Critical chain project scheduling problem in DTRTP environment

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Abstract
There are various optimal execution modes with the same minimum project duration in the DTRTP. The impact of different optimal execution modes on the critical chain scheduling was analyzed. Two filtering indexes including total duration and resource tightness were proposed to select the optimal execution modes with better project performance.

Keywords: DTRTP, Total Duration, Resource Tightness

INTRODUCTION

The DTRTP was firstly introduced by De Reyck (1998), De Reyck et al. (1998) and Demeulemeester et al. (2000). In construction and software development projects, it often occurs that the activities could be executed in different modes. The work content of each activity can be represented as the product of the duration and the corresponding resource requirements which should be no less than the given content. The DTRTP is a special generalization of the multi-mode resource-constrained project scheduling problem (MRCPSP). There are various efficient execution modes with the same minimum project duration in the DTRTP which are known as optimal execution modes. Often, projects in practice are affected by many uncertainties in the execution phase, which leads to project delay, increased costs and frequent changes. The projects will produce different performance in the execution phase under uncertainty.

Critical Chain Buffer Management (CC/BM) was put forward by Goldratt, which has been gained comprehensive attention in project management. There are plenty of academics who looked into CC/BM, of which Ching-Wen Chang and Ruo-Nan Li objectively reviewed relative merits and research achievements of critical chain method both at home and abroad. Despite multi-mode project scheduling problem is very important in both theory and practice, existing research on the CC/BM is mainly in single-mode background. Few scholars studied the CC/BM combined with multi-mode project scheduling problem, and less researchers focus on CC/BM in the DTRTP. Unlike the single-mode project scheduling problem, there are various optimal execution modes in the DTRTP. We can use CC/BM to insert feeding buffers into the baseline schedule for constructing homologous project schedule. The execution time and resource requirements of each activity vary in optimal execution modes, and the impact of various uncertain factors differs in the project duration which will ultimately lead to different performances in the execution phase. Therefore, when CC/BM is applied to the DTRTP, the key issue is to select the execution modes which will produce better performance.

In this paper, we first chose an instance, using a branch-and-bound procedure (Demeulemeester et al. (2000)) to get all the optimal execution modes. After that, CC/BM was applied to insert feeding buffers for constructing the project schedule. Then we figured out the Timely Project Completion Probability (TPCP) of different optimal execution modes. By computer simulations, the impact of different optimal execution modes on the critical chain scheduling was analyzed, the indexes and corresponding strategy to select the optimal execution modes making performance better have also been given. Finally, we do simulations on hundreds of randomly constructed DTRTP instances to verify the effectiveness of the indexes and strategy.

THE DTRTP AND RELEVANT ALGORITHMS

The DTRTP Problem

Project managers usually assign the work content (each activity will be assigned an individual given content) by creating the Work Breakdown Structure (WBS). The pre-assigned work content of each activity can be represented as the product of execution time and corresponding resource requirements which should be no less than the given content. Since the execution time of each activity in practice is uncertain, a series of execution modes will be obtained. When we compare the execution modes with each other in all these execution modes, we will discover the same execution time or an equal amount of resources required in two execution modes. At this time, the execution mode whose work content (the product of execution time and corresponding resource requirements) is less will be chose for an efficient execution mode. For example, each activity in the project is assigned a given work content W_i. When the execution mode is M_m, the activity execution time is d_{im}, the amount of renewable resources is r_{im}, then the product of d_{im} and r_{im} should be equal to or larger than the given work content W_i. If a project activity has two different execution modes M_m and M_n, whose execution time or resource requirements is identical, we should choose the execution mode M_m (the product of execution time and corresponding resource requirements is less) as the efficient execution mode. To get all efficient execution modes, we can order the execution modes in ascending execution time. To solve the DTRTP problem, after finding all efficient execution modes, we should construct the project schedules for them. The objective of the DTRTP is to find a project schedule with an as short project length as possible.

