))} \quad (4)$$

Both the supplier and the retailer aim to maximize their own profits.

4 Model Analysis

In Section 3, we proposed the model for the two-stage dual-retailer channel supply chain. In this section, we analyze the model. The interaction between the supplier and the retailer are analyzed as a Stackelberg game with the supplier as the leader. A backward induction procedure is applied to solve the problem. In the backward induction procedure, the retailer makes decisions first, then the supplier makes decisions based on the retailer's decisions.

For any \mathbf{p} , which, in turn, determines the annual demand rate \mathbf{D} , the retail store i 's optimal order size is the EOQ order quantity $q_i = \sqrt{2s_i D_i(\mathbf{p})/h_i}$. The resulting ordering and holding

cost is $\sqrt{2s_i h_i D_i(\mathbf{p})}$. Thus, the retailer's yearly profit function (3) can be rewritten as

$$\pi^r(\mathbf{p}) = \sum_{i=1}^2 (p_i - w_i) D_i(\mathbf{p}) - \sqrt{2s_i h_i D_i(\mathbf{p})} \quad (5)$$

For any \mathbf{w} charged by the supplier, the retailer's objective is to choose \mathbf{p} in order to maximize its yearly profit. In general, this profit function fails to exhibit any known structural properties to ensure a unique optimal solution. However, we can show that this function is strictly concave in \mathbf{p} in most, if not all, realistic markets in which sales-to-inventory ratios are not excessively low and demand elasticities are not excessively large (in absolute value). More specifically, we introduce the following conditions: Let $I_i = p_i D_i$, which is the total gross income for selling the product in store i at price p_i , and let $V_i = \sqrt{2h_i s_i D_i}$ denote the optimal total inventory and setup cost for the product selling in store i . We use e_i to denote the absolute price elasticity of store i 's demand. In the linear demand example, $e_i = (\alpha_i + \theta_{ij}) p_i / D_i$. We assume that

$$e_i \leq 4 \frac{I_i}{V_i}, \quad (6)$$

This condition has been discussed in Bernstein and Federgruen (2003, 2004), which show that this condition is satisfied in virtually all realistic markets. According to the empirical data that the authors refer to, we assume the following strengthened condition:

$$e_i \leq 0.4 \frac{I_i}{V_i}. \quad (7)$$

We can get condition (7) from the data, originally published in Tellis (1988), in Bernstein and Federgruen (2003). We believe this condition (7) is also satisfied in most realistic markets. However, Bernstein and Federgruen (2003) assume only $e_i \leq 4I_i/V_i$ because it is sufficient to prove their results.

Theorem 1. *When (6) applies, the retailer's problem is strictly jointly concave in \mathbf{p} .*

A closed-form solution for \mathbf{p} in terms of \mathbf{w} cannot be obtained from solving the first order conditions of the retailer's problem. However, the strict concavity of the retailer's problem guarantees a one-to-one mapping between \mathbf{p} and \mathbf{w} . Therefore, we can obtain \mathbf{w} represented by

(8) in terms of \mathbf{p} by solving the first order conditions of the retailer's problem:

$$w_i(\mathbf{p}) = -\frac{\sqrt{h_i s_i}}{\sqrt{2D_i}} - \frac{(\alpha_j + \theta_{ji})\theta_{ij} - (\alpha_j + \theta_{ji})\theta_{ji}}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_j - \frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_i - \frac{\theta_{ji}k_j + (\alpha_j + \theta_{ji})k_i}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}}, \text{ for } i, j = 1, 2 \text{ and } i \neq j \quad (8)$$

From (8), we obtain

$$\frac{\partial w_i(\mathbf{p})}{\partial p_i} = -\frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} + \frac{\sqrt{h_i s_i}(\alpha_i + \theta_{ij})}{(2D_i)^{\frac{3}{2}}} \quad (9)$$

and

$$\frac{\partial w_i(\mathbf{p})}{\partial p_j} = -\frac{(\alpha_j + \theta_{ji})(\theta_{ij} - \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} - \frac{\sqrt{h_i s_i}\theta_{ij}}{(2D_i)^{\frac{3}{2}}} \quad (10)$$

From (9), we see that p_i increases with w_i , which is intuitive; if the product has a higher purchase cost, it will have a higher retail price. However, the retail price in one store may increase or decrease with the wholesale price of the other store. That is, $\partial w_i / \partial p_j$ can be positive or negative. It depends on the difference between the cross-price effect parameters θ_{ij} and θ_{ji} . If $\theta_{ij} \geq \theta_{ji}$, which shows that demand of store j is less sensitive to the price change in store i than store i to that of store j , p_j decreases as w_i increases. However, if $\theta_{ij} < \theta_{ji}$, $\partial w_i / \partial p_j$ can be positive or negative.

Substituting $w_i(\mathbf{p})$ into the supplier's problem, we have the following:

$$\begin{aligned} \pi^s(\mathbf{p}) = & -\sqrt{2SH[D_1(\mathbf{p}) + D_2(\mathbf{p})]} - \left(\frac{\theta_{ji}k_j + (\alpha_j + \theta_{ji})k_i}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} + c_i \right) D_i(\mathbf{p}) - \frac{1}{2}\sqrt{2s_i h_i D_i(\mathbf{p})} \\ & + \sum_{i=1}^2 \left(-\frac{(\alpha_j + \theta_{ji})\theta_{ij} - (\alpha_j + \theta_{ji})\theta_{ji}}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_j - \frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_i \right) D_i(\mathbf{p}). \end{aligned} \quad (11)$$

If we assume the following condition:

$$\max \left(\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})}, \frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2} \right) \leq \frac{39\sqrt{2}}{2} \approx 27, \quad (12)$$

where $\alpha_{\max} = \max(\alpha_1, \alpha_2)$, $\alpha_{\min} = \min(\alpha_1, \alpha_2)$, $\theta_{\max} = \max(\theta_{12}, \theta_{21})$, and $\theta_{\min} = \min(\theta_{12}, \theta_{21})$, we have Theorem 2.

