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The Periodic Vehicle Routing Problem with the Joint Replenishment Planning under Supply Chain Environment

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Abstract

This study tries to integrate the joint replenishment problem and the periodic vehicle routing problem under supply chain environment. A mathematical model is developed to minimize the total operating costs which includes inventory cost, ordering cost, and transportation cost. A heuristic algorithm based on the Tabu search is then constructed to find the best combination of vehicle service types. The model and solution algorithm are verified by numerical examples. Finally, a sensitively analysis is also conducted by testing different vehicle loading capacities. The policies of joint replenishment are compared and discussed on the basis of cost viewpoint.

Keywords: Joint Replenishment Problem, Periodic VRP, Tabu Search, Combination of Vehicle Service.

I. Introduction

In this competition environment, organizations of manufacturing and distribution are forced to optimize their operating processes and reduce overall cost in each function of organization. The modern transportation system can distribute various products all over the world in an efficient way to meet customers' different requirements. In addition, the supplier and retailers are forced to work together to reduce the overall costs which include the inventory cost, the ordering cost, and the transportation cost. The vendor managed inventory (VMI) systems become one of the effective approaches to integrate all requirements. In a VMI system, the vendor is responsible for all the product replenishments. The vendor can also decide when and how many products should be distributed (Dong and Xu, 2002). It is expected that the VMI system can reduce not only the retailers' inventory costs but also the delivery cost of supplier. Matthew *et. al.* (1999) further indicate that implementation of VMI system can reduce the inventory cost of supply chain and increases the service level. This research will assume a VMI environment operating under joint replenishment policy by single vendor.

Another important logistic issue in a supply chain is the product delivery operations. A typical distribution center handles thousands of products in daily operations. Goods are quickly delivered to their destinations through different vehicle routings. Different size of vehicle fleet, different vehicle routing plans, and different service zones will impact the cost of distribution center. For most of the distribution centers, the cost of vehicle routings is the major part of the transportation cost. It is also believed that a joint replenishment is a critical consideration factor for a successful distribution center.

The delivery operations under joint replenishment policy can be combined with the periodic vehicle routings. The problem (PVRP) is more complicated than the traditional vehicle routing problem (VRP). PVRP can be defined as: Find the minimal vehicle routing cost in a cycle period, given number of vehicle, loading capacity of vehicle, delivery quantities, and service dates for each customer (Christofides and Beasley, 1984). To solve the PVRP, it is necessary first to find how many service days in a repeat cycle. For each service day in a cycle, it is a typical vehicle routing problem. Obviously, the computation time of a PVRP is proportional to days in a cycle period.