Optimal Execution Mode and Generation of Corresponding Project Schedule

In order to solve the DTRTP problem, many scholars proposed relevant algorithms, including both deterministic algorithms and heuristic algorithms. A depth-first branch-and-bound procedure for the deterministic DTRTP have been presented in Demeulemeester et al. (2000). Ranjbar and Kianfar proposed a genetic algorithm (uses the resource utilization to produce cross-point) combined with local search algorithm to solve the DTRTP problem in 2007. To deal with the uncertainty of project scheduling and implementation, Long and ohsato developed a fuzzy critical chain method to insert the project buffer in the end of project schedule in 2008. Ranjbar et al. (2009) adopted the scatter search algorithm which uses the re-connection of path as a solution to solve the DTRTP problem. In addition, few scholars have been devoted to the study of scheduling policies in DTRTP environment (Tian and Demeulemeester (2013, 2014)).

Take the instance shown in figure 1 as an example, we will describe how to use the branch and bound algorithm to find all the optimal execution modes and corresponding project schedules. Project network diagram is displayed by Active-On-Node (AON).The start node and end node of project are dummy nodes, whose execution time and resource requirements are both zero. There is only a single renewable resource is considered.

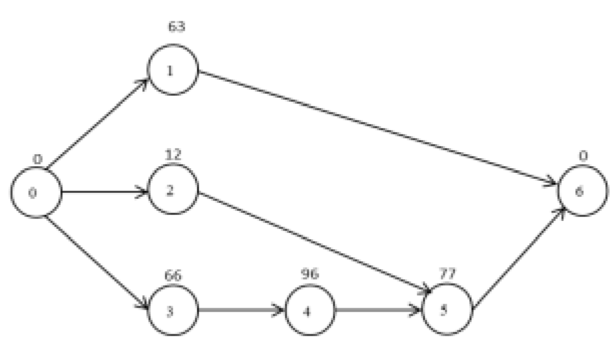

Figure 1 - The DTRTP instance

Efficient execution modes

There are seven activities and only one renewable resource (20 units available per period) in the project network, where activity 0 and activity 6 are dummy activities. The work content of each activity has been assigned. The amount of resources required of each activity is between 1 and 20. Then choosing the efficient execution modes from all the execution modes and order them in ascending execution time.

Optimal execution modes and baseline schedule

There are thousands of efficient execution modes in the DTRTP. In this simple instance, you can find out there are totally 196560 groups of efficient execution modes. In this paper, in order to get all execution modes and corresponding project schedule, a branch-and-bound procedure (Demeulemeester et al. (2000)) has been used. It may occurs that various optimal execution modes has the same minimum project duration in the DTRTP. The same thing also happened in this instance. As shown in figure 2, the two different optimal execution modes has the same minimum project duration.

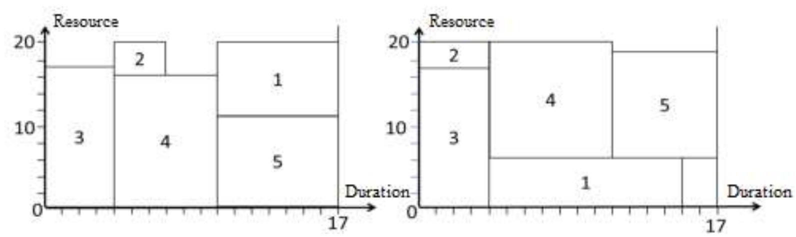

Figure 2 - Two different optimal execution modes with the same minimum duration

Using the branch and bound algorithm, it can calculate that this instance has a total of 15 groups of optimal execution modes. At the same time, this algorithm has been used to generate the corresponding feasible baseline schedule. The minimum duration of these execution modes is 17.

CRITICAL CHAIN PROJECT SCHEDULE IN THE DTRTP

After finding all the optimal execution modes and corresponding baseline schedule by branch and bound algorithm, we identify the critical chain of all these 15 groups of optimal execution modes. Then the CC/BM is applied to insert the feeding buffer in the baseline schedule and thus generate the critical chain project schedule. In this paper we apply the C&PM to calculate the buffer size, which means 50 percent of the length of non-critical chain. After inserting into the feeding buffers, it may lead to secondary resource conflict. The general approach is to delay the start time of activities in the corresponding critical chain. Although this method can solve the secondary resource conflict, it often leads to a break of critical chain. It can be discovered that there exists a break in the critical chain schedule of the sixth group optimal execution modes.