Theorem 2. *When conditions (7) and (12) apply, the supplier's objective function is strictly jointly concave in \mathbf{p} .*

From the previous discussion, we know that condition (7) holds for almost all industries.

Although conditions (7) and (12) are sufficient to guarantee strict concavity, our numerical experiments will show that they are not necessary.

Theorem 3. *When conditions (2), (7), and (12) apply, a unique equilibrium exists for the Stackelberg game.*

Proof of Theorem 3. Because the retailer's objective function is strictly concave, there is one unique optimal retail price \mathbf{p} and one order quantity \mathbf{q} for any given wholesale price \mathbf{w} . The strict concavity of the supplier's objective function guarantees that there is only one unique optimal solution for \mathbf{w} and Q . \square

The unique equilibrium, however, does not have a closed-form solution. The numerical analysis in Section 5 will provide insights about the properties of the solution.

Next, we turn our attention to the numerical analysis in order to gain more insights under different business scenarios.

5 Numerical Analysis

In Section 4, we demonstrated the existence of a unique set of optimal joint decisions, under certain conditions. However, the complexity of the models prevented us from finding closed-form solutions. Instead, we carry out numerical analysis to investigate the impact of cross-price effect on joint decisions, which is one of the main concerns of this paper, as well as the impact of self-price sensitivities and production and inventory costs on joint decisions.

5.1 Parameter Settings

Suppose that the products sold at the two retail stores have different configurations, therefore, their unit costs are different. We assume that the product in the online store, store 1, has a lower cost and a higher self-price sensitivity than the product in the off-line store, store 2. It is common for products in off-line stores to be customized, thus, have higher unit costs. Online customers have higher price sensitivity because it is easy for them to compare prices and switch to other sellers. In addition, the empirical studies in Degeratu et al. (2000) suggest that online customers are more price sensitive than off-line customers. The authors also examined the combined effect of price and promotion on price-sensitivity. When the combined effect is

Table 1: The parameter sets

	Set 1 (base case)		Set 2 (high cost difference)	
	Store 1	Store 2	Store 1	Store 2
Self-price sensitivity, α	15	11	15	11
Cross-price effect, θ	2	2	2	2
Potential market size, k	1000	1000	1000	1000
Production cost, c	25	35	5	35
Supplier's setup cost, S	1000	1000	1000	1000
Supplier's unit holding cost, H	1	1	1	1
Retailer's setup cost, s	100	200	100	200
Retailer's unit holding cost, h	1.5	2	1.5	2

considered, online customers are less price-sensitive. In our experiments, we only consider the effect of price, thus, it is reasonable to assume a higher price sensitivity for the online store.

The substitutability between the products in the two stores is caused by both product differentiation and store differentiation. We do not identify these effects separately. If the products sold are exactly the same in both stores, the substitutability is only caused by store differentiation. In this paper, the combined substitutability is represented by θ_{ij} in the demand function. A common approach is to assume that the product sold in one store has the same cross-price effect to the price changes of the product as in the other store (see, for example, Choi, 1996). Thus, we assume $\theta_{12} = \theta_{21} = \theta$.

Table 1 shows the first two cases that we will analyze. The base case is chosen to be reasonably realistic. The high cost difference case is identical to the base case, except that store 1 has a much lower unit production cost. By comparing these two cases, we will highlight the impact of the production cost difference on optimal joint decisions. Observe that the unit production costs range from \$5 to \$35. Based on this we will restrict the numerical analysis to retail prices in the range from \$1 to \$150 for both products.

Next, we turn to the numerical analysis and investigate the impact of the different parameters.

5.2 Cross-price Effect

In a multi-retailer channel supply chain, understanding the degree to which the products in different channels are substitutable is crucial for both the manufacturer and the retailer. The

