The optimal reservation value with market movements

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Abstraction

Despite the topic of real estate liquidity has aroused wide attention and research, the relationship between reservation values and liquidity is still little known in terms of theory. This paper investigates the problem that a seller of a house considers auction as the main form of sale mechanism to price its reservation value based on the market conditions, the demand changes and the likelihood of successful sale. We measure market value movements and market liquidity, deriving the seller's optimal price. Our results show that seller's reservation value moves more slowly than bidders not only in the upward market but also in the downward market. Meanwhile, market liquidity is greater in the upward market than in the downward market. In addition, the influencing factors that seller's reservation value changes with market also include the inherent value of house and the number of participants arrived.

Keywords: pricing, an auction process, market movement

INTRODUCTION

The real estate market is an important one that receives much concern from industry and academia while liquidity is a key topic of real estate research. Comparing with a public market, liquidity in a private asset market is less. Hot real estate market exhibits a rising trend with greater trading volume and higher price than norm, and it is much easy to sell assets (greater liquidity). Just the opposite obviously occurs in cold markets. In terms of demand and liquidity, Fisher(2003) derive a constant-liquidity index which captures changes in asset market price over time with holding constant expected time on the market. And from the experience view, whether the movement in the underlying buyers' reservation is greater than the underlying seller side, including relative movement either away from, or toward, one another. This is an indisputable fact that both reservation prices may move in the same direction. However, an interesting topic about who moves more quickly has not been studied in theory.

Relative to negotiated sale, auction is the main alternative form of sale mechanism in real estate market. In this case, auction is no longer one-to-one match like negotiation process but multiple matches. An auction sometimes presents many buyers bidding for a single house, sometimes also can present many buyers bidding for multiple housing units (Lu Han, 2014). The equivalence with the Dutch auction is strategically equivalent to the first-priced sealed-bid auction. The English open ascending auction also equivalent to the second-price sealed-bid auction when values are private. Adams et al (1992) compare these two sale mechanisms by modeling negotiated sale as a 'slow Dutch auction' in real estate. Lusht (1996) contributes to the literature on more house sales volume in an auction by controlling for a house's characteristics. English auctions for houses are used more frequently in Australia. In general, if auction is as a way to sell houses, there, sometimes, exist only 2~3 bidders when the auction takes place, in spite of a great number of onlookers in a real auction. In common with markets such as Ireland (Stevenson, 2015), English open outcry auction is also a main sale mechanism. Maye (1995) and Lusht (1996) show that English auction is the logical choice for modeling real estate auction.

Just as Myerson（1981）analyzes in his study that an auction design, from revenue maximization perspective, may not only involve selecting auction formats but also involve the seller's reservation value. Sometimes the seller can increase the expected revenue by optimizing his reservation value. A seductive issue in the literature concerns whether a seller reveals his reservation price. Secret reservation price and public reservation price are two manners of reservation price set by seller. No matter which manner is chosen, the highest bidder can win the objective if and only if his bid is not smaller than the seller's reservation price.

While the role of concealing reservation price in increasing the price obtained in a common-value auction such as Vincent (1995), revealing the reservation value may lead to that there is no bid during an auction because the potential buyers will feel discouraged if their potential maximum bids are lower than the reservation values. (Goetzmann W,2006)A seller in real estate auction will usually set a secret reservation price as the minimum bid that he is willing to accept.

The nondisclosure of the reservation value can increase revenue obtained on the premise of existing a greater number of bidders（Vincent1995). The number of bidders pays a significant role in the most auction literature, particularly in those papers that have considered a higher sale price obtained. The work of Goeree and Offerman(2002,2003) also illustrate that revenue will increase with the number of bidders. Comparing with the effect of revealing more information, the effect of the number of bidders is stronger. To some degree, the majority of existing literatures (Ching and Fu 2003, Ong et al. 2005, Ooi, Sirmans and Turnbull 2006) that relate directly to the issue of real estate show some form of evidence the number of bidders has some influence on either the probability of a sale being achieved or the price obtained.

