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A mathematical model for dynamic cellular manufacturing systems with production planning and labor assignment

Seyed Vahid Daei Niaki[a,*], Esmaeil Mehdizadeh[a], Reza Tavakkoli-Moghaddam[b]

[a] Department of Industrial Engineering, Islamic Azad University, Qazvin Branch, Qazvin, Iran

[b] Department of Industrial Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran

1- vahidsh789@yahoo.com 2- emehdi@qiau.ac.ir 3- tavakoli@ut.ac.ir

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Abstract

Cellular Manufacturing (CM) includes some machine cells in which each cell is responsible for processing a family of similar parts. Reduction in product life cycle and variation in product mix and demands creates dynamic condition in the manufacturing systems. Because of the effect of production planning purposes in reconfiguration and cell formation in dynamic conditions, integrating these concepts is an important issue. On the other hand, considering worker requirements are such critical parameters in implementation of cell formation phases that increase the designing efficiency of cellular manufacturing systems and make proposed model more applicable. This paper presents a comprehensive multi-objective mixed integer mathematical programming model which considers cell formation problem and production planning seamlessly. In addition this model follows determination of production optimum policy (like determining quantity of production, inventory and subcontracting parts) and workers optimum assignment to manufacturing cells. The advantages of the proposed model are as follows: considering dynamic system reconfiguration, multi period production planning, operation sequence, alternative process plans for part types, machine and worker flexibility, duplicate machines, machine capacity, available time of workers and worker assignment. The aim of the proposed model is to minimize inter and intra-cell movement costs, machine and reconfiguration costs, setup costs, production planning costs (holding, backorder and subcontracting costs) and workers hiring, firing, training and salary costs, as well as minimizing summation of machines idle times as a second objective. To verify model validation, computational results are presented by solving some numerical examples, using Lingo optimization software in small sized problems.

Keywords: Dynamic cellular manufacturing systems, Cell formation problem, Multi-Objective mathematical programming, Production planning, Worker assignment

1. Introduction

Competitive pressure in global market, costumers innumerous and various requests, variation in demands for product mix with medium volume-medium variety and reducing product life cycle and presentation time to market have been forced producers to improve efficiency of manufacturing process and activities. Therefore implementation of Just in Time (JIT) seems a necessary action.

A current approach in developing manufacturing systems which is able to quick adaption to demand variations without necessity to lots of reinvestment is Group Technology (GT). GT is a production philosophy which assigns products to part families and required machines to cells by using similarity benefits in designing phase.

One of the most important applications of GT in manufacturing context is Cellular Manufacturing Systems (CMS). CMS is one of the efficient systems in manufacturing environment for products with high volume and variety which prepare growth and development context in global markets with incorporating job shop and flow shop benefits.

Cellular Manufacturing (CM) includes some machine cells in which each cell is responsible for processing a family of similar parts.

Reduction in product life cycle and variation in product mix and demands creates dynamic condition in the manufacturing systems. Defresha and Chen [1] presented a comprehensive mathematical model for designing a dynamic cellular manufacturing system based on tools requirements for parts and tools availability on machines. Kioon et al. [2] proposed a production planning and dynamic cell formation integrated model with aims such as minimizing inter and intra-cell material handling cost, inventory and production costs, reconfiguration costs and machine operation and overhead costs. Safai and Tavakkoli-Moghaddam [3] studied effect of parts subcontracting on reconfiguration with adding machine constant costs and backorder costs. Mahdavi et al. [4] presented a mixed integer mathematical programming model for designing dynamic cellular manufacturing systems with considering production planning and worker assignment. Papaioannou and Wilson [5] studied a literature review of cell formation problem concentrating formulations proposed in the last decade.

The goal of this paper is presenting an integrated model for production planning and cell formation in dynamic cellular manufacturing systems with multi-objective approach that concentrating on workers optimum assignment, selecting a production plan with least costs, reducing inventory and trade-off between parts outsourcing and reconfiguration for planning horizon. Therefore this paper follows that with proposing a comprehensive and integrated multi-objective model which considers most of effective and essential parameters in designing cellular manufacturing systems in dynamic environment, solve a multi-period cell formation problem that determine production planning policy and worker optimum assignment simultaneously with aiding multi-objective mathematical programming approaches.