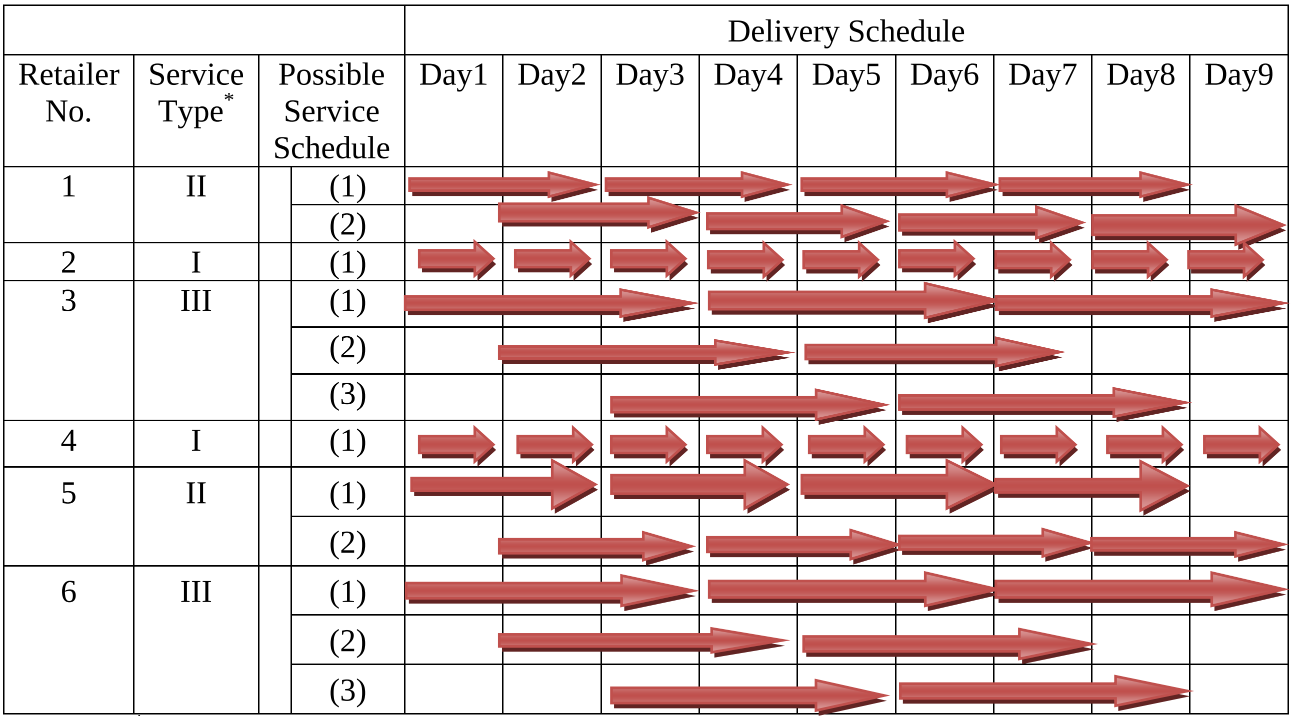
This research will focus on the periodic vehicle routing problem based on the VMI environment. The delivery quantities of each customer should be generated first by using the formula of joint replenishment in Copra and Meindl (2007). Based on the delivery quantities of each customer, this study starts to find all possible sets of delivery schedule in a cycle. Each set of delivery schedule should be solved as one periodic vehicle routing problem. The objective of this research is to find the optimal set of delivery schedule and the associated information, such as daily routing arrangements and the transportation cost. Sections of this paper are organized as follows: The mathematical model and solution algorithm are described in section II. Section III describes an illustration problem and it's solution. The costs comparison and a sensitivity analysis are also conducted in this section. The concluding remarks are summarized in the final section IV.

II. The Model and Solution Algorithm

2.1 Problem Statement

The periodic vehicle routing problem in a VMI environment is the major focus in this research. The joint replenishment concept is also applied in the periodic vehicle routing problem. Since the ordering cycles for retailers are different, the delivery schedule becomes more complicated. For better understanding the delivery schedule in the periodic vehicle routing problem, a simple illustration example is shown in Figure 1. In Figure 1, there are 6 retailers having three different service types, *i.e.* I, II, and III, in 9 days time horizon. Each service type still has some possible service schedules. By considering all possible service schedules for each retailer, we can generate all possible delivery schedules in this periodic vehicle routing problem. In this case, the total possible set of delivery schedule is 36 ($1^A*2^B*3^C = 1^2*2^2*3^2 = 36$, where A, B, and C represent the number of retailer for service type I, II, and III, respectively). In any set of delivery schedule, the repeat cycle is 6 days.

To solve this periodic vehicle routing problem, we need to find the minimal transportation (vehicle routing cost) from one of these 36 sets of delivery schedule. In addition, each set of delivery schedule is consist of 6 (6 days in one repeat cycle) independent vehicle routing problems. It is obviously that the theoretical best solution is not so easy to find, when the problem scale is large. Therefore, this research will propose a solution algorithm using the Tabu logic.



Remarks: *Service type I - Everyday, Service type II - Every other day,
Service type III - Every 3 days. : One delivery day

Figure 1 Possible Delivery Schedule

2.2 Model Assumptions and Definition of Variables

This research is based on the following assumptions:

(a) Only one distribution center (supplier or vendor) and several given retailers are considered in a VMI environment. Locations of the distribution center and retailers are given and fixed.

(b) Retailers accept the joint replenishment policy. The requirement (delivery quantity) of each retailer is given and fixed. Ordering cycle of a retailer can be different from other retailers. In the distribution center, the shortage situation is not considered.