According to the above consideration, the article is mainly structured as follows. We adopt a more general auction model to compare sell side reservation value movement and buy side reservation value movement when the states of market changes. The first stage is regarded as the benchmark market in which present a normal state (average liquidity) in asset market. In addition, the seller set an initial reservation price in the benchmark market. In the next stage, the second stage market can be divided into upward market and downward market two scenes. Given the change value of buyer reservation value, seller in both scenes selects his optimal changes on the basis of utility maximization principle. We derive how reservation value movement relates to such factors as market conditions, the intrinsic value, the timing of the auction, the state of the market and the number of bidders.

RESERVATION VALUE RELATIVE MOVEMENT OVER TIME

Consistent with Kyungmin Kin(2012), we assume that no trade occurs when sellers' reservation price is less than the buy side, $WTP<WTA$. The left-hand side means buyers' willingness to pay to a seller who is randomly selected from the population, whereas the minimum price that sellers are willing to accept is depicted on the right-hand side.

Given a monopoly of a single indivisible house faces a lot of buyers with unit demands plants to take English auction as a way to transaction houses. Consistent with the assumptions in John G (1981), the optimal strategy of each bidder is to submit his reservation value in an English auction if he knows his own v but not that of other competitors and the bid $b(v) = v$ (willingness-to-pay) is considered in our model. We assume that the intrinsic value of auction house is v_s ($v_s > 0$) known by seller. According to the market environment, the seller will set and conceal a private reservation value which is higher than the intrinsic value of the house. On the other hand, the seller's low price is not known by buyers, so all arrived buyers will bid. Gan Q (2013) mentions that buyers enter the market according to a homogeneous Poisson process with constant arrival rate λ. Then, the number of buyers in [0, T] is Poisson distributed with the mean λT. That is, for $T > 0$, $\Pr(n) = e^{-\lambda T}(\lambda T)^n / n!$ $(n = 0,1,2...)$, where λ and n are Poisson distributed parameters.

Important assumptions of the model are that all participants are risk neutral and they only care about expected revenue. The first one about risk neutral hypothesis implied the utility function of participants is linear. Without loss of generality, the estimate value of the object from each bidder is independent, denoted v_i. Even if the bidder knows others' valuation information, he or she doesn't change his/her valuation. All bidders are symmetrical, even if their valuations follow the same probability distribution and each of them doesn't know the valuations from others. A bidder knows private information follow the common cumulative distribution function $F(v)$ and the corresponding probability density function $f(v)$ on a positive and finite support $[L, H]$, with $0 \le L < H$. We assume $F(v)$ is a strictly increasing differentiable function, also $F(L) = 0$ and $F(H) = 1$. Let $J(v) = v - f(v)/(1 - F(v))$ as virtual value function that is a strictly increasing function. According to Myerson et al, (1981), the function $J(v)$ is very important in auction theory. In general, the inherent value of house is less than the lowest threshold of bidders' valuation L, which we assume $v_s < L$ in order to calculate conveniently. On other thing to note is the bid price cumulative distribution $F(v)$ is independent of the bid arrival process governed by parameter λ, and both $F(v)$ and λ are independent of time.

In our model, there are two stages. A seller enters real estate to auction a house and regards the moment as the first stage market. We assume the first stage market is also as a benchmark. Potential buyers bid to win the house in an auction. According to the current marker environment, the seller needs to set the optimal reservation price r in order to gain the reasonable revenue $r - v_s$ even if the trade price is seller's reservation price in the second-price auction, with the inherent value of house v_s. In the next moment, the real estate market may be upward market or down market in the second stage as the market is becoming more and more cold or hot. All the buyers will improve or reduce valuation Δr_b and bid price becomes $v \pm \Delta r_b$ as market changes and news arrives. Meanwhile, the seller also corresponding revises its reservation price because of the profit maximization. As a result, seller's reservation price becomes $r^* \pm \Delta r_s$ in the second stage market, with a positive amount $\Delta r_s > 0$. In another word, the seller's reservation price changes at the same direction with that of the reservation prices of these bidders. Our model mainly analyzes market liquidity and compares the motion range of seller's reservation price and the motion range of bidders' reservation price. That is, we will compare Δr_s with Δr_b below.