Results show there are such problems (a break of critical chain or an overflow of non-critical chain) in all the 15 groups of critical chain project schedules. To solve this problem, this paper undertake the rearrangement algorithm proposed by Yan Zhao to obtain rearrangement project schedule which will be more reasonable. In this instance, there are nine groups of optimal execution modes can obtain a rearrangement project schedule after using the rearrangement algorithm. Others are unable to solve the problem by the rearrangement algorithm.

THE IMPACT OF DIFFERENT OPTIMAL EXECUTION MODES

Indexes of Screening Optimal Execution Modes

After applying the CC/BM to the DTRTP, we could generate different critical chain project schedules based on different optimal execution modes, which will produce different performance in the execution phase under uncertainty. Then we analyzed the impact of different optimal execution modes on the critical chain project schedules by computer simulations, and proposed two indexes that may influence project performance to help project managers screening optimal execution modes.

In the DTRTP, each activity has a pre-assigned work content, which can be transformed into the product of activity duration and resource requirements. The activities could be executed in different modes, corresponding to different amounts of resources and thus different activity duration. Therefore, the generated critical chain project schedules are distinct. Activity duration and the amount of resource required determine the project schedule and performance. This paper puts forward two indexes in terms of execution time and resource requirements, and analysis the relationship between the two indexes and project performance evaluation indexes by computer simulations.

The first index is total duration (the sum of each project activity duration). As an example, there is a project with 5 activity nodes. The start activity node and end node are dummy nodes. The activity duration of other nodes are d_{im} , d_{jm} , d_{km} .Then the total duration (DS) is $d_{im} + d_{jm} + d_{km}$. When we choose M_m as the execution mode, the amount of activity nodes are n , the execution time of activity i is d_{im} , then DS of this project is $\sum_i^n d_{im}$.

Resources tightness represents the resource loading of project activities. When we choose M_m as the execution mode, the amount of activity nodes is n , the execution time of activity i is d_{im} , the resource requirements of activity i is r_{im} , the amount of renewable resource available per period is R , the minimum project duration is S , then the resources tightness of this project is $\sum_i^m d_{im} * r_{im} / (R * S)$. Generally, the larger the resources tightness, the greater the resource loading, and the greater possibility of a delay in project execution phase.

The Setting of Experimental Parameters

Simulation experiments is divided into two parts: single project simulations and multiple project simulations. Single project simulation experiments take the project shown in figure 1 as an example to simulate the execution of critical chain project schedule. In multiple project simulations, we first randomly generated 100 instances by RanGen software, then calculated the optimal execution modes and corresponding critical chain project schedules of each project, finally we do simulations for the execution of the project schedule. The setting of relevant experimental parameters is displayed in table 1.

Table 1 - Relevant experiment parameters

Control parameters	**Parameter values or probability distribution**
The distribution of duration	Logarithmic normal distribution
Standard deviation ∂	0.3
Project duration	The sum of minimum duration and 30 percent of critical chain length
Scheduling method	Parallel scheduling
Scheduling strategy	Roadrunner
Amount of simulation runs	1000
Project performance index	TPCP

When doing simulations for the execution of critical chain schedule, in order to express the uncertainty of activity execution time, we assume that activity execution time are logarithmic normally distributed. The activity duration is the expected value of each activity's execution time. Then we randomly assign the execution time to each activity according to this probability distribution. Finally, for each project schedule, we generate 1,000 simulating schedules and execute them. Assume the duration of activity i is d_{im}, which is the expected value of execution time, and the corresponding normal distribution standard deviation is ∂, then the corresponding expected value of normal distribution execution time is $u(i) = \ln(d_{im}) - \partial^2 / 2$. When simulating the execution time of activity i by MATLAB, we can use this formula: $\log nrnd(u(i), \partial)$.