(c) The periodic vehicle routing problem is considered in this model including the vehicle routings and distances. Goods are delivered from the distribution center to all retailers through different vehicle routings.

(d) All vehicles have the same loading capacity and overloading is not acceptable. In each vehicle routing, the location of distribution center is the starting point and also the ending point.

(e) The transportation cost of vehicle is proportional to distance. The unit distance cost is fixed and given.

Before the model development, several variables should be defined as follows.

Variables for inventory system:

A: a set of all retailers i. $i = 1, 2, \ldots.., a$ and a is the total number of retailer
D_i: annual requirement for retailer i. D_i is fixed and given
C: unit purchasing cost, constant
H: unit holding cost per year, a constant
E: ordering cost per joint order for vendor, a constant
e_i: ordering cost per order for retailer, a constant
N: the maximal order number for all retailers in a year, order number of the retailer with highest delivery frequency
m_i: the multiplier of retailer i comparing with the retailer with highest delivery frequency
f_i: delivery frequency for retailer i, $f_i = N / m_i$
q_i: delivery quantity for retailer i, $q_i = (D_i / N)\, m_i$
y_{ik}: If the retailer i is served by the kth service type, then $y_{ik} = 1$. Otherwise, $y_{ik} = 0$.

Variables in vehicle routing:

D: a set of all delivery days, $D = \{1, 2, \ldots, d\}$, d is total number of delivery in a year, $d = N$
M: a set of delivery number in one repeat cycle, $M = \{1, 2, \ldots, m\}$, m is the maximal delivery number in one repeat cycle
V: a set of activated vehicle r, $r \in V$, $V = \{1, 2, \ldots, v\}$, v is the maximal number of vehicle
P: a set of all service types, $P = \{1, 2, \ldots, k\}$, k is the maximal number of service type
p_i: a set of service day which can satisfied retailer i using the delivery frequency f_i, $p_i \subseteq P$, $i \in A$
S: a subset of A, $S \subseteq A$, $S \neq \phi$, $S \neq A$
a_{kt}: If the kth service type is in the tth delivery within m, then $a_{kt} = 1$. Otherwise, $a_{kt} = 0$.
U: Loading capacity of vehicle, a constant
G: delivery cost per unit distance, a constant
d_{ij}: delivery distance between node i and node j, Node(i, j)
x_{ijtr}: decision variable, If the vehicle r go through Node(i, j) in the tth delivery in m, then $x_{ijtr} = 1$. Otherwise, $x_{ijtr} = 0$.

2.3 Mathematical Model

In this subsection, an integrated mathematical model combining a joint replenishment inventory model and a periodic vehicle routing model is proposed. The joint replenishment inventory model considers the carrying costs and the ordering costs. The periodic vehicle routing model considers the delivery cost which is calculated by travel distance multiplying delivery cost per unit distance. The objective function and associated constraints are listed as follows.

$$Minimize \quad TC = Z_1 + Z_2 + Z_3$$
$$= (\sum_{i=1}^{a} \frac{H \cdot C \cdot D_i}{2 \cdot f_i}) + (N \cdot E + \sum_{i=1}^{a} (f_i) \cdot e_i) + (G \cdot \sum_{t=1}^{m} \left[\sum_{i=0}^{a} \sum_{j=0}^{a} \sum_{r=1}^{v} d_{ij} \cdot x_{ijtr} \right] \cdot \left[\frac{d}{m} \right]) \tag{1}$$