Suppose three market conditions are respectively the benchmark market, the upward market and the downward market ($i = B, U, D$). When seller's reservation price is r_i and the arrival rate of customer flow is λ , seller expected revenue is made up of three parts:

1. When the number of bidders is $n = 0$, seller's revenue is the intrinsic value of the house v_s with the probability $e^{-\lambda T}$.
2. When the number of bidders is $n = 1$, seller's revenue is $v_s F_i(r_i) + r_i(1 - F_i(r_i))$, with the probability $\lambda T e^{-\lambda T}$.
3. When the number of bidders is $n > 1$, $v_s F_i^n(v)(r_i) + n\int_r^H [F_i(v) + vF_i(v) - 1]F_i^{n-1}(v)dv$ is as seller's revenue, with the probability $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$.

According to the description above, we may know seller's expected profit as

$$\begin{aligned}\prod_k^j(r) &= v_s e^{-\lambda T} + \lambda T e^{-\lambda T}[v_s F_i(r_i) + r_i(1 - F_i(r_i))] \\ &\quad + \sum_{n=2}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!}\{v_s F_B^n(r) + n\int_r^H [F_i(v) + vF_i'(v) - 1][F_i(v)]^{n-1}dv\} \\ &= v_s e^{-\lambda T[1-F_B(r)]} + \lambda T\int_r^H [F_i(v) + vF_i'(v) - 1]\cdot e^{-\lambda T[1-F_i(v)]}dv,\end{aligned} \tag{1}$$

where the market enters the k stage and market condition is j ($j = base, up, down$; $k = 1, 2$). It can be seen that the expected profit $\prod_k^j(r)$ increases with v_s and λT .

Based on Equation (1), a buyer who has submitted a bid, if the auction is successful, will have a chance to win the auction house. However, in the second bid auction, the winner only pays the second highest price to the seller. The function $F_i(r_i)$ is explained as the probability that a particular buyer's bid less than the reservation value set by the seller. That is, it is no clinch a deal if any buyer's bid is lower than the seller's reservation value. To make sure his holding value v_s , the seller is always better off by waiting more time when the auction is failure. A buyer, if only a bid from him, will win the auction object and ends up only paying the reservation price set by the seller when his offer is higher than the reservation value with the probability of winning $1 - F_i(r_i)$. The optimal reservation value of revenue maximization is $r_i^* = J^{-1}(0)$ (Krishna2002, section 5).

The benchmark market

The first stage market is regarded as a benchmark market before market experiences rising or falling, which seller confirms an optimal reservation price. The seller will be according to the expected profit maximization to set the optimal reservation price, that is

$$\max_r \prod_1^{base}(r) = v_s e^{-\lambda T(1-F(r))} + \lambda T\int_r^H [F(v) + vF'(v) - 1]\cdot e^{-\lambda T[1-F(v)]}dv, \quad L \le r \le H . \tag{2}$$

The probability of a successful auction for the seller is always 1 because the reservation price r is lower than the boundary value L . Therefore, the profit when the reservation price r equal to the boundary value L must be higher than when the reservation price r is lower than the boundary value L, seller will not chose r which is lower than L; On the other hand, when r is higher than H, the house almost be left without anybody to care for it and the auction will be ended in failure. The seller's expected profit is zero at this moment. Therefore, the reservation price r has a constraint $L \le r \le H$.

Proposition 1 If $f(L) \le 1/(L - v_s)$, then r^* satisfies $1 - r^* f(r^*) + v_s f(r^*) - F(r^*) = 0$; If $f(L) > 1/(L - v_s)$, then $r^* = L$.

Proof: Taking the second derivative of seller's expected profit with respect to the reservation price r leads to

$$\left.\frac{\partial^2 \prod_1^{base}(r)}{\partial r^2}\right|_{r=r^*} = [\lambda T f(r)]^2 e^{-\lambda T(1-F(r))}[v_s - (r - \frac{1-F(r)}{f(r)})] + \lambda T \cdot [(v_s - r) f'(r) - 2f(r)] \mathrm{e}^{-\lambda T(1-F(r))} < 0 \tag{3}$$

Thus it can be seen the function $\prod^{base}(r)$ is concave, the optimal reservation price is in stationary point or boundary point. Then, we will analyze boundary conditions and discuss the optimal reservation price chosen by the seller in the first phase.