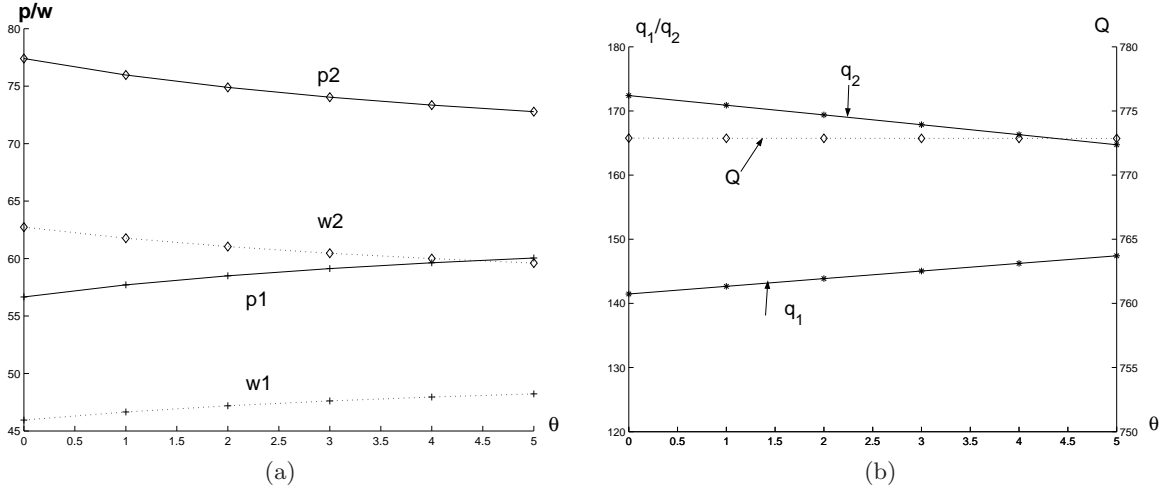


Figure 1: The impact of cross-price effect on optimal decisions

cross-price effect has impacts on the pricing, the production, and the profitability of both stores and products, and we expect considering it will benefit the supply chain members.

5.2.1 Impact of cross-price effect

The cross-price effect is represented by θ . A higher θ indicates that the consumers consider products sold in the two retail stores as more and more substitutable (less and less differentiated), which can indicate that the consumers have weaker preference for one retailer channel over the other (less store differentiation). Product category is one factor that affects retail store differentiation. For example, store differentiation is low for books but high for clothes. The optimal decisions for each members under different cross-price effects are shown in Figure 1.

We observe a clear price convergence as shown in Figure 1a. As two products become more substitutable (θ increases), their retail prices become closer to each other, and so do their wholesale prices. The retail price in the off-line store which is higher at $\theta = 0$ decreases, while the retail price in the online store, which is lower at $\theta = 0$, increases with θ . The wholesale prices demonstrate the same trend. The retail margin, the difference between retailer price and wholesale price, increases for products in the online store and decreases for products in the off-line store. These are intuitively appealing results. If the consumers have weaker preference to purchase from one store rather than the other store (larger θ), the prices in different stores get closer.

The EOQ for the off-line store (q_1) decreases and that for online store (q_2) increases with

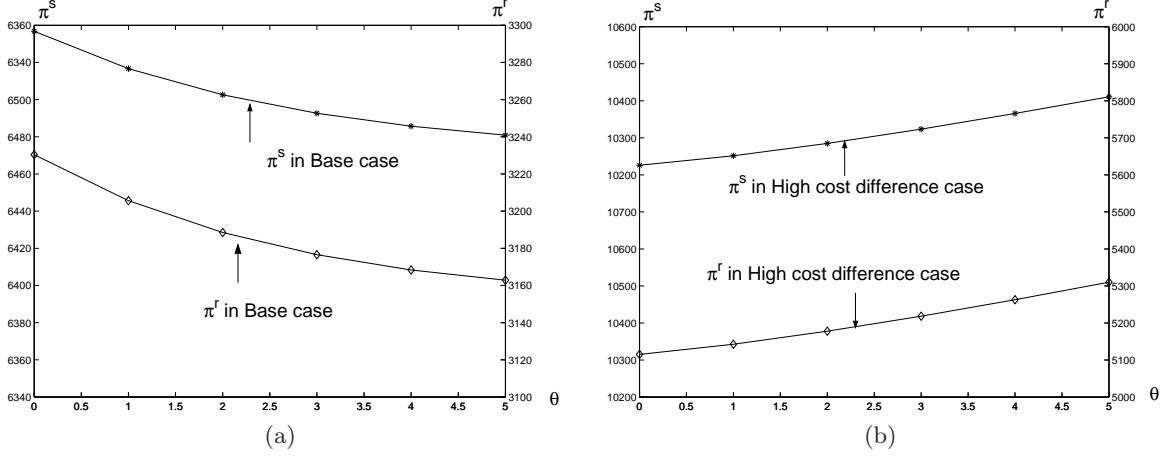


Figure 2: The impact of cross-price effect on optimal profits

θ as shown in Figure 1b. In an EOQ model, demand changes in the same direction as order quantity. Thus, this result indicates that more consumers choose to buy online if the two products become more substitutable. The supplier's EOQ (Q) is relatively stable with respect to θ , so does demand for the supplier as the sum of the demand from the two stores.

The supplier's and the retailer's profits decrease with θ in the base case as shown in Figure 2a. They increase with θ in the high cost difference case as shown in Figure 2b. We find that the production costs c_i and the self-price sensitivities α_i play a role. In the base case, the production costs c_1 and c_2 are close, the difference between self-price sensitivities α_1 and α_2 play a key role in the impact of substitution (store substitution in this scenario) on the profits. Increased θ indicates that the customers have weaker preference for one store over the other. Thus, the impact of self-price sensitivities is reduced by the increased substitution of products (θ). Therefore, the low production cost difference leads to fiercer inter-product competition when θ increases, which damages both the supplier's and the retailer's profits.

In the high cost difference case, the impact of a larger difference in production costs c_1 and c_2 dominates that of the self-price sensitivity on the profits. When the impact of different self-price sensitivities is reduced by the increased substitution, the high production cost difference allows the members to take advantage of the fact that customers treat the products as more similar and more substitutable; thus, both the retailer and the supplier earn higher profits.

The above observations about the price convergence effect and the profit trends have implications for the pricing strategies of products sold in online and off-line stores. For products

that are highly substitutable, such as books, the price difference in the two channels should be small. In addition, when a retailer has different product configurations for different retail channels, it should take the design of the products into consideration. The larger difference between production costs benefits both members.