This paper is organized as follows: in section 2 problem formulation of the proposed model is presented. Computational results from solving proposed model on small-scale examples is followed to verify model validation in section 3. The paper ends with conclusions.

2. Problem formulation

In this section a multi-objective mixed integer mathematical programming model is presented which surveys cell formation problem in cellular manufacturing systems. Main constraints of the proposed model include machine capacity, investment on machines, workers available times and minimum cell size.

2.1. Notations

2.1.1. Indices

p Index for part types ($p = 1,\dots,$ P)

j Index operations belong to part p ($j = 1,\dots,$ O_p)

m Index for machine types ($m = 1,\dots,$ M)

c Index for manufacturing cells ($c = 1,\dots, C$)

h Index for periods ($h = 1,\dots, H$)

w Index for worker types ($w = 1,\dots, W$)

2.1.2. Input parameters

P Number of part types

C Maximum number of cells that can be formed

M Number of machine types

H Number of time periods

O_p Number of operations required for part p

W Number of worker types

D_{ph} Demand for part type p in period h

λ_{ph} Unit subcontracting cost of part type p in period h

γ_{ph} Unit holding cost of part type *p* in period *h*

ρ_{ph} Unit backorder cost of part type *p* in period *h*

α_m Maintenance and overhead costs of machine type *m*

B_m Variable cost of machine type *m* for each unit time

S_m Installing cost of machine type *m*

R_m Removing cost of machine type *m*

a_{jpm} 1 if operation *j* of part type *p* can be done on machine type *m*; 0 otherwise

t_{jpm} Processing time required to perform operation *j* of part type *p* on machine type *m*

C_{jpm} Setup cost for operation *j* of part type *p* on machine type *m*

B_p^{inter} batch size for inter-cell movements of part *p*

B_p^{intra} batch size for intra-cell movements of part *p*

γ^{inter} inter-cell movement cost per batch

γ^{intra} intra-cell movement cost per batch

Lc Lower bound for cell size in terms of the number of machine types

Uc Upper bound for cell size in terms of the number of machine types

Uw Upper bound for cell size in terms of the number of workers

A_m The number of available machines of type *m*

A_w The number of available workers of type *w*

LB Lower bound for subcontracting parts

UB Upper bound for subcontracting parts

$UB_{Lmachin}$ Maximum number of machines which a worker can serve

P_{mw} 1 if worker type *w* is ready to work on machine type *m* or be able to acquire capability of working on machine by training; 0 otherwise

ϕ_{mw} Training cost per time unit of worker *w* for attaining performing skill on machine type *m* for a worker who can get necessary skill of working on machine by training

T_m Required time for training a worker who serves machine type *m* in terms of time unit

H_{wh} Hiring cost of worker type *w* within period *h*

F_{wh} Firing cost of worker type *w* within period *h*

S_{wh} Salary cost of worker type *w* within period *h*

T_{mh} Available time for machine type *m* in period *h*

T_{wh} Available time for worker type *w* in the period *h*

A An arbitrary big positive number

2.1.3. Decision variables

N_{mch} Number of machines type *m* allocated to cell *c* in period *h*

N^{+}_{mch} Number of machines type *m* added in cell *c* in period *h*

N^{-}_{mch} Number of machines type *m* removed from cell *c* in period *h*

L_{wch} Number of workers of type *w* allocated to cell *c* during period *h*

L^{+}_{wch} Number of workers of type *w* added to cell *c* during period *h*

L^{-}_{wch} Number of workers of type *w* removed from cell *c* during period *h*

Q_{ph} Number of demand of part type *p* to be produced in period *h*

S_{ph} Number of demand of part type *p* to be subcontracted in period *h*

I_{ph} Inventory level of part type *p* at the end of period *h*; $I_{p0} = I_{pH} = 0$