The seller's expected profit is found by maximizing equation (2) with respect to reservation price r. This yields the fist-order condition

$$\frac{\partial \prod_1^{base}(r)}{\partial r} = \lambda T f(r) v_s e^{-\lambda T[1-F(r)]} - \lambda T[F(r) + r f(r) - 1] e^{-\lambda T[1-F(r)]} \tag{4}$$

So we can see the boundary conditions are as follows:

$$\begin{cases} \left.\dfrac{\partial \prod_1^{base}(r)}{\partial r}\right|_{r=L} = \lambda T e^{-\lambda T(1-F(L))}[1 - (L - v_s) f(L)] \\ \left.\dfrac{\partial \prod_1^{base}(r)}{\partial r}\right|_{r=H} = \lambda T e^{-\lambda T(1-F(L))}[(v_s - H) f(H)] < 0 \end{cases} \tag{5}$$

We know that $v_s < L$, so there are two situations needing us to discuss the optimal value of reservation price in terms of the equation (5). If $f(L) \le 1/(L - v_s)$, then $\left.\frac{\partial \prod_1^{base}(r)}{\partial r}\right|_{r=L} \ge 0$, the optimal reservation price r^* satisfies $\left.\frac{\prod_1^{base}(r)}{\partial r}\right|_{r=r^*} = 0$, correspondingly, $1 - r^* f(r^*) + v_s f(r^*) - F(r^*) = 0$. The seller's reservation price achieves the optimal value in the stationary point. If $f(L) > 1/(L - v_s)$, then $\left.\frac{\partial \prod_1^{base}(r)}{\partial r}\right|_{r=L} < 0$, therefore, arbitrary $r \in [L, H]$, implies $\frac{\partial \prod_1^{base}(r)}{\partial r} \le 0$, $r^* = L$. The seller's reservation price achieves the optimal value in the boundary point.

This shows that the seller in the first phase has to consider the auction reservation price and the likelihood of a successful auction. If the probability that bidders submit the lowest price is above $1/(L-\nu_s)$, then the seller attaches great importance to the likelihood of successful auction and decides to sell the house at the lowest price L, which is as his reservation price. In addition, the seller pays more attention to setting the reservation price in real estate auction if the probability that bidders submit the lowest price is relative smaller, below $1/(L-\nu_s)$.

The upward market

Assuming that the second stage market is a hot real estate market, we regard the bidder party as a lead and the bidder's reservation price will increase as the market becomes more and more hot. The seller's reservation price will accordingly follow the bidder's reservation price increase in the same direction although the seller in the first stage (the benchmark market) has selected an optimal reservation price r^*. In order to maximize his expected revenue, the seller will adjust his reservation price as the optimal price over upward period.

The new optimal reservation price, reset by seller, is denoted by r_u. In order to compare the two sides of the reservation price movements, we assume that the motion range of bidder's reservation price and the motion range of the seller's reservation price are Δr_b and Δr_s, respectively. We let Δr_b is exogenous given and satisfies $r^* + \Delta r_b \in [L + \Delta r_b, H + \Delta r_b]$. Due to the density function, in the first stage $f(\nu)$ and $\nu_u = \nu + \Delta r_b$, according to the theorem (Michael, 2004, P76), we can know the density function in the second phase is $f_U(\nu_u) = f(\nu_u - \Delta r_b)$, and the corresponding discretion function is $F_U(\nu_u) = F(\nu_u - \Delta \mathrm{r}_b)$. The seller's reservation is changed to $r_u = r^* + \Delta r_s$, so we can see that the seller chooses the optimal reservation price is equal to select an optimal movement value Δr_s.

The seller chooses an optimal move to maximize his expected revenue:

$$\begin{aligned}\max_{\Delta r_s} \prod\nolimits_2^U(\Delta r_s) &= \nu_s e^{-\lambda T} + \lambda T e^{-\lambda T}[\nu_s F_U(r_u) + r_u(1 - F_U(r_u))] \\ &\quad + \sum_{n=2}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!}\{\nu_s F_U^n(r_u) + n\int_{r_u}^{H}[F_U(\nu_u) + \nu_u F'(\nu_u) - 1]F_U^{n-1}(\nu_u)d\nu_u\} \\ &= \nu_s e^{-\lambda T[1 - F(r^* + \Delta r_s - \Delta r_b)]} + e^{-\lambda T}\lambda T[\Delta r_b(1 - F(r^* + \Delta r_s - \Delta r_b))] \\ &\quad + \lambda T\int_{r^* + \Delta r_s - \Delta r_b}^{H}[F(\nu) + \nu F'(\nu) - 1]\cdot e^{-\lambda T[1 - F(\nu)]}d\nu,\end{aligned} \tag{6}$$

where, $\max\{L + \Delta r_b - r^*, 0\} \le \Delta r_s \le H + \Delta r_b - r^*$.