We have analyzed the impact of the cross-price effect for the base case, the high cost difference case, and many other cases. The general pattern of price convergence and the qualitative behavior of the optimal decisions and optimal profit is the same in all cases that we have analyzed. In the remainder of this section, we will report additional numerical results for the base case.

5.2.2 Benefit from considering cross-price effect

Researchers such as Zhu and Thonemann (2003) show that firms benefit from considering the cross-price effect. However, in our model firms cannot always benefit from this consideration. When making decisions on the prices and the inventory replenishment without considering the cross-pricing effect, the firms take the demands as $D_i^w = k_i - \alpha_i p_i$ instead of the true demands $D_i = k_i - \alpha_i p_i + \theta_i(p_j - p_i)$. We use the superscript w to distinguish the demands without considering the cross-price effect from the true demands. The profits without considering θ are calculated by the firms' decisions on prices and the true demands in the market. From Figure 3, we can see that both members benefit from considering θ in the base case, and both suffer slightly from considering θ in the high cost difference case.

In the base case, the firms benefit from considering θ because the joint decisions are made taking the fierce competition between two channels into consideration, which we have discussed in Section 5.2.1. The competition between two channels is subtle in the high cost difference case, which makes no big difference in profitability of the supply chain members.

5.2.3 Summary of Our Key Findings

We summarize our findings about the impact of the cross-price effect on each member's optimal decisions and performance in Table 2, where a notation of \uparrow (\downarrow) indicates an increase (decrease) of a quantity, and a notation of $--$ indicates no change.

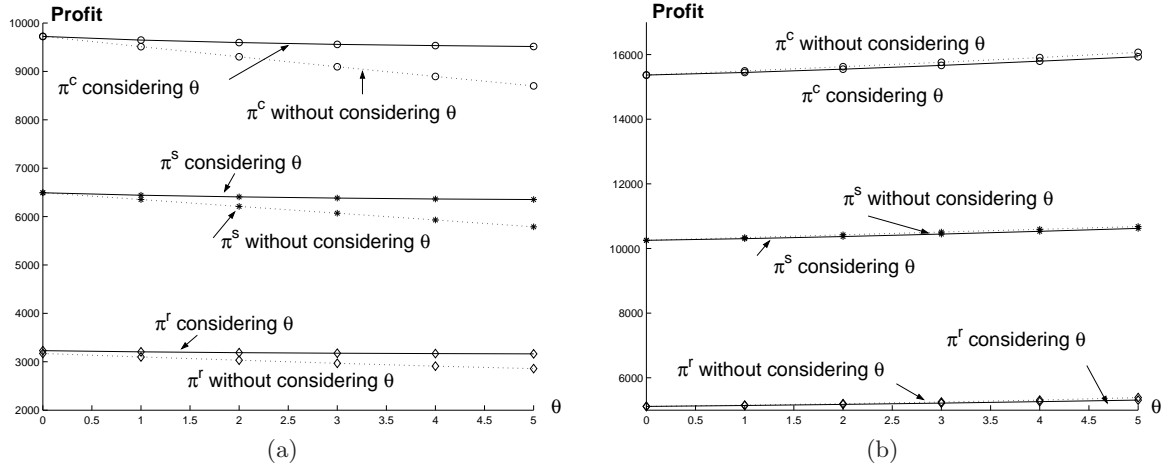


Figure 3: The profits with or without considering the cross-price effect

Table 2: Summary of the impact of cross-price effect on optimal decisions and profits

	The online store ($i = 1$)	The off-line store ($i = 2$)	The supplier
Retail price (p_i)	\uparrow	\downarrow	N/A
Demand and EOQ (D_i and q_i)	\uparrow	\downarrow	N/A
Wholesale price (w_i)	\uparrow	\downarrow	N/A
Total demand ($D_1 + D_2$) and Q	N/A	N/A	--
Profits	\downarrow or \uparrow		\downarrow or \uparrow

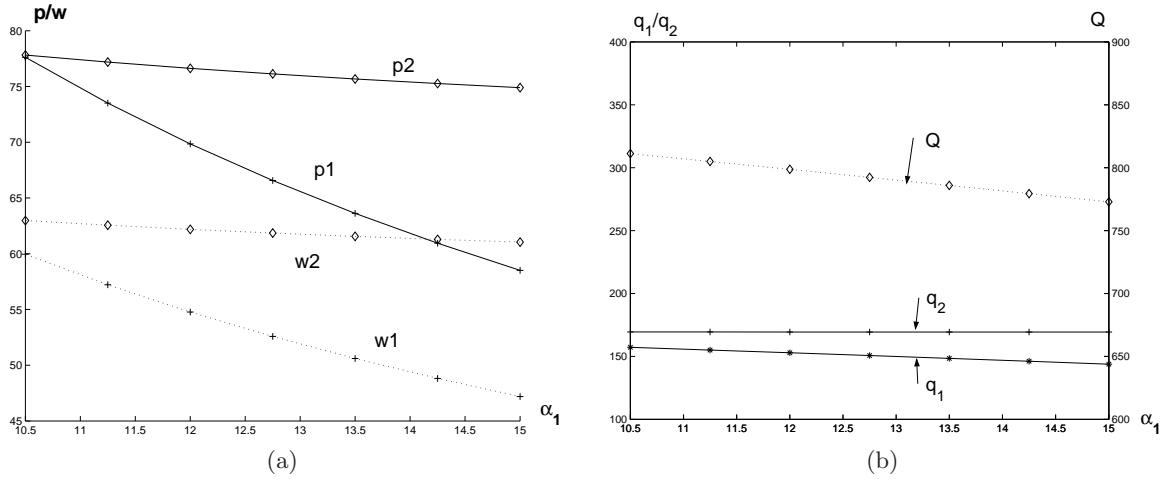


Figure 4: The impact of α_1 on optimal decisions

5.3 Impact of Self-price Sensitivity

Recall that the self-price sensitivity is represented by α_i in the demand function. For the dual-retail channel supply chain, α_1 and α_2 have a symmetric impact on optimal decisions. When α_i increases, so that the demand for store i becomes more sensitive to its own price, the retail price and the wholesale price in this store decrease, which is intuitive. The retail price and the wholesale price of the other store are less sensitive, although they do decrease as well because of the price convergence effect. These observations are shown in Figure 4a.