Subject to:

$$N \geq \left[\sum_{i=1}^{a} D_i \right] \Big/ v \cdot U \qquad \forall i \in A \tag{2}$$

$$\sum_{j=1}^{a} \sum_{r=1}^{v} x_{0jtr} \leq v \qquad \forall t \in M \tag{3}$$

$$\sum_{i=1}^{a} \sum_{j=0}^{a} \frac{D_i \cdot m_i}{N} \cdot x_{ijtr} \leq U \qquad \forall t \in M \;, \quad \forall r \in V \tag{4}$$

$$\sum_{k \in p_i} y_{ik} = 1 \qquad \forall i \in A \tag{5}$$

$$\sum_{j=0}^{a} \sum_{r=1}^{v} x_{ijtr} - \sum_{k \in p_i} a_{kt} \cdot y_{ik} = 0 \qquad \forall i \in A \;, \quad \forall t \in M \tag{6}$$

$$\sum_{i=0}^{a} x_{ihtr} - \sum_{j=0}^{a} x_{hjtr} = 0 \qquad \forall h \in A, \forall t \in M, \forall r \in V \tag{7}$$

$$\sum_{i \subset S} \sum_{j \subset S} x_{ijtr} \leq |S| - 1 \qquad \forall t \in M, \forall r \in V, \forall S \subseteq A \tag{8}$$

$$x_{ijtr} \in (0,1) \;, \quad y_{ik} \in (0,1) \; \forall i, j \in A, \quad \forall k \in P, \quad \forall t \in M, \quad \forall r \in V \tag{9}$$

$$m_i \geq 1 \;, \quad m_i \in \text{INT} \;, \quad N \in \text{INT} \quad \forall i \in A \tag{10}$$

The objective function indicated in equation (1) includes three terms: the annual carrying costs (Z_1), the annual ordering costs (Z_2), and the delivery costs in the periodic vehicle routing system (Z_3). For cost comparison purpose, the first term and second term are included in the objective function, which come from Copra and Meindl (2007). Equation (2) ensures all delivery requirements of retailers can be satisfied. In equation (3), the number of vehicle required in this system is no more than the given fleet size, *i.e.* v. Restriction of the vehicle loading capacity is presented in equation (4). Equation (5) makes sure that each retailer can be served by one and only one service type. Equation (6) ensures each retailer should be served by the kth service type in the tth delivery. Equation (7) ensures that the same vehicle drive to and out of a retailer. Sub-tour is restricted in equation (8). The integer constraints and the feasible range constraints are described in equation (9)-(10).

2.4 Solution Algorithm

A solution algorithm including two phases is developed by using the Tabu logic. Phase one constructs the initial routings using sweeping method. Phase two performs the routing improvement. Three traditional neighborhood improvement approaches are used in phase two: the 1-1 internal exchanges, the 1-1 external exchanges, and the external 1-0 insertion exchanges. The flow chart of this solution algorithm is presented in Figure 2.

