

# Evaluation of different designs of end-of-life products using linear physical programming

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## Abstract

This paper presents a linear physical programming approach for evaluating the End-Of-Life (EOL) processing options for products with design alternatives. The main objective of this study is to determine the strategy for satisfying the components demands for minimizing the total cost and disposal weight and maximizing the quality level and material sales revenue.

**Keywords:** Sensor-embedded products, Design alternatives, Linear physical programming

## INTRODUCTION

Awareness about environmental issues has been rising as a result of the increase in the use of virgin resources and has led to several legislations that enforces manufacturers' responsibility beyond the products' useful life. This has led to a growing interest in the End-Of-Life (EOL) management techniques for the processing of products. There are many advantages to EOL management such as reduction in the use of virgin resources, decrease in the use of landfills and cost savings coming from the reuse of EOL products, disassembled components and recycled materials. There are several product recovery techniques such as remanufacturing, refurbishing, repairing and recycling (Thierry et al., 1995). All the recovery techniques involve disassembly operations up to a certain level. Of all the recovery operations, remanufacturing and disassembly are considered to be the most complex ones, due to the lack of information about the quality and quantity of EOL products and their components (Ondemir and Gupta, 2013). When there is no information available about the components' quality, comprehensive testing is needed to gather that information. After testing, if an EOL product is found not suitable for remanufacturing, the time and resources spent on determining that are wasted. However, emerging information technology devices, such as sensors and radio-frequency identification (RFID) tags, ease the EOL recovery decision making by reducing the uncertainty about the quality of returned products.

Sensor Embedded Products (SEPs) eliminate the uncertainties related to the conditions of the EOL products by providing life-cycle information (Ondemir and Gupta, 2014). A SEP contains sensors which monitor the product's use cycle and record its dynamic life cycle data (Ondemir et al., 2012). This information includes the content of each product, and its condition which enables

the estimation of remaining useful life of components (Vadde et al., 2008). Once this data is obtained, the optimal recovery decisions can be made without actual disassembly or inspection operations.

The received EOL products are available in different design alternatives. EOL products do not show uniform qualities since they originate from various sources where they are subjected to different working conditions. As a result, it is highly likely that each EOL product has its own quality condition exhibiting unique remanufacturing needs. Hence, finding the EOL products with minimal recovery costs becomes a crucial problem. Design alternatives are evaluated to choose the best design that satisfies the optimization criterion/criteria.

## PHYSICAL PROGRAMMING

Messac, Gupta and Akbulut (Messac 1996) proposed a new optimization technique known as linear physical programming. It addresses issues related to multiple objective optimization such as problem formulation, nature of the obtainable solutions and the algorithm. Most of the real world decision-making problems are characteristically multi-objective and there are various tools to solve them. One such popular tool is goal programming. It treats each objective as a goal and attempts to achieve preset target values for these goals. The goals are weighed according to decision maker's preferences. But the greater challenge here is to accurately determine the weights that reflect the decision maker's true preferences.

Linear Physical Programming (LPP) avoids this task of choosing weights. In Physical Programming (PP), decision maker has some concrete idea about the objectives, which can be represented in physically meaningful objectives or constraints or decision variables. Many models have been developed using PP (See, for example, (Ilgin and Gupta, 2012).

In LPP, there are four hard classes viz. "Must be smaller" (Class 1-H), "Must be larger" (Class 2-H), "Must be equal" (Class 3-H) and "Must be in the range" (Class 4-H) and four soft classes viz. "Smaller is better" (Class 1-S), "Larger is better" (Class 2-S), "value is better" (Class 3-S) and "Range is better" (Class 4-S). Soft classes are illustrated in Figure 1.