B_{ph} backorder level of part type *p* in period *h*; $B_{p0} = B_{pH} = 0$

Y_{ph} 1 if $Q_{ph} > 0$; 0 otherwise

Y'_{ph} 1 if $I_{ph} > 0$ and equals to 0 if $B_{ph} > 0$

p_{mw} 1 if worker type *w* is used to work on machine type *m*; 0 otherwise

X_{jpmwch} 1 if operation *j* of part type *p* is done on machine type *m* with worker *w* in cell *c* in period *h*; 0 otherwise

2.2. Mathematical model

$$\text{Min } Z_1 = \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} (\alpha_m N_{mch}) + \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{m=1}^{M}\sum_{w=1}^{W} (\beta_m Q_{ph} t_{jpm} X_{jpmwch}) +$$

$$\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{m=1}^{M} (S_m N^{+}_{mch} + R_m N^{-}_{mch}) + \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{w=1}^{W} (H_{wh} L^{+}_{wch} + F_{wh} L^{-}_{wch}) +$$

$$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} [Q_{ph} / B_p^{inter}]\, \gamma^{inter}\, \left|\sum_{m=1}^{M} X_{(j+1)\,pmwch} - \sum_{m=1}^{M} X_{jpmwch}\right|$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} [Q_{ph} / B_p^{intra}]\, \gamma^{intra}\, \left[\sum_{m=1}^{M} \left| X_{(j+1)\,pmwch} - X_{jpmwch}\right| -\right.$$

$$\left.\left|\sum_{m=1}^{M} X_{(j+1)\,pmwch} - \sum_{m=1}^{M} X_{jpmwch}\right|\right] + \sum_{h=1}^{H}\sum_{p=1}^{P} [\gamma_{ph} I_{ph} + \rho_{ph} B_{ph} + \lambda_{ph} S_{ph}]$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{m=1}^{M}\sum_{w=1}^{W} ([Q_{ph} / B_p^{inter}]\, C_{jpm} X_{jpmwch}) + \sum_{m=1}^{M}\sum_{w=1}^{W} (p_{mw} \varphi_{mw} T_m)$$

$$+ \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{w=1}^{W} (S_{wh} L_{wch}) \qquad \textbf{(1-1)}$$

$$\text{Min } Z_2 = \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{m=1}^{M} \left[N_{mch} T_{mh} - \sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} Q_{ph} t_{jpm} X_{jpmwch}\right] \qquad \textbf{(1-2)}$$

St :

$$\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} X_{jpmwch} t_{jpm} Q_{ph} \le N_{mch} T_{mh} \quad \forall\ m, c, h \quad (2)$$

$$\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{m=1}^{M} X_{jpmwch} t_{jpm} Q_{ph} \le L_{wch} T_{wh} \quad \forall\ w, c, h \quad (3)$$

$$D_{ph} = Q_{ph} + I_{p(h-1)} - B_{p(h-1)} - I_{ph} + B_{ph} + S_{p(h-1)} \quad \forall\ p, h \quad (4)$$

$$\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{j=1}^{Op}\sum_{w=1}^{W} X_{jpmwch} \le A\, Q_{ph} \quad \forall\ p, h \quad (5)$$

$$N_{mch} = N_{mc(h-1)} + N^{+}_{mch} - N^{-}_{mch} \quad \forall\ m, c, h \quad (6)$$

$$L_{wch} = L_{wc(h-1)} + L^{+}_{wch} - L^{-}_{wch} \quad \forall\ w, c, h \quad (7)$$

$$\sum_{m=1}^{M} N_{mch} \ge Lc \quad \forall\ c, h \quad (8)$$

$$\sum_{m=1}^{M} N_{mch} \le Uc \quad \forall\ c, h \quad (9)$$

$$\sum_{w=1}^{W} L_{wch} \le Uw \quad \forall\ c, h \quad (10)$$

$$\sum_{c=1}^{C} N_{mch} \le A_m \quad \forall\ m, h \quad (11)$$

$$\sum_{c=1}^{C} L_{wch} \le A_w \quad \forall\ w, h \quad (12)$$

$$p_{mw} \le P_{mw} \quad \forall\ w, m \quad (13)$$

$$\sum_{w=1}^{W} p_{mw} = 1 \quad \forall\ m \quad (14)$$

$\sum_{m=1}^{M} \quad p_{mw} \; \leq \; UB_{LMachin} \quad \forall \; w$ (15)