From Figure 4b, we see that the EOQ of store i decreases with α_i because D_i decreases with α_i but is insensitive to α_j . The total demand and the supplier's EOQ decrease with respect to both α_i and α_j .

Both the supplier's and retailer's profits decrease with α_i . See Figure 5. This result is intuitive because when one product becomes more price-sensitive, the firms that manufacture and sell this product can only achieve a lower profit. This observation holds in a supply chain environment.

6 Concluding Remarks

In this paper, we consider joint pricing and inventory/production decision problems for the members in a two-stage dual-channel retailer supply chain. The supplier in the chain purchases or manufactures its product under an EOQ model and sells the product through an exclusive

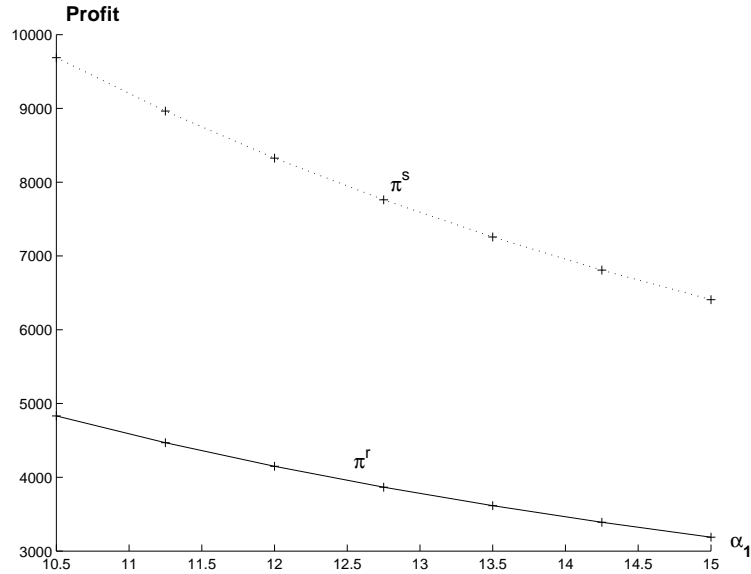


Figure 5: The impact of α_1 on optimal profits

dual-channel retailer. A typical dual-retailer channel in the e-commerce era includes an online store and an off-line store. Because of economies of scale, the Internet delivery distribution center is established specially to supply online customers. Thus, each retailer channel has its own warehouse. We model the inventory management in each of them using an EOQ model with a cross-pricing effect. The products sold in the online store and the off-line store might have different configurations and be seen as substitutes by consumers, which can be considered as a multiple product problem on the retailer's side. However, on the supplier side, the product is from one supplier or from the same product line; thus, we consider a joint setup cost in the supplier's problem.

Our main purpose in this paper is to investigate how the supplier and the retailer in such a supply chain system make their joint pricing and inventory/production decisions in equilibrium and to determine the impact of the cross-price effect on the decisions and on each member's and the supply chain's performance. We build price-sensitive EOQ joint decision supply chain models in order to understand the interactions between pricing and production/inventory decisions at both the supplier and the retailer levels. We formulate Stackelberg games that enable the supplier as the leader to take into account the reactions of the downstream retail stores when making decisions.

We show that a unique Nash equilibrium exists for the Stackelberg games when certain

conditions apply. One critical condition concerns the relationship between the demand price elasticity and the sales-to-inventory value of the product. The condition is shown to be virtually satisfied in most, if not all, realistic markets in which sales-to-inventory ratios are not excessively low and demand elasticities are not excessively large (in absolute value). Furthermore, numerical experiments show that a unique Nash equilibrium appears to exist even if some of or all the conditions do not apply.

The impact of the cross-price effect between products sold in the dual-retailer channel is one of the main issues in this paper. The cross-price effect in such a channel represents impacts of both store differentiation (consumer preference on store types) and product differentiation (different product configurations). We observe a price convergence effect when the cross-price effect becomes stronger. The observation implies that pricing for different product categories for the online store and the off-line store must be done strategically. For products such as books, where consumers may not have a strong preference for either online or off-line shopping, the prices converge; that is, prices are getting closer as the cross-price effect increases; while for products such as apparel, where consumers may strongly prefer to shop off-line because of the additional information from the shopping experience, the prices might be quite far apart. Furthermore, when the products sold in different channels have different configurations, for example, a hard-cover edition of a book is available only in the off-line store, this should be taken into consideration when making pricing decisions.