As described above, the seller's reservation price $r_u = r^* + \Delta r_s$ has to be changing in buyer's reservation price interval ($L + \Delta r_b \le \Delta r_s + r^* \le H + \Delta r_b$). If not, the auction will be failure or the seller has failed to achieve the profit maximization. That is, the constraint of the seller's price change range is $\max\{L+\Delta r_b - r^*, 0\} \le \Delta r_s \le H + \Delta r_b - r^*$.

Proposition 2 $\Delta r_s^* < \Delta r_b$ if $f(L) \le 1/(L-v_s)$; $\Delta r_s^* = \Delta r_b$ if $f(L) > 1/(L-v_s)$.

Proof: Taking the second derivative of seller's expected profit with respect to the motion range of the seller's reservation price Δr_s results in

$$\begin{aligned}\frac{\partial^2 \prod_2^U(\Delta r_s)}{\partial \Delta r_s{}^2} &= [\lambda T f(r^* + \Delta r_s - \Delta r_b)]^2 e^{-\lambda T[1-F(r+\Delta r_s - \Delta r_b)]}\{v_s + \frac{[1-F(r^*+\Delta r_s - \Delta r_b)]}{f(r^*+\Delta r_s - \Delta r_b)} - (r^* + \Delta r_s - \Delta r_b)\} \\ &+ \lambda T e^{-\lambda T[1-F(r^*+\Delta r_s - \Delta r_b)]} f'(r^*+\Delta r_s - \Delta r_b)[v_s - (r^* + \Delta r_s - \Delta r_b)] \\ &- \Delta r_b \lambda T e^{-\lambda T} f'(r^* + \Delta r_s - \Delta r_b) - 2\lambda T e^{-\lambda T[1-F(r^*+\Delta r_s - \Delta r_b)]} f(r^* + \Delta r_s - \Delta r_b) < 0\end{aligned} \tag{7}$$

then, the above objection function $\prod_2^U(\Delta r_s)$ is concave.

Thus it shows that the optimal increase value is obtained either at the stationary point or the boundary point. According to the conclusion of proposition 2, we divide two cases $f(L) \le 1/(L-v_s)$ and $f(L) > 1/(L-v_s)$ to discuss the seller's optimal pricing in the upstream market below.

Taking the first derivative of seller's expected profit with respect to the motion range of the seller's reservation price Δr_s leads to:

$$\begin{aligned}\frac{\partial \prod_2^U(\Delta r_s)}{\partial \Delta r_s} &= v_s \lambda T e^{-\lambda T[1-F(r^*+\Delta r_s - \Delta r_b)]} f(r^* + \Delta r_s - \Delta r_b) - \lambda T e^{-\lambda T} \Delta r_b f(r^* + \Delta r_s - \Delta r_b) \\ &- \lambda T e^{-\lambda T[1-F(r^*+\Delta r_s - \Delta r_b)]}\{F(r^* + \Delta r_s - \Delta r_b) + (r^* + \Delta r_s - \Delta r_b) f(r^* + \Delta r_s - \Delta r_b) - 1\}.\end{aligned} \tag{8}$$

If $f(L) \le 1/(L-v_s)$, r^* satisfies $1 - r^* f(r^*) + v_s f(r^*) - F(r^*) = 0$, then

$$\left.\frac{\partial \prod_2^U(\Delta r_s)}{\partial \Delta r_s}\right|_{\Delta r_s = \Delta r_b} = -\lambda T e^{-\lambda T} \Delta r_b f(r^*) \le 0. \tag{9}$$

From the above inequality, we can know $\Delta r_s^* < \Delta r_b$;