$\sum_{c=1}^{C} \sum_{m=1}^{M} \sum_{w=1}^{W} \quad a_{jpm} \, X_{jpmwch} \; = \; Y_{ph} \quad \forall \; j \, , \, p \, , h$ (16)

$LB \; \leq \; S_{ph} \; \leq \; UB \quad \forall \; p \, , h$ (17)

$I_{pH} - B_{pH} = 0 \quad \forall \; p$ (18)

$Q_{ph} \; \leq \; A \; Y_{ph} \quad Q_{ph} \; \geq \; Y_{ph} \; \forall \; p \, , h$ (19)

$I_{ph} \; \leq \; A \; Y'_{ph} \quad B_{ph} \; \leq \; A \; (1\text{-}Y'_{ph}) \; \forall \; p \, , h$ (20)

$N_{mch} \, , \, N^{+}_{mch} \, , \, N^{-}_{mch} \, , \, L_{wch} \, , \, L^{+}_{wch} \, , \, L^{-}_{wch} \, , \, Q_{ph} \, , \, S_{ph} \, , \, I_{ph} \, , \, B_{ph} \geq 0$ and are integer

$X_{jpmwch} \, , \, , \, p_{mw} \, , \, Y_{ph} \, , \, Y'_{ph} \; \epsilon \; \{ \, 0 \, , \, 1 \, \}$ (21)

The first objective function includes several cost terms as follows:

(The first term) Machine constant cost: Constant cost consists of maintenance and overhead cost of machines. (The second term) Machine operation cost: Operation (variable) cost is sum of total load allotted to each machine in each cell. (The third term) Machine relocation cost: The cost of installing added machines in a period and removing deleted machines from a period. (The fourth term) Hiring and firing cost: With adding worker to a cell (because of worker shortage in this cell) from a period to the next one, its hiring cost, and with removing worker from a cell (because the cell not needed to this worker) from a period to the next one, its firing cost, will be accrued to the model. (The fifth term) Inter-cell material handling cost: This cost will be incurred to the model whenever all required operations for manufacturing a part is not processed within a cell and it is needed to move to another cell for processing its next operation except the cell which it is assigned. (The sixth term) Intra-cell material handling cost: If two consecutive operations which are required for processing a part will be done within a cell but on different machines, then we need an intra-cell movement. (The seventh term) Production planning cost: This term consists of inventory, backorder and subcontracting costs for all parts in all periods for the planning horizon. (The eighth term) Set up cost: This term calculates set up cost of each production batch on different machines.

(The ninth term) Worker training cost: This term minimizes training cost according to labor present skills. As regards it is a time consuming procedure, considering time cost of workers training is a matter identic real world situation. (The tenth term) Salary cost: It refers to the salary paid for workers in the planning horizon. (The second objective function) Minimizing summation of machines idle time: Machine idle times for each cell include difference between sum of available times and sum of machines busy times for that cell. This objective function computes summation of all these idle times in all cells and then minimizes them. Constraint (2) ensures capacity of machines is not exceeded and demands will be satisfied. Constraint (3) assures that available times for workers are not exceeded. Constraint (4) is material balance well known equation which creates equivalency for all parts quantity level between two consecutive periods. Constraint (5) shows that if a part has not been produced in a period or Q_{ph}=0 none of its operation should have been dedicated to a machine, cell and worker. Constraint (6) balances number of each machine types in each cell and each period. Constraint (7) balances number of available workers between two consecutive periods. Constraints (8) and (9) indicate lower and upper bound for cell size respectively. Constraint (10) represents minimum number of workers that is assigned to each cell in each period. Constraint (11) guarantees number of machine types allotted to all cells in each period will not exceed number of available machines from that type in this period. Constraint (12) shows that in each period, total number of workers allotted to all cells from type *w* should not be more than number of available workers from type *w* in that period. Constraint (13) ensures that worker type *w* must have allocated to a machine which is able to work on it. Constraint (14) guarantees that each machine can be served only by one worker. Constraint (15) controls upper bound for number of machines which worker w serves them. Constraint (16) ensures that if a partial portion of part demands must be produced in a specific period, each required operation for processing that part on its related machine in each period just could have been assigned to one cell and be done only by one worker who is able to work on that machine. Constraint (17) indicates lower and upper bound for subcontracting quantity for each part in each period. Constraint (18) expresses that inventory and backorder level must be zero at the end of periods. Constraint (19) is supplementary for constraint 16. If necessary operations for processing parts in equation 16 can be done, then some portion of demand could be produced in that specific period. Constraint (20) ensures that inventory and backorder cannot happen simultaneously. Constraint (21) determines the type of decision variables.