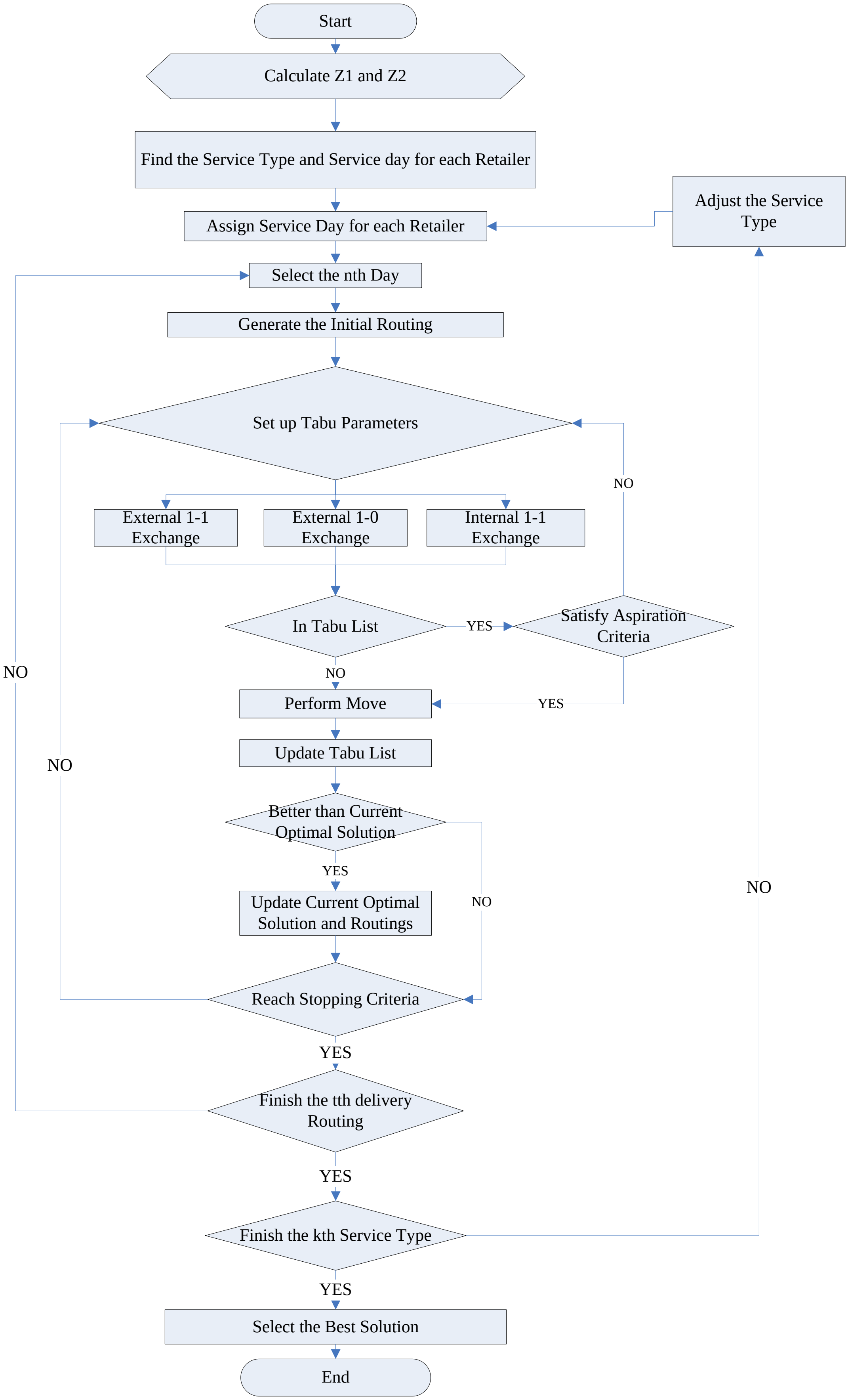


Figure 2   Flow Chart of the Solution Algorithm

III. Illustration Problem and Analysis

This section presents an illustration problem and the solution using the solution algorithm proposed in section II. In this research, the objective function focuses on the overall costs including the carrying cost, the ordering cost, and the delivery cost. Therefore, the overall cost under joint replenishment policy is also compared with the non-joint replenishment policy (or tradition replenishment policy). Finally, a sensitivity analysis is also presented by considering different loading capacity of vehicle.

3.1 Basic Problem Data and Parameters of Tabu Search

This problem has one distribution center as the single vender which serves 15 independent retailers. The detailed data of 15 retailers is presented in Table 1. Since there are four different service types in the system, the total possible set of delivery schedule is 768 (i.e. $1^A*2^B*3^C*4^D = 1^7*2^6*3^1*4^1= 768$). In addition, the non-repeat set of delivery schedule can be reduced to 64 by examining each set of delivery schedule. For each set of delivery schedule, the repeat cycle is 12 days.

Table 1 Detailed Data for 15 Retailers

Retailer No.	Coordinates		Annual Demand	Order Frequency (Original)	Service Type	Order Frequency (In One Year)	Order Quantity (Each Time)
	X	Y					
0 *	30	40					
1	37	52	5260	21	2	29	182
2	49	49	31105	51	1	57	546
3	52	64	2802	16	2	29	97
4	20	26	38011	57	1	57	667
5	40	30	36200	55	1	57	636
6	21	47	3055	16	2	29	106
7	17	63	20240	42	1	57	356
8	31	62	565	7	4	15	38
9	52	33	4217	19	2	29	146
10	51	21	19330	41	1	57	340
11	42	41	8414	27	1	57	148
12	31	32	2886	16	2	29	100
13	5	25	3822	18	2	29	132
14	12	42	17749	39	1	57	312
15	36	16	1095	10	3	19	58

Remarks: * : 0 represents the distribution center.