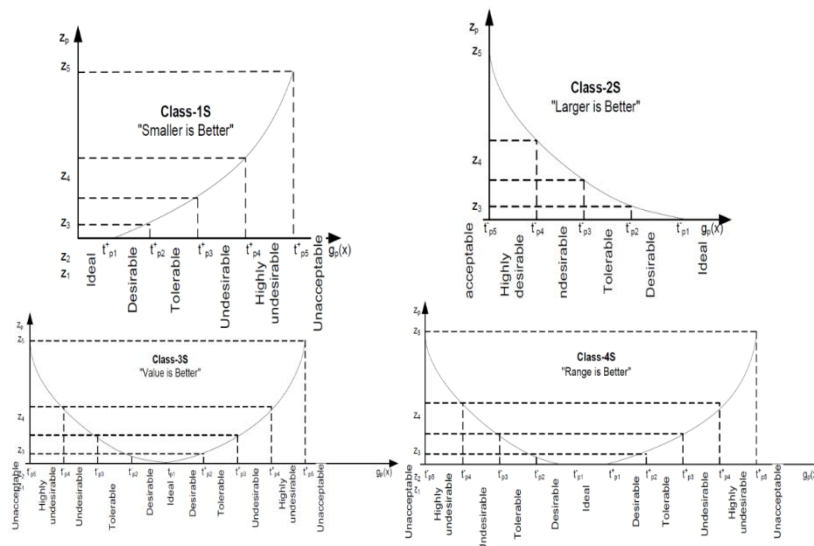


Figure 1: LPP Soft class functions

## ARTODTO SYSTEM

This paper deals with an Advanced-Repair-To-Order-Disassembly-To-Order (ARTODTO) system which receives sensors embedded EOL products. Once the EOL product is received, all the data captured by the sensors is stored in a database. The extra information such as remaining lives of components is determined by means of this life cycle data and data retrieval mechanisms. Based on the remaining lives of components, they are divided into different bins known as life bins. For example, life bin 1 may contain components of remaining lives of at least one year, life bin 2 may contain components of remaining lives between one and three years and life bin 3 may contain components of remaining lives of at least three years. Based on the remaining lives of components, the ARTODTO system repairs the products to meet the products demands, disassembles the components to meet the components demands and recycles the materials to meet the materials demands. Once the EOL products are acquired, some products are repaired to meet the products demands. Some products are disassembled and the conditions of the components are determined. The disassembled components can be operable and non-operable. The operable components are used to meet the components demands while the non-operable components are either recycled to meet the materials demands or are disposed of.

## NOMENCLATURE

Variables	Definition
$MV$	Material value;
$Q$	Quality level;
$QDIS$	Total number of components disposed of;
$TC$	Total cost;
$TDC$	Total disassembly cost;
$TRPC$	Total repair cost;
$TOPC$	Total outside procurement cost;
$TDIC$	Total disposal cost;
$TRC$	Total recycling cost;
$THC$	Total holding cost;
$cdis_{jb}$	Number of components $js$ in remaining-life-bin $b$ that are disposed of;
$d_{u,s}^{+,-}$	Positive/Negative deviation from the $s^{th}$ range limit of objective $u$ ;
$def_{ijb}$	1 if components $j$ in EOL product $i$ is disassembled because of remaining life deficiency and placed in remaining-life-bin $b$ during repair, zero otherwise;
$l_{jb}$	Number of components $js$ purchased for remaining-life-bin $b$ ;
$rep_{imjb}$	1 if a component $j$ from life-bin $b$ needs to be used to repair EOL product $i$ in order to make a product for life-bin $m$ , zero otherwise;
$fd_j$	Number of non-functional components $js$ that are disposed of;
$fr_j$	Number of non-functional components $js$ that are recycled;
$r_{jb}$	Number of components $js$ in remaining-life-bin $b$ that are recycled;
$rp_{ij}$	1 if component $j$ in EOL product $i$ is disassembled during repair, zero otherwise;
$s_i$	1 if EOL product $i$ is stored, zero otherwise;
$sc_{jb}$	Number of components $js$ in remaining-life-bin $b$ that are stored;
$sm_k$	Amount of material $k$ stored;
$\phi$	The model objective to be minimized;
$w_i$	1 if EOL product $i$ is recycled, zero otherwise;

$x_i$	1 if EOL product $i$ is disassembled for components, zero otherwise;
$x_{ijb}$	1 if component $j$ in EOL product $i$ is disassembled and placed in remaining-life-bin $b$ , zero otherwise;
$y_i$	1 if EOL product $i$ is repaired, zero otherwise;
$y_{im}$	1 if EOL product $i$ is repaired to make a product for remaining-life-bin $m$ , zero otherwise;
$z_i$	1 if EOL product $i$ is disposed of, zero otherwise;