Demand for the store with the lower price (usually the online store) increases, but the demand for the off-line store decreases when there is a strong cross-price effect (a larger substitution factor). The inventory decisions in the retail stores follow the same trend that demand has. Demand for the supplier (the total demand) is relatively insensitive to the cross-price effect, and the same is true for the EOQ for the supplier. The difference between the production (purchasing) costs and the difference between self-price sensitivities play an important role when examining the impact of cross-price effect on the supplier's and the retailer's profits. If the difference between production costs is large and dominates, the chain can take advantage of the fact that customers treat the products in the two stores as more alike and more substitutable; thus, the chain can obtain more profit. If the difference between production costs is small and the difference between the self-price sensitivities dominates, the competition among products increases with substitution, which causes the supplier, the retailer, and the chain as well to lose

profit. These observations shed light on a supplier's product configuration for different retailer channels and the retailer's store management decisions. In their decision-making, it is important to examine the mixed controlling effect of product costs and price sensitivities.

The impact of self-price sensitivities on optimal decisions and profits is straightforward. When customers become more price-sensitive to the product in one channel, both the supplier's and retailer's profits will decrease.

In this paper, we have shown that unique optimal decisions exist in a two-stage dual-retailer channel supply chain without coordination efforts. We further offer numerical insights on the impact of the cross-price effect and other parameters on optimal decisions, each member's performance, and the chain-wide performance. Although some results in this paper are intuitive, now we have a competition model between channels that could be extended to allow us to explore other research directions such as different inventory management alternatives. In addition, channels with different contract types and coordination efforts offer an interesting area for further research, as do models with more products in all supply chain stages. However, more elaborate analytical methods are needed to solve these supply chain models with added complexity.

Appendix: Proofs

Proof of Theorem 1. We use D_i to denote $D_i = k_i - \alpha_i p_i + \theta_{ij}(p_j - p_i)$. The second derivatives of function (5) are given by

$$\begin{aligned}\frac{\partial \pi_{RMI}^{r^2}}{\partial p_1^2} &= -2(\alpha_1 + \theta_{12}) + \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2 \sqrt{2h_2 s_2 D_2}}{4D_2^2} \\ \frac{\partial \pi_{RMI}^{r^2}}{\partial p_2^2} &= -2(\alpha_2 + \theta_{21}) + \frac{\theta_{21}^2 \sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{(\alpha_2 + \theta_{21})^2 \sqrt{2h_2 s_2 D_2}}{4D_2^2} \\ \frac{\partial \pi_{RMI}^{r^2}}{\partial p_1 \partial p_2} &= \theta_{12} + \theta_{21} - \frac{\theta_{12}(\alpha_1 + \theta_{12}) \sqrt{2h_1 s_1 D_1}}{4D_1^2} - \frac{\theta_{21}(\alpha_2 + \theta_{21}) \sqrt{2h_2 s_2 D_2}}{4D_2^2}\end{aligned}$$

From assumption (7), we can verify $1/10(\alpha_1 + \theta_{12}) \geq (\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1} / 4D_1^2$ and $1/10\theta_{21} \geq \theta_{21}^2 \sqrt{2h_2 s_2 D_2} / 4D_2^2$. Therefore, we have

$$\begin{aligned}\alpha_1 + \theta_{12} + \theta_{21} &> \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2 \sqrt{2h_2 s_2 D_2}}{4D_2^2} \\ \implies 2(\alpha_1 + \theta_{12}) &> \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2 \sqrt{2h_2 s_2 D_2}}{4D_2^2} \\ \implies \frac{\partial \pi_{RMI}^{r^2}}{\partial p_1^2} &< 0\end{aligned}$$

By applying the same reasoning, we can show $\partial \pi_{RMI}^{r^2} / \partial p_2^2 < 0$ as well.

Next, we verify that the determinant of the Hessian matrix is positive, or equivalently

$$\begin{aligned}\frac{\partial \pi_{RMI}^{r^2}}{\partial p_1^2} \frac{\partial \pi_{RMI}^{r^2}}{\partial p_2^2} &> \left(\frac{\partial \pi_{RMI}^{r^2}}{\partial p_1 \partial p_2} \right)^2 \\ \iff \left[(\alpha_1 + \theta_{12}) \left(1 - \frac{(\alpha_1 + \theta_{12}) V_1}{4D_1^2} \right) + \theta_{21} \left(1 - \frac{\theta_{21} V_2}{4D_2^2} \right) \right] &\left[\theta_{12} \left(1 - \frac{\theta_{12} V_1}{4D_1^2} \right) + (\alpha_2 + \theta_{21}) \left(1 - \frac{(\alpha_2 + \theta_{21}) V_2}{4D_2^2} \right) \right] \\ &> \left[\theta_{12} \left(1 - \frac{(\alpha_1 + \theta_{12}) V_1}{4D_1^2} \right) + \theta_{21} \left(1 - \frac{(\alpha_2 + \theta_{21}) V_2}{4D_2^2} \right) \right] &\left[\theta_{12} \left(1 - \frac{(\alpha_1 + \theta_{12}) V_1}{4D_1^2} \right) + \theta_{21} \left(1 - \frac{(\alpha_2 + \theta_{21}) V_2}{4D_2^2} \right) \right]\end{aligned}$$

Since the objective function is shown to be strictly concave in \mathbf{p} under conditions (2) and (6). \square