2.3. Linearization of the proposed model

The proposed model is a mixed integer programming model which is considered nonlinear because of some absolute terms in objective function.

First we try to linearize absolute existent in the fifth term by using two auxiliary binary variables Z^1_{jpwch} and Z^2_{jpwch}.This is done by replacing absolute term with phrase ($Z^1_{jpwch}+Z^2_{jpwch}$) and adding the following constraint:

$$Z^1_{jpwch} - Z^2_{jpwch} = \sum_{m=1}^{M} X_{(j+1)pmwch} - \sum_{m=1}^{M} X_{jpmwch} \qquad (22)$$

Therefore the fifth term of first objective function will change to the following term:

$$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} [Q_{ph} / B_p^{inter}]\, \gamma^{inter} (Z^1_{jpwch} + Z^2_{jpwch})$$

As respects previous equation, is yet nonlinear because of two decision variables multiplication, nonnegative variable e^1_{jpwch} was replaced with that, as follows:

$$e^1_{jpwch} = Q_{ph}(Z^1_{jpwch} + Z^2_{jpwch})$$

So two following constraints will be added to the model:

$$e^1_{jpwch} \geq Q_{ph} - M (1 - Z^1_{jpwch} - Z^2_{jpwch}) \qquad \forall\ j, p, w, c, h \qquad (23)$$

$$e^1_{jpwch} \leq Q_{ph} + M (1 - Z^1_{jpwch} - Z^2_{jpwch}) \qquad \forall\ j, p, w, c, h \qquad (24)$$

Theses equations shows that, if two consecutive operations of a part is done in two different cells, $e^1_{jpwch} = Q_{ph}$ and $e^1_{jpwch}=0$ otherwise

Finally the fifth term of the first objective was replaced by following linear term:

$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W}$ [e^1_{jpwch} / B_p^{inter}] γ^{inter}

For linearizing the sixth term of the first objective function, we used two binary variables y^1_{jpmwch} and y^2_{jpmwch}. So the following constraint must be added to the model:

$$y^1_{jpmwch} - y^2_{jpmwch} = X_{(j+1)pmwch} - X_{jpmwch} \quad \forall\ j, p, m, w, c, h \quad (25)$$

Consequently the sixth term of the objective function will be altered as follows:

$$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} [Q_{ph} / B_p^{intra}]\ \gamma^{intra}\{ \sum_{m=1}^{M}(y^1_{jpmwch} + y^2_{jpmwch}) - (Z^1_{jpwch}+Z^2_{jpwch})\}$$

As regards previous equation, is yet nonlinear because of two decision variables multiplication, nonnegative variable e^2_{jpwch} was replaced such as following:

$$e^2_{jpwch} = Q_{ph} \{ \sum_{m=1}^{M}(y^1_{jpmwch} + y^2_{jpmwch}) - (Z^1_{jpwch} + Z^2_{jpwch}) \}$$