The cost data for inventory is summarized as follows: Ordering cost for vendor is $5,000. Ordering cost for retailer is $1,000. Product price is $100 per unit. Annual carrying cost is $10 per unit. Delivery cost is $100 per unit distance. Loading capacity is 1,000 unit per vehicle.

The parameters for Tabu search is obtained by using the Taguchi experiment design. The critical parameters settings are described as follows: Length of Tabu list is 5. The maximal number of iteration (stopping criteria) is 1000. The execution probability for three proposed neighborhood exchanges are: (1) 0.4 for the external 1-0 insertion exchanges, (2) 0.4 for the 1-1 external exchanges, and (3) 0.2 for 1-1 internal exchanges.

3.2 Problem Solution and Cost Comparison

Using the proposal solution algorithm, the total delivery distance for each non-repeat set of delivery schedule is listed in Table 2. By observing data in Table 2, the optimal delivery schedule can be found in #105 which has the minimal annual delivery distance: 14,191.9574. Based on the objective function presented in equation (1), the optimal value of objective function is 4,239,309 which is calculated in equation (11).

$$
\begin{aligned}
TC &= \sum_{i=1}^{a} \frac{H \cdot C \cdot D_i}{2 \cdot f_i} + N \cdot E + \sum_{i=1}^{a} (f_i) \cdot e_i + G \cdot \sum_{t=1}^{m} \left[\sum_{i=0}^{a} \sum_{j=0}^{a} \sum_{r=1}^{v} d_{ij} \cdot x_{ijtr} \right] \cdot \left[\frac{d}{m} \right] \\
&= \sum_{i=1}^{15} \frac{10 \cdot 100 \cdot D_i}{2 \cdot f_i} + N \cdot 5000 + \sum_{i=1}^{15} (f_i) \cdot e_i + 100 \cdot \sum_{t=1}^{12} \left[\sum_{i=0}^{15} \sum_{j=0}^{15} \sum_{r=1}^{5} d_{ij} \cdot x_{ijtr} \right] \cdot \left[\frac{57}{12} \right] \\
&= [1{,}928{,}113.43] + [285{,}000 + 607{,}000] + [100 * 2{,}987.7805 * 4.75] \\
&= [1{,}928{,}113] + [892{,}000] + [1{,}419{,}196] = 4{,}239{,}309 \qquad (11)
\end{aligned}
$$

Table 2 Delivery Distances for 64 Sets of Delivery Schedule

Set ID *	Distance in One Cycle (A)	Annual Distance (B) **	Set ID *	Distance in One Cycle (A)	Annual Distance (B) **	Set ID *	Distance in One Cycle (A)	Annual Distance (B) **
1	3096.0992	14706.4712	40	3069.7159	14581.1505	79	3027.5478	14380.8521
2	3081.4152	14636.7222	41	3006.9637	14283.0776	80	3045.4136	14465.7146
3	3045.2744	14465.0534	42	3018.3504	14337.1644	97	3005.383	14275.5693
4	3009.3109	14294.2268	43	3040.2426	14441.1524	98	3021.2329	14350.8563
5	3058.1048	14525.9978	44	3040.7475	14443.5506	99	3069.5733	14580.4732
6	3050.7143	14490.8929	45	3012.6606	14310.1379	100	3051.0966	14492.7089
7	3028.9145	14387.3439	46	3030.2523	14393.6984	101	3071.4841	14589.5495
8	3001.2414	14255.8967	47	3049.9539	14487.281	102	3064.7818	14557.7134
9	3083.7682	14647.899	48	3102.3774	14736.2927	103	3085.2986	14655.1684
10	3083.0379	14644.4302	65	3085.1917	14654.6606	104	3074.7182	14604.9115
11	3043.9119	14458.5815	66	3072.8481	14596.0285	**105**	**2987.7805**	**14191.9574**
12	3001.4697	14256.9811	67	3019.2436	14341.4071	106	3012.8851	14311.2042
13	3081.5444	14637.3359	68	3048.3048	14479.4478	107	3051.2676	14493.5211
14	3118.2784	14811.8224	69	3084.4225	14651.0069	108	3060.5361	14537.5465
15	3032.7339	14405.486	70	3025.6623	14371.8959	109	3008.7764	14291.6879
16	3008.1141	14288.542	71	3014.0468	14316.7223	110	3043.4221	14456.255
33	3007.883	14287.4443	72	3080.6415	14633.0471	111	3066.1592	14564.2562
34	2999.1455	14245.9411	73	3205.5897	15226.5511	112	3070.9665	14587.0909
35	3067.167	14569.0433	74	3035.9146	14420.5944			
36	3002.7133	14262.8882	75	3042.7855	14453.2311			
37	3014.9005	14320.7774	76	3000.7817	14253.7131			
38	3052.3301	14498.568	77	3064.6279	14556.9825			
39	3076.1883	14611.8944	78	3076.2346	14612.1144			