Proof of Theorem 2. The second derivatives of the supplier's objective function are

$$\begin{aligned} \frac{\partial \pi_{RMI}^{s2}}{\partial p_1^2} &= -4(\alpha_1 + \theta_{12}) + \frac{1}{2} \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} + \frac{1}{2} \frac{\theta_{21}^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} \\ &\quad + \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{\partial \pi_{RMI}^{s2}}{\partial p_2^2} &= -4(\alpha_2 + \theta_{21}) + \frac{1}{2} \frac{(\alpha_2 + \theta_{21})^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} + \frac{1}{2} \frac{\theta_{12}^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} \\ &\quad + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial \pi_{RMI}^{s2}}{\partial p_1 \partial p_2} &= 2(\theta_{12} + \theta_{21}) - \frac{1}{2} \frac{(\alpha_2 + \theta_{21}) \theta_{21} \sqrt{2s_2 h_2 D_2}}{4D_2^2} - \frac{1}{2} \frac{(\alpha_1 + \theta_{12}) \theta_{12} \sqrt{2s_1 h_1 D_1}}{4D_1^2} \\ &\quad + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}] [-(\alpha_1 + \theta_{12}) + \theta_{21}]}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \end{aligned} \quad (15)$$

Let us first prove $\frac{\partial \pi_{RMI}^{s2}}{\partial p_1^2} < 0$. From the retailer's problem, we know

$$-2(\alpha_1 + \theta_{12}) + (\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1} / 0.4D_1^2 + \theta_{21}^2 \sqrt{2h_2 s_2 D_2} / 0.4D_2^2 < 0,$$

that is to say,

$$-\frac{1}{10}(\alpha_1 + \theta_{12}) + \frac{1}{2}(\alpha_1 + \theta_{12})^2 \sqrt{2h_1 s_1 D_1} / 4D_1^2 + \frac{1}{2}\theta_{21}^2 \sqrt{2h_2 s_2 D_2} / 4D_2^2 < 0$$

.

Thus, as long as we can show $-\frac{39}{10}(\alpha_1 + \theta_{12}) + \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \leq 0$, we will have $\frac{\partial \pi_{RMI}^{s2}}{\partial p_1^2} < 0$.

$$\begin{aligned} -\frac{39}{10}(\alpha_1 + \theta_{12}) + \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} &\leq 0 \\ \iff \frac{39}{10}(\alpha_1 + \theta_{12}) [2SH(D_1 + D_2)]^{\frac{3}{2}} &\geq S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2 \\ \iff (D_1 + D_2)^{\frac{3}{2}} &\geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_1 + \theta_{12})} [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2. \end{aligned}$$

$$\iff [k_1 + k_2 - (\alpha_1 + \theta_{12} - \theta_{21})p_1 - (\alpha_2 + \theta_{21} - \theta_{12})p_2]^{\frac{3}{2}} \geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_1 + \theta_{12})} [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2$$

The assumption, $e_i \leq 0.4D_i p_i / \sqrt{2S_i H_i D_i}$ where $e_i = (\alpha_i + \theta_{ij})p_i / D_i$, connects parameters and prices together. Since we are considering shared setup cost in the supplier's problem, we can

apply the assumption in the form $p_1/D_1 \leq 0.4D_1p_1/\sqrt{HSD_1}$ assuming that each product takes half of the setup cost.

$$\begin{aligned} \frac{(\alpha_1 + \theta_{12})p_1}{D_1} &\leq \frac{0.4D_1p_1}{\sqrt{HSD_1}} \\ \iff (\alpha_1 + \theta_{12})\sqrt{HSD_1} &\leq 0.4D_1^2 \\ \iff (\alpha_1 + \theta_{12})^2HSD_1 &\leq 0.4^2D_1^4 \\ \iff (\alpha_2 + \theta_{21})^2HSD_2 &\leq 0.4^2D_2^4 \end{aligned}$$

If we denote $\theta_{\min} = \min(\theta_{21}, \theta_{12})$ and $\alpha_{\min} = \min(\alpha_2, \alpha_1)$, we have

$$\begin{aligned} HS(D_1 + D_2)(\alpha_{\min} + \theta_{\min})^2 &\leq 0.4^2(D_1^4 + D_2^4) \\ \implies HS(D_1 + D_2)(\alpha_{\min} + \theta_{\min})^2 &\leq 0.4^2(D_1 + D_2)^4 \\ \implies \sqrt{HS(D_1 + D_2)}(\alpha_{\min} + \theta_{\min}) &\leq 0.4(D_1 + D_2)^2 \\ \implies \frac{\sqrt{HS}}{\frac{0.4}{(\alpha_{\min} + \theta_{\min})}} &\leq (D_1 + D_2)^{\frac{3}{2}} \end{aligned}$$

In order to have $\frac{\partial \pi_{RMI}^{s^2}}{\partial p_2^2} < 0$, we need $(D_1 + D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_2 + \theta_{21})} [-(\alpha_2 + \theta_{21}) + \theta_{12}]^2$. So if $\frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2} \leq \frac{39\sqrt{2}}{2} \approx 27$, $\frac{\partial \pi^{r^2}}{\partial p_1^2} \leq 0$ and $\frac{\partial \pi^{r^2}}{\partial p_2^2} \leq 0$. So it means for some industries and some parameters, the inequality may not hold.