(1)The scheduling method adopted in the execution phase of project is parallel scheduling. The Gating-task of critical chain can’t start no earlier than the project scheduled time, others should adopt roadrunner strategy to execute, which means other activities will begin as early as possible.

(2)The project duration has an effect on the timely project completion probability. For all the optimal execution modes with the same minimum duration, the sum of minimum duration (the project minimum duration S of this instance shown in figure 1 is 17) and 30 percent of critical chain length is regarded as the project duration.

(3)Timely Project Completion Probability(TPCP）is used to evaluate the project performance. Corresponding to the project schedule, we generate 1,000 simulating schedules and execute them to calculate the average TPCP.

Single Project Simulations

According to the experimental environment setting in section 4.2, we do simulations on the critical chain project schedules (corresponding to the 15 groups of optimal execution modes) and rearrangement project schedules(9 groups of optimal execution modes can get more reasonable rearrangement schedules) to calculate average TPCP. Results is displayed in table 2.

Table 2 - The average TPCP of project schedules and rearrangement schedules

Group No.	the average TPCP of project schedules	the average TPCP of rearrangement schedules
1	0.3660	-
2	0.3800	-
3	0.3650	-
4	0.2990	-
5	0.8300	0.8400
6	0.5200	0.8730
7	0.1150	-
8	0.0530	-
9	0.8390	0.8220
10	0.6980	0.7810
11	0.6310	0.7990
12	0.4790	0.8270
13	0.5460	0.8100
14	0.5170	0.7750
15	0.6530	0.6950

In this instance, there are nine groups of optimal execution modes can obtain a rearrangement project schedule after using the rearrangement algorithm. Others are unable to solve the problem by the rearrangement algorithm. These optimal execution modes in this instance which can't obtain a rearrangement project schedule have smaller average timely project completion probability. When problems (a break of critical chain or an overflow of non-critical chain) can be solved, we can calculate the average TPCP of rearrangement schedules. The optimal execution modes which have greater project performance are group 5,6,9,10,11,12,13,14,15. It also can be find out that the rearrangement project schedule corresponding to the sixth group of optimal execution modes having the best project performance. The results are given in table 3, including the average TPCP of rearrangement schedules and relevant index (DS,RF) values.

Table 3 - The average TPCP of rearrangement schedules and relevant index values

Group No.	TPCP	DS	RF
5	0.8400	30	0.9471
6	0.8730	32	0.9471
9	0.8220	33	0.9471
10	0.7810	36	0.9441

11	0.7990	37	0.9471
12	0.8270	38	0.9353
13	0.8100	39	0.9353
14	0.7750	39	0.9471
15	0.6950	45	0.9529
Mean value	0.8024	-	-

As it can be seen from table 3, when total duration or resources tightness is relatively small, the corresponding average TPCP will be relatively large.

Following the 80/20 Rule, we propose two Indicators range: $[ds_{\min}, ds_{\min} + 0.2*(ds_{\max} - ds_{\min})]$ and $[rf_{\min}, rf_{\min} + 0.2*(rf_{\max} - rf_{\min})]$. $ds_{\min}$ and $ds_{\max}$ represents the minimum and maximum of the value of DS. $rf_{\min}$ and $rf_{\max}$ represents the minimum and maximum of the value of RF.

When the value of DS is included in $[ds_{\min}, ds_{\min} + 0.2*(ds_{\max} - ds_{\min})]$, it means that the value of DS is between 30 and 33 in this instance. The relevant optimal execution modes satisfy this condition are group 5,6,9. It can be discovered that their average TPCP is larger than the average value of all optimal execution modes. Owing to the constant work content, when total duration (DS) is relatively small, the corresponding resource requirements will be relatively large. Once an individual activity using less time to complete, the resource occupied by this activity will be quickly released. Then the flexibility of project schedule will increase, as a result, the project schedule will deal with uncertainty better and produce greater project performance.