Parameters	Definition
$I$	Set of EOL products on hand;
$B$	Set of remaining-life-bins (for components);
$J$	Set of components dealt with;
$M$	Alias for $B$ (for products);
$K$	Set of material types dealt with;
$b, i, j, k, m$	Running numbers;
$u$	Criterion;
$s$	Ranges of criterion;
$\alpha$	Destructive disassembly cost factor;
$a_{ij}$	1 if component $j$ of EOL product $i$ is functional;
$\beta_{ij}$	The highest life-bin that component $j$ of EOL product $i$ can be placed in;
$f_{ij}$	1 if component $j$ of EOL product $i$ is nonfunctional;
$c_{jb}$	Outside procurement cost of a component $j$ for life-bin $b$ ;
$c_{aj}$	Assembly cost of a component $j$ ;
$c_{dj}$	Disassembly cost of a component $j$ ;
$c_{dsj}$	Disposal cost of a component $j$ ;
$ch_j$	Holding cost of a component $j$ ;
$crc_j$	Recycling cost of component $j$ ;
$dc_{jb}$	Demand for component $j$ in remaining-life-bin $b$ ;
$dp_m$	Demand for product in remaining-life-bin $m$ ;
$dm_k$	Demand for material $k$ ;
$dfc_{imj}$	1 if component $j$ of EOL product $i$ is remaining-life-deficient for life bin $m$ ;
$h$	Unit EOL product holding cost;
$mh_k$	Unit holding cost for material $k$ ;
$mis_{ij}$	Binary parameter taking 1 if component $j$ is missing in EOL product $i$ , zero otherwise;
$prc_k$	Unit sales price of material $k$
$t_{u,s}^{+,-}$	Positive/Negative limit to the $s^{th}$ range of objective $u$ ;
$\varpi_{u,s}^{+,-}$	Positive/Negative deviation weight of the $s^{th}$ range of objective $u$ ;

## PROBLEM FORMULATION

In LPP, once the system criteria are determined, the desirability ranges of each criterion are defined. Using these range boundaries and Linear Physical Programming Weights (LPPW) algorithm, the weights for each criterion are generated. These weights and deviational variables define the objective function for the problem. Therefore, the objective function can be written as follows:

$$\min \phi = \sum_{u \in U} \sum_{s \in \{2,3,4,5\}} (\varpi_{us}^+ d_{us}^+ + \varpi_{us}^- d_{us}^-) \quad (1)$$

The system goals and constraints are defined as the constraints to the model.

## Constraints

### Class 1S: Smaller is better

The first criterion of the system is related to the total cost (TC). The mathematical expression is written as follows:

$$TC - d_{1,s}^+ \leq t_{1,s}^+, \quad s = 2, \dots, 5 \quad (2)$$

$$TC \leq t_{1,5}^+ \quad (3)$$

The second criterion of the system is related to the number of disposed items (QDIS). The mathematical expression is written as follows:

$$QDIS - d_{2,s}^+ \leq t_{2,s}^+, \quad s = 2, \dots, 5 \quad (4)$$

$$QDIS \leq t_{2,5}^+ \quad (5)$$

### Class 2S: Larger is better

The third criterion is related to the material value (MV). The mathematical expression is written as follows:

$$MV - d_{3,s}^- \geq t_{3,s}^-, \quad s = 2, \dots, 5 \quad (6)$$

$$MV \geq t_{3,5}^- \quad (7)$$

The fourth criterion of the system is related to the quality level. The mathematical expression is written as follows:

$$Q - d_{4,s}^- \geq t_{4,s}^-, \quad s = 2, \dots, 5 \quad (8)$$

$$Q \geq t_{4,5}^- \quad (9)$$

Total Cost (TC) is the sum of total disassembly cost (TDC), total repair cost (TRC), total outside procurement cost (TOPC), total disposal cost (TDIC), total recycling cost (TRC) and total holding cost (THC). The total cost function can be written as follows:

$$TC = g_1 = TDC + TRPC + TOPC + TDIC + TRC + THC \quad (10)$$

$$TDC = \sum_{i \in I, j \in J} x_i (a_{ij} cd_j + f_{ij} \alpha cd_j) + (z_i + w_i)(a_{ij} + f_{ij}) \alpha cd_j \quad (11)$$

$$TRPC = \sum_{i \in I, j \in J} [rp_{ij} (a_{ij} (cd_j + ca_j) + f_{ij} (\alpha cd_j + ca_j) + mis_{ij} ca_j) \quad (12)$$

$$y_{im}(f_{ij} + mis_{ij} + dfc_{imj}) \leq rp_{it}, \quad \forall i, m, \{j, k \mid k \in P_j\} \quad (13)$$

$$TOPC = \sum_{j \in J, b \in B} c_{jb} l_{jb} \quad (14)$$

$$TDIC = \sum_{j \in J} cds_j (\sum_{b \in B} cdis_{jb} + \sum_{i \in I} z_i (a_{ij} + f_{ij}) + fd_j) \quad (15)$$

$$TRC = \sum_{j \in J} crc_j (\sum_{b \in B} r_{jb} + \sum_{i \in I} w_i (a_{ij} + f_{ij}) + fr_j) \quad (16)$$

$$THC = h \sum_{i \in I} s_i + \sum_{j \in J} ch_j \sum_{b \in B} sc_{jb} + \sum_{k \in K} mh_k sm_k \quad (17)$$