Remarks: * : Use the original set ID number (*i.e.* ID number from 1 to 786)
** : (B)= (A)*57/12, where 12 is the length of a repeat cycle, 57 is the annual delivery frequency.

This research focuses on the joint replenishment policy and goods are delivered in the periodic vehicle routings. It is interesting to compare the system's overall cost with cost of non-joint replenishment policy (*i.e.* the tradition replenishment policy or the independent replenishment policy). The overall cost, as defined in equation (1), includes the carrying cost, the ordering cost, and the delivery cost. Table 3 indicates the difference between two policies using the same problem data. In Table 3, only the annual carrying cost of the joint replenishment policy is higher than the cost of non-joint replenishment policy. For the other two costs, *i.e.* the annual ordering cost and annual delivery cost, the joint replenishment policy is significantly better than the non-joint replenishment policy (i.e. -56.95% and -28.35%, respectively). From the overall cost view point, the joint replenishment policy still is a better choice.

Table 3 Cost Comparison on Joint and Non-Joint Replenishment Policy

	Replenishment Policy		Comparison	
	(A) Non-Joint	(B) Joint	Deviation **	Percentage ***
(1)Annual Carrying Cost	1,391,079	1,928,113	537,034	27.85%
(2) Annual Ordering Cost	1,400,000	892,000	-508,000	-56.95%
(3)Annual Delivery Distance	18,215	14,191	-4,023	-28.35%
(4)Annual Delivery Cost	1,821,540	1,419,195	-402,344	-28.35%
Overall Cost *	4,612,619	4,239,309	-373,310	-8.81%

Remarks: *: Overall Cost = (1) + (2) + (4), **: (B)-(A), ***: [(B)-(A)]/(B)×100%

3.3 Sensitivity Analysis on Vehicle Loading Capacity

This subsection focuses on comparing different loading capacities of vehicle. It is believed that higher loading capacity can reduce routings (*i.e.* number of vehicle required), however, the larger vehicle requires more purchasing budget and higher fuel consumption rate. In Table 4, the annual delivery distance, the number of vehicle required, and average utilization of all vehicle are compared on the basis of three different loading capacities (*i.e.* 700, 1000, and 1300). Based on the data provided in Table 4, a good decision on vehicle selection can be made if the real world cost data can be collected, such as the fixed cost of each vehicle, the fuel consumption rate of each vehicle, and life time of each vehicle.

Table 4 Delivery Distance and Vehicle Requirement for Different Loading Capacities

	Loading Capacity of Vehicle		
	700	1000	1300
Distance in One Cycle	3413.3	2987.7	2630.04
Annual Distance	16213.4	14191.9	12492.7
Vehicle Required	7	5	4
Average Loading (%)	78.85%	77.28%	74.3%

IV. Conclusion

This research proposes a mathematical model for the periodic vehicle routing problem under VMI environment and joint replenishment policy. A solution algorithm based on Tabu logic is presented and verified by an illustration problem. The solution results indicate that the proposed algorithm is effective and efficient in the solution process. From the solution of the example problem, it is also shown that the joint replenishment policy is better than the traditional non-joint replenishment policy. It is also believed that the model, algorithm, and application suggested in this paper can help the management to make a better decision.

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