In order to prove $\frac{\partial \pi_{RMI}^{s^2}}{\partial p_1^2} \frac{\partial \pi_{RMI}^{s^2}}{\partial p_2^2} > (\frac{\partial \pi_{RMI}^{s^2}}{\partial p_1 \partial p_2})^2$, we have to show

$$\begin{aligned} &[-4(\alpha_1 + \theta_{12}) + \frac{1}{2} \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} + \frac{1}{2} \frac{\theta_{21}^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} + \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}]_* \\ &[-4(\alpha_2 + \theta_{21}) + \frac{1}{2} \frac{(\alpha_2 + \theta_{21})^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} + \frac{1}{2} \frac{\theta_{12}^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}] \\ &> \\ &[2(\theta_{12} + \theta_{21}) - \frac{1}{2} \frac{(\alpha_2 + \theta_{21})\theta_{21} \sqrt{2s_2 h_2 D_2}}{4D_2^2} - \frac{1}{2} \frac{(\alpha_1 + \theta_{12})\theta_{12} \sqrt{2s_1 h_1 D_1}}{4D_1^2} \\ &\quad + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}] [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}] \end{aligned}$$

This can be written as $(A + B)(C + D) > (E + F)^2$, where

$$\begin{aligned}
A &= -\frac{1}{10}(\alpha_1 + \theta_{12}) + \frac{1}{2} \frac{(\alpha_1 + \theta_{12})^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} + \frac{1}{2} \frac{\theta_{21}^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} \\
B &= -\frac{39}{10}(\alpha_1 + \theta_{12}) + \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \\
C &= -\frac{1}{10}(\alpha_2 + \theta_{21}) + \frac{1}{2} \frac{(\alpha_2 + \theta_{21})^2 \sqrt{2s_2 h_2 D_2}}{4D_2^2} + \frac{1}{2} \frac{\theta_{12}^2 \sqrt{2s_1 h_1 D_1}}{4D_1^2} \\
D &= -\frac{39}{10}(\alpha_2 + \theta_{21}) + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} \\
E &= \frac{1}{20}(\theta_{12} + \theta_{21}) - \frac{1}{2} \frac{(\alpha_2 + \theta_{21})\theta_{21} \sqrt{2s_2 h_2 D_2}}{4D_2^2} - \frac{1}{2} \frac{(\alpha_1 + \theta_{12})\theta_{12} \sqrt{2s_1 h_1 D_1}}{4D_1^2} \\
F &= \frac{19}{20}(\theta_{12} + \theta_{21}) + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}] [-(\alpha_1 + \theta_{12}) + \theta_{21}]}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}.
\end{aligned}$$

In the retailer's problem, it is shown that $-A > E$ and $-C > E$ hold for all industries. Next, we show the condition under which $-B > F$ and $-D > F$ hold.

$$\begin{aligned}
(D_1 + D_2)^{\frac{3}{2}} &\geq \frac{\sqrt{SH}(\alpha_1 + \theta_{12} - \theta_{21})(\alpha_1 + \alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_1 - \theta_{21})} \\
\implies \frac{39\sqrt{2}}{5}(\alpha_1 - \theta_{21})(D_1 + D_2)^{\frac{3}{2}} &\geq \sqrt{SH}(\alpha_1 + \theta_{12} - \theta_{21})(\alpha_1 + \alpha_2) \\
\implies \frac{39\sqrt{2}}{5}(\alpha_1 + \theta_{12})(D_1 + D_2)^{\frac{3}{2}} - \sqrt{SH}[\alpha_1 + \theta_{12} - \theta_{21}]^2 & \\
\geq \frac{39\sqrt{2}}{5}(\theta_{12} + \theta_{21})(D_1 + D_2)^{\frac{3}{2}} + \sqrt{SH}(\alpha_2 + \theta_{21} - \theta_{12})(\alpha_1 + \theta_{12} - \theta_{21}) & \\
\implies \frac{39\sqrt{2}}{5}(\alpha_1 + \theta_{12})(D_1 + D_2)^{\frac{3}{2}} - \sqrt{SH}[\alpha_1 + \theta_{12} - \theta_{21}]^2 & \\
> \frac{19\sqrt{2}}{10}(\theta_{12} + \theta_{21})(D_1 + D_2)^{\frac{3}{2}} + \sqrt{SH}(\alpha_2 + \theta_{21} - \theta_{12})(\alpha_1 + \theta_{12} - \theta_{21}) & \\
\implies \frac{39}{10}(\alpha_1 + \theta_{12}) - \frac{S^2 H^2 [-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} & \\
> \frac{19}{20}(\theta_{12} + \theta_{21}) + \frac{S^2 H^2 [-(\alpha_2 + \theta_{21}) + \theta_{12}] [-(\alpha_1 + \theta_{12}) + \theta_{21}]}{[2SH(D_1 + D_2)]^{\frac{3}{2}}} & \\
\implies -B > F &
\end{aligned}$$

Thus, we need $(D_1 + D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_1 + \theta_{12} - \theta_{21})(\alpha_1 + \alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_1 - \theta_{21})}$ for $-B > F$ and

$(D_1 + D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_2 + \theta_{21} - \theta_{12})(\alpha_1 + \alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_2 - \theta_{12})}$ for $-D > F$. We have $\frac{\sqrt{HS}}{(\alpha_{\min} + \theta_{\min})^{0.4}} \leq (D_1 + D_2)^{\frac{3}{2}}$ from the assumption. Therefore, as long as $\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})} \leq 27$, $\frac{\partial \pi_{RMI}^{s^2}}{\partial p_1^2} \frac{\partial \pi_{RMI}^{s^2}}{\partial p_2^2} > (\frac{\partial \pi_{RMI}^{s^2}}{\partial p_1 \partial p_2})^2$ because $(A + B)(C + D) > (E + F)^2$.

In summary, the supplier's problem can be proved to be strictly concave when

$$\max\left(\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})}, \frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2}\right) \leq \frac{39\sqrt{2}}{2} \approx 27$$

□

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