Number of disposed items (QDIS) is mathematically expressed as follows:

$$QDIS = g_2 = \sum_{j \in J} (\sum_{b \in B} cdis_{jb} + \sum_{i \in I} z_i (a_{ij} + f_{ij}) + fd_j) \quad (18)$$

Material value (MV) is mathematically expressed as follows:

$$MV = g_3 = \sum_{k \in K} prc_k (dm_k + sm_k) \quad (19)$$

Quality level (Q) is defined as the difference between the highest life bin a component can be placed in, and the life bin it is actually placed in. It is divided into two terms,  $Q^1$  and  $Q^2$ . They are mathematically expressed as follows:

$$Q = Q^1 + Q^2 \quad (20)$$

$$Q^1 = \sum_{i \in I, j \in J, b \in B} x_{ijb} (\beta_{ij} - b) \quad (21)$$

$$Q^2 = \sum_{i \in I, j \in J, b \in B} rep_{imjb} (b - m) + \sum_{i \in I, m \in M, j \in J} (a_{ij} y_{im} - \sum_{b \in B} rep_{imjb}) (\beta_{ij} - m) \quad (22)$$

All constraints of the system belong to hard classes.

1. An EOL product is disassembled, repaired, disposed of, recycled or left untouched (stored). Therefore,

$$x_i + y_i + z_i + w_i + s_i = 1, \forall i \quad (23)$$

2. Complete disassembly implies that all the components of a product are disassembled if that product is to be disassembled and a component can be placed in only one bin after disassembly. Therefore,

$$\sum_{b \in B} x_{ijb} = x_i a_{ij}, \forall i, j \quad (24)$$

3. EOL product is repaired to produce only one product for only one life-bin. Therefore,

$$\sum_{m \in M} y_{im} = y_i, \forall i \quad (25)$$

4. Product demand is met by repaired EOL products. The number of products produced by repairing EOL products in product life-bin  $m$  should at least be equal to the product demand.

$$\sum_{i \in I} y_{im} = dp_m, \forall m \quad (26)$$

5. Component demand is satisfied by recovered and procured operable components. Recovered components are obtained from the disassembled and repaired EOL products. For each life bin  $b$  and component  $j$ , the number of recovered and procured components must be at least equal to the components demand after components used in repair, recycled, stored, and disposed of are taken out. Therefore,

$$\sum_{i \in I} (x_{ijb} + def_{ijb}) - \sum_{i \in I, m \in M} (rep_{imjb}) + l_{jb} - r_{jb} - sc_{jb} - cdis_{jb} = dc_{jb}, \forall b, j \quad (27)$$

6. Non-functional, missing and remaining-life-time deficient components must be replaced with components having remaining life time that is sufficient for producing a product for product-life-bin  $m$ . Therefore,

$$\sum_{\{b \in B, m \in M | b \geq m\}} rep_{imjb} = y_{im} (f_{ij} + mis_{ij} + dfc_{imj}), \quad (28)$$

$$\forall i, j, m$$

## NUMERICAL EXAMPLE

An example is considered to illustrate the formulated methodology. The ARTODTO system receives sensors embedded EOL refrigerators with two design alternatives. All the received EOL refrigerators may not be in good condition. There may be some missing or broken components. All the use phase information is captured by the sensors embedded in the refrigerators. Based on the remaining lives, components are separated into three life-bins. The first life-bin holds components whose remaining life is between one and two years. The second life-bin holds components whose remaining life is between two and three years. The third life-bin holds components with remaining life of three years or more. The ARTODTO system receives 200 EOL refrigerators that are used to meet all the demands. Some portion of the details of EOL refrigerators received is displayed in Table 1. Tables 2, 3 and 4 provide different costs associated with the EOL refrigerators.