And following constraints should be added to the mathematical model:

$$e^2_{jpwch} \geq Q_{ph} - M \{ 1 - \sum_{m=1}^{M}(y^1_{jpmwch} + y^2_{jpmwch}) + (Z^1_{jpwch} + Z^2_{jpwch}) \} \quad \forall\ j, p, w, c, h \quad (26)$$

$$e^2_{jpwch} \leq Q_{ph} + M \{ 1 - \sum_{m=1}^{M}(y^1_{jpmwch} + y^2_{jpmwch}) + (Z^1_{jpwch} + Z^2_{jpwch}) \} \quad \forall\ j, p, w, c, h \quad (27)$$

Eventually the sixth term of the first objective is replaced by following linear term

$$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{Op}\sum_{w=1}^{W} [e^2_{jpwch} / B_p^{intra}]\ \gamma^{intra}$$

For linearizing second and eighth terms in the first objective, second objective function and also constraints 2 and 3, we used nonnegative variable f_{jpmwch} for replacing two decision variables which were multiplied in these terms as following:

$$f_{jpmwch} = Q_{ph}\, X_{jpmwch}$$

And then two following constraints must be added to the model:

$$f_{jpmwch} \geq Q_{ph} - M (1 - X_{jpmwch}) \quad \forall\ j, p, m, w, c, h \quad (28)$$

$$f_{jpmwch} \leq Q_{ph} + M (1 - X_{jpmwch}) \quad \forall\ j, p, m, w, c, h \quad (29)$$

3. Numerical examples

In this section, results from solving two numerical examples in small sized problems are presented to check performance accuracy and verifying validation of the proposed model in case of single objective. This examples were solved by branch-and-bound (B&B) algorithm using Lingo 9.0 optimization software on a PC includes Intel® Core^{TM} i7 with 1.73 GHz processor and 4 GB RAM.

Example 1. First example consists of 2 cells, 3 parts, 3 machines, 2 workers and 2 periods. It is assumed that each part has 2 operations which must be processed on its required machines according to operation-part-machine incident matrix. Also each operation can be done by two alternative workers which cause more flexibility in assignment process. Necessary information for machines and parts are shown in Table 1 and 2 respectively. Table 3 indicates information that is required for workers. The data set related to operation-part-machine incidence matrix is shown in table 4. This matrix delineates processing time and set up cost respectively for each part operations on different machine types. Minimum and maximum cell size (lower and upper bound for number of machines allowed in each cell) for first example are considered 1 and 3 respectively. Maximum cell size in terms of number of workers and maximum number of machines which a worker can serve them are determined 2. Whereas the proposed model is a two objective mathematical model, we transformed it to a single objective model by weighted method before solving it. Weight value for each objective function depends on decision maker viewpoint. So we considered 0.8 for the first and 0.2 for the second objective function. The objective function value and production plan for examples No. 1 and 2 are shown in table 5 and 6 respectively. Likewise part families, machine groups and assigned workers to each cell are presented in table 7 for examples 1 and 2.

Example 2. Second example consists of 2 cells, 5 parts, 5 machines, 3 workers and 2 periods. It is supposed that each part has 3 operations which must be processed on its required machines according to operation-part-machine incident matrix. After linearizing, the proposed model includes 870 variables and 1676 constraints with 0.07 hours computational time for the first example. Also example No. 2 comprises 4491 variables and 8326 constraints. It is clear that obtaining the exact solution for this problem size in a reasonable time is computationally intractable. Hence we interrupted the LINGO software after 10 hours and the best solution so far is reported in Table 5, 6 and 7.