If $f(L) > 1/(L-v_s)$, r^* satisfies $r^* = L$, then

$$\begin{aligned}\left.\frac{\partial \prod_2^U(\Delta r_s)}{\partial \Delta r_s}\right|_{\Delta r_s = \Delta r_b} &= v_s \lambda T e^{-\lambda T[1-F(L)]} f(L) - \lambda T e^{-\lambda T} \Delta r_b f(L) - \lambda T e^{-\lambda T[1-F(L)]}(F(L) + Lf(L) - 1) \\ &= -\lambda T e^{-\lambda T} \Delta r_b f(L) + \lambda T e^{-\lambda T}\{v_s f(L) - Lf(L) + 1\} < 0.\end{aligned} \tag{10}$$

From the above inequality, we can know $\Delta r_s^* < \Delta r_b$. In addition, according to the above constraint condition $\Delta r_b = L - r^* + \Delta r_b \le \Delta r_s^*$, we gain $\Delta r_s^* = \Delta r_b$.

Conclusion: When the probability of the low price offered by bidder is low, comparing with the change range of the bidder's reservation price, the change range stems from the seller is less in the rising market. We can see the seller does not choose a high price because he is more concerned with the likelihood of successful auction. On the basis of the seller's price already presented in the first stage, the seller will slightly increase his reservation price. In contrast, when the probability of the low price offered by the bidder is high, the change range of the seller's reservation price is consistent with that of the bidder's reservation price. It is because that the seller worries the buyer depresses auction price, which leads to fail to gain his expected revenue with loss of generality as the market becomes more and more hot. In another word, the seller has to pay more attention to pricing in order to ensure that the reservation value as the transaction price is also possible.

The downward market

Next, let us assume that the second stage market is a cold real estate market. The seller's reservation value turns into $r_d = r^* - \Delta r_s$. We can see that the seller choose the optimal reservation price is equal to select an optimal motion range Δr_s. The seller chooses a reduction amount to maximize his expected revenue:

$$\begin{aligned}
\max_{\Delta r_s} \prod\nolimits_2^D (\Delta r_s) &= v_s e^{-\lambda T} + \lambda T e^{-\lambda T}[v_s F_D(r_d) + r_d(1 - F_D(r_d))] \\
&\quad + \sum_{n=2}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} \{ v_s F_D^n(\mathrm{r_d}) + n \int_{r_d}^{H} [F_D(v_d) + v_d F_D'(v_d) - 1] F_D^{\,n-1}(v_d) dv_d \} \\
&= v_s e^{-\lambda T[1-F(r^*-\Delta r_s+\Delta r_b)]} + e^{-\lambda T} \lambda T [\Delta r_b (F(r^* - \Delta r_s + \Delta r_b) - 1)] \\
&\quad + \lambda T \int_{r^*-\Delta r_s+\Delta r_b}^{H} [F(v) + vF'(v) - 1] \cdot e^{-\lambda T[1-F(v)]} dv
\end{aligned} \tag{11}$$

where, $\max\{r^* - H + \Delta r_b, 0\} \le r \le r^* - L + \Delta r_b$.

Intuitively, the description of the above equation is similar to the upward market. Beginning with the benchmark market, the seller will gradually reduce his reservation price as the market becomes more and more cold. After changing, the reservation price $r^* - \Delta r_s$ follows the bidder's interval and satisfies $L - \Delta r_b \le r^* - \Delta r_s \le H - \Delta r_b$. That is, considering about the likelihood of a successful auction and the profit maximum of seller, the reduction range of seller's reservation price Δr_s satisfies the constraint condition $\max\{r^* - H + \Delta r_b, 0\} \le r \le r^* - L + \Delta r_b$.

Proposition 3 If $f(L) < 1/(L - v_s - \Delta r_b)$, then $\Delta r_s^* < \Delta r_b$; If $f(L) \ge 1/(L - v_s - \Delta r_b)$, then $\Delta r_s^* = \Delta r_b$.

Firstly, let us show the objection function on the reduction movement is concave below. Taking the second derivative of seller's expected profit with respect to the reduction range of the seller's reservation price Δr_s results in:

$$\frac{\partial^2 \prod_2^D(\Delta r_s)}{\partial \Delta r_s^{\ 2}} < 0. \tag{12}$$

Then, the above objection function $\prod_2^D(\Delta r_s)$ is concave.