Likewise, in all these nine groups of optimal execution modes, when the value of RF is included in $[rf_{\min}, rf_{\min} + 0.2*(rf_{\max} - rf_{\min})]$, it means that the value of RF is between 0.9353 and 0.9388 in this instance. The relevant optimal execution modes satisfy this condition are group 12 and group 13. It also can be discovered that their average TPCP is larger than the average value of all optimal execution modes. In general, the smaller the resources tightness, the smaller the resource loading, therefore the timely project completion possibility will be larger.

Multiple Project Simulations

In order to validate the effectiveness of proposed indexes further in this paper, we randomly generated 100 instances (with a pre-assigned amount of work content of each activity) by RanGen software to do simulations. These 100 instances are all with 12 activities and only one single renewable resource (10 units available per period). Order Strength (OS) which represents project complexity is a control parameter used to generate networks, the given OS is 0.5 in the experiments. We assign a work content which is randomly distributed with $[10, 50]$ for each activity of these 100 instances. For each project, based on finding the optimal execution modes with a branch and bound algorithm, then we construct the critical chain project schedule and rearrangement project schedules with the CC/BM described in this paper. Finally, we execute the generate1000 simulating schedules and calculate simulation results.

Except the TPCP, this paper also calculate the following three indexes:

(a) The amount of instances satisfy the condition that the average TPCP of all selected optimal execution modes (the value of DS is included in $[ds_{\min}, ds_{\min} + 0.2*(ds_{\max} - ds_{\min})]$ is larger than the average value of all optimal execution modes.

(b) The amount of instances satisfy the condition that the average TPCP of all selected optimal execution modes (the value of RF is included in $[rf_{\min}, rf_{\min} + 0.2*(rf_{\max} - rf_{\min})]$ is larger than the average value of all optimal execution modes.

(c) The amount of instances satisfy the condition that the average TPCP of all selected optimal execution modes (the value of DS is included in $[ds_{\min}, ds_{\min} + 0.2*(ds_{\max} - ds_{\min})]$ and the value of RF is included in $[rf_{\min}, rf_{\min} + 0.2*(rf_{\max} - rf_{\min})]$ is larger than the average value of all optimal execution modes.

Through analyzing the average TPCP of critical chain project schedules and rearrangement schedules, we will find out that all the optimal execution modes in these 100 instances which can obtain a rearrangement project schedule produce greater project performance.

By single project simulations, we can initially speculated that when the total duration or resources tightness is relatively small, the corresponding average TPCP will be relatively large. Statistics(in this session) show that there are 82 instances satisfy the condition that when the total duration is relatively small, the corresponding average TPCP will be relatively large, 77 instances satisfy the condition that when resources tightness is relatively small, the corresponding average TPCP will be relatively large, and 73 instances satisfy the condition that when the total duration and resources tightness are both relatively small, the corresponding average TPCP will be relatively large. As can be seen, most of the projects can use the two indexes (DS, RF) to select a better optimal execution modes which will produce better project performance.

CONCLUSION

There are various optimal execution modes with the same minimum project duration in the DTRTP. Most of existing research on DTRTP aim at developing algorithms for finding a project schedule with an as short project length as possible. But the problem of how to select the execution mode making project performance (such as TPCP) better is ignored. Based on getting all execution modes with the same minimum project duration with a branch-and-bound procedure, this paper applies the CC/BM to the DTRTP, and tries to propose some filtering indexes and screening strategies to select the optimal execution modes and corresponding baseline schedule with better project performance under uncertainty. By computer simulations, it can be concluded that:

(1) Total duration (DS) and resources tightness (RF) are proposed as the indexes to select the optimal execution modes. And the effectiveness of the two indexes were verified by large amount of simulated experiments.

(2) Firstly, we can choose the execution modes which can obtain a rearrangement project

schedule from all the optimal execution modes with the same minimum project duration. Then the two indexes can be used to select the optimal execution modes. We can select the execution modes whose value of DS and RF is both relatively small if it exists. Otherwise, we can select the execution modes whose value of DS (or RF) is relatively small. The selected optimal execution modes will generally produce better project performance under uncertainty.

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