4. Conclusion

In this paper a new multi-objective mixed integer mathematical programming model is presented which comprehensively considers solving the integrated multi-period cell formation problem, production planning (optimum production policy like production, inventory and subcontracting quantity level) and workers optimum assignment to manufacturing cells simultaneously, in a dynamic cellular manufacturing system. Some advantages of the proposed model are as considering multi-period production planning, dynamic system reconfiguration, operation sequence, alternative process plans, machine and worker flexibility, duplicate machines, machine capacity, available time of workers and workers assignment. The first objective function includes minimizing sum of miscellaneous costs like inter and intra-cell material handling cost, machine and reconfiguration cost, set up cost, inventory, backorder and subcontracting cost and workers hiring, firing, training and salary cost, as well as minimizing summation of machines idle times as a second objective. Performance and validity of the proposed model are illustrated by solving 2 numerical examples. As it is found in the second example in compare with the first one, with increasing problem scales, required computational time for solving the problem is increased significantly. So because of NP-hard nature of cell formation problem we cannot use exact optimization methods for large scale problems in a reasonable time. Hence applying different types of heuristic and meta-heuristic algorithms for solving the proposed model in the real world large sized problems seems an important issue. Also considering other critical issues in cell formation phase, like tools assignment and layout for machines and cells sounds an appropriate choice for future researches.

Table 1- information required for machines in example 1

Machines information								
Machine type	α_m	β_m	S_m	R_m	T_m	A_m	T_{m1}	T_{m2}
1	1200	8	400	300	30	2	500	500
2	1500	4	600	375	45	2	500	500
3	1800	6	500	450	25	2	500	500

Table 2- information required for parts in example 1

Parts information																
Part type	D_{p1}	D_{p2}	LB_{p1}	LB_{p2}	UB_{p1}	UB_{p2}	λ_{p1}	λ_{p2}	γ_{p1}	γ_{p2}	ρ_{p1}	ρ_{p2}	B_p^{inter}	B_p^{intra}	γ^{inter}	γ^{intra}
1	0	150	0	75	0	300	3	3	1	1	14	14	50	5	25	5
2	60	80	30	40	120	160	6	6	2	2	12	12	50	5	30	6
3	120	100	60	50	240	200	9	9	3	3	10	10	50	5	15	3

Table 3- machine-worker incidence matrix and the workers information for example 1

	machine			(φ_{mw}) *machine*										
Worker	1	2	3	1	2	3	H_{w1}	H_{w2}	F_{w2}	S_{w1}	S_{w2}	A_w	T_{w1}	T_{w2}
1	1	0	1	0	1000	5	200	200	150	500	500	2	500	500
2	0	1	1	1000	5	0	200	200	150	500	500	2	500	500

Table 4- operation-part-machine incidence matrix includes processing time and setup cost for example 1

Machine	*part 1*		*part 2*		*Part 3*	
	O_1	O_2	O_1	O_2	O_1	O_2
1	0.4,6	0,0	0.3,5	0,0	0,0	0.1,7
2	0.2,8	0,0	0,0	0.4,6	0.3,7	0,0
3	0,0	0.3,7	0.2,8	0,0	0.1,5	0,0

Table 5- objective function value and its components for examples 1 and 2

	Total	Holding	Sub-contracting	Inter-cell movement	Intra-cell movement	Constant cost	Variable cost	Setup	Training	Salary	Worker relocation	Idle times
Example1	8449.6	150	1380	500	150	5400	408	570	225	1000	300	1916
Example2	16722	120	3696	3	2502	7800	1401	1229	-------	1500	200	2762

Table 6- production plan for examples 1 and 2

		period 1					**Period 2**				
		Part 1	Part 2	Part 3	Part 4	Part 5	Part 1	Part 2	Part 3	Part 4	Part 5
Example 1	Subcontracting		80	100							
	Backorder										
	Holding	150									
	Production	150	60	120							
	Demand		60	120			150	80	100		
Example 2	Subcontracting		80	219	60	35					
	Backorder			120		70					
	Holding				30						
	Production		60		150		150		1		145
	Demand		60	120	120	70	150	80	100	90	110

Table 7- parts, machines and workers assignment to cells for examples 1 and 2

		parts		*machines*		*workers*	
		Cell 1	*Cell 2*	*Cell 1*	*Cell 2*	*Cell 1*	*Cell 2*
Example 1	*Period 1*	1,2,3	2	1,2,3	1	1,2	1
	Period 2	-------	-------	-------	-------	-------	-------
Example 2	*Period 1*	2,4	-------	1,5	-------	3	-------
	Period 2	------	1,3,5	-------	1,2,3	-------	1,2,2

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