We know from the above the reduction range may achieve the optimal value when it is either in stationary point or in boundary point. According to the conclusion of proposition 2, we divide two cases to discuss the seller's optimal pricing in the downward market below. Taking the first derivative of seller's expected profit with respect to the reduction range of the seller's reservation price Δr_s leads to:

$$\begin{aligned}\frac{\partial \prod_2^D(\Delta r_s)}{\partial \Delta r_s} &= -v_s \lambda T e^{-\lambda T[1-F(r^*-\Delta r_s+\Delta r_b)]} f(r^*-\Delta r_s+\Delta r_b) - \Delta r_b \lambda T e^{-\lambda T} f(r^*-\Delta r_s+\Delta r_b) \\ &\quad + \lambda T e^{-\lambda T[1-F(r^*-\Delta r_s+\Delta r_b)]}\{F(r^*-\Delta r_s+\Delta r_b)+(r^*-\Delta r_s+\Delta r_b) f(r^*-\Delta r_s+\Delta r_b)-1\}.\end{aligned} \tag{13}$$

If $f(L) \le 1/(L-v_s)$, r^* satisfies $1-r^* f(r^*)+v_s f(r^*)-F(r^*)=0$ .Then

$$\left.\frac{\partial \prod^D(\Delta r_s)}{\partial \Delta r_s}\right|_{\Delta r_s=\Delta r_b} = -v_s \lambda T e^{-\lambda T[1-F(r^*)]} f(r^*) - \lambda T e^{-\lambda T}\Delta r_b f(r^*) + \lambda T e^{-\lambda T[1-F(r^*)]}\{F(r^*)+r^* f(r^*)-1\} < 0. \tag{14}$$

By the above inequality, we can know the result $\Delta r_s^* < \Delta r_b$;

If $f(L) > 1/(L-v_s)$, r^* satisfies $r^* = L$. Then

$$\left.\frac{\partial \prod^D(\Delta r_s)}{\partial \Delta r_s}\right|_{\Delta r_s=\Delta r_b} = \lambda T e^{-\lambda T}\{Lf(L)-v_s f(L)-\Delta r_b f(L)-1\} \tag{15}$$

If $f(L) \ge 1/(L-v_s-\Delta r_b)$, then $\Delta r_b \le \Delta r_s^*$. And according to the constraint condition $\Delta r_s^* \le r^*-L+\Delta r_b = \Delta r_b$, so we get the relationship $\Delta r_s^* = \Delta r_b$;

If $f(L) < 1/(L-v_s-\Delta r_b)$, then $\Delta r_s^* < \Delta r_b$;

Conclusion: In the downward market, the sell side reservation value moves slower than the bid side reservation value if the probability of low price offered by bidder is low. It is because the seller believes since the probability of low price offered by bidder is low, then he may slightly improve a little reservation value. Even if the final transaction price of the auction house is this reservation value, the seller also gains a little higher revenue. The reduction range of seller's reservation price should be consistent with the reduction range due to the bidder if the probability of low price offered by the bidder is high. It means that the seller pays more attention to the likelihood of successful trading. Therefore, the seller's reservation price drop very low, even reach the boundary point L in order to ensure the realization of trading. In another word, when the market is in a downward phase, this comes at the expense of lowering the seller's reservation price to a minimum, so as to achieve a quick sale state.

CONCLUSIONS

In this paper we have showed the results consistent with Fisher et al. that optimal reservation value set by seller changes with market movements and changes in the same direction with bidder's reservation price moves. I show that the bidder's reservation price moves more quickly than the seller whether the market is rising or falling. On the other hand, we can see that the important factors that influence the two parties' reservation price movement include the low-price probability, the inherent value of house and the number of participants arrived. As described above, we can know that the number of bidders/bids is generally significant not only in the likelihood of successful sale but also with respect to the sale price achieved. The seller's optimal reservation price increases with the number of bidders. The role of house inherent value is highlighted in the findings that the increase speed of seller's reservation price is greater than the decrease speed and ensure the seller's benefit if there is only one bid. The probability of low price offered by bidders directly influences the seller's optimal price in different periods.

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