Table 1: Received EOL refrigerator details

EOL Refrigerator	Design	Remaining life of components (j) (years)				
		Cabinet (1)	Compressor (2)	Condenser (3)	Expansion valve (4)	Evaporator (5)
1	1	-	-	-	2.03	1.54
2	2	-	4.67	-	3.68	2.79
3	1	3.03	-	-	5.17	4.49
4	1	1.29	5.84	1.68	2.59	4.72
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
197	2	5.20	1.43	-	-	-
198	1	6.76	-	5.16	2.98	4.25
199	2	6.42	5.87	2.96	2.30	4.92
200	2	-	-	-	2.51	3.56

Table 2: Disassembly and Assembly Costs

Components ( <i>j</i> )	Disassembly Costs (\$/unit)		Assembly Costs (\$/unit)	
	Design 1	Design 2	Design 1	Design 2
Cabinet (1)	0.00	0.00	0.00	0.00
Compressor (2)	1.80	1.50	1.80	1.50
Condenser (3)	0.90	1.00	.90	1.00
Expansion valve (4)	0.50	0.60	0.50	0.60
Evaporator (5)	2.00	3.00	2.00	3.00

Table 3: Outside Procurement and Holding Costs

Components ( <i>j</i> )	Outside Procurement Costs (\$/unit)			Holding Costs (\$/unit)
	Bin 1	Bin 2	Bin 3	
Cabinet (1)	25.00	38.47	50.00	5.00
Compressor (2)	102.52	155.63	210.50	10.50
Condenser (3)	50.00	80.26	102.58	16.00
Expansion valve (4)	18.28	26.35	35.00	2.50
Evaporator (5)	94.23	141.48	186.56	10.50

Table 4: Recycling and disposal costs

Components ( <i>j</i> )	Recycling Costs (\$/unit)	Disposal Costs (\$/lb)
Cabinet (1)	10	0.23
Compressor (2)	10	0.23
Condenser(3)	16	0.23
Expansion valve (4)	4	0.23
Evaporator (5)	15	0.23

The component demands are given in Table 5 and material yields, demands, holding costs and sales prices are given in Table 6. Table 7 provides the desirability ranges for each criterion.

Table 5: Components demands

Components ( <i>j</i> )	Remaining Life Bins		
	Bin1	Bin2	Bin3
Cabinet (1)	15	12	15
Compressor (2)	10	11	13
Condenser (3)	14	0	10
Expansion valve (4)	9	4	3
Evaporator (5)	13	15	7

Products demands are assumed to be 10, 12 and 10 for the remaining life bins 1, 2 and 3 respectively.



Table 6: Material yields, demands, holding costs and sale prices

Components (j)		Plastic	Steel
Yield (lbs.)	Cabinet (1)	-	10.00
	Compressor (2)	5.00	-
	Condenser (3)	-	-
	Expansion valve (4)	-	-
	Evaporator (5)	12.00	-
Demand (lbs)		240.00	400.00
Holding cost(\$/lb)		2.40	0.50
Sale price (\$/lb)		12.00	2.50

Table 7: Desirability ranges for each criterion

	Total Cost (\$)	Number of disposed items	Material Value (\$)	Quality Level
<b>Ideal</b>	$\leq 7000$	$\leq 0$	$\geq 6900.00$	$\geq 1970$
<b>Desirable</b>	(7000.00, 7450.00]	(0, 65]	[5420.00, 6900.00)	[1500, 1970)
<b>Tolerable</b>	(7450.00, 8300.00]	(65, 95]	[4200.00, 5420.00)	[1000, 1500)
<b>Undesirable</b>	(8300.00, 10000.00]	(95, 140]	[2700.00, 4200.00)	[500, 1000)
<b>Highly Undesirable</b>	(10000.00, 12000.00]	(140, 200]	[2150.00, 2700.00)	[0, 500)
<b>Unacceptable</b>	$> 12000.00$	$> 200$	$< 2150.00$	$< 0$

## RESULTS

The LPPW algorithm was coded and run using MATLAB. The mathematical model was constructed and solved using LINGO 13.0. Table 8 displays the values of the performance measures after solving the model.

Table 8: Values of the performance measures and objective

Objectives	Description	Aspiration Levels	Value
	Total cost	Desirable	7239.03
	Number of disposed items	Undesirable	134
	Material value	Tolerable	4642.58
	Quality level	Highly undesirable	482
Other measures	Total disassembly cost	-	\$740.35
	Total repair cost	-	\$242.49
	Total outside procurement cost	-	187.64
	Total recycling cost	-	\$795.12
	Total holding cost	-	\$4759.57
	Total disposal cost	-	\$513.86

80 EOL products are disassembled in order to meet the components demands (50 of design 1, 30 of design 2), 30 EOL products are repaired to meet the products demands (all of design 1), 17 EOL products are recycled (7 of design 1 and 10 of design 2), 23 EOL products are disposed of (11 of design 1, 12 of design 2) and 50 (6 of design 1, 44 of design 2) are stored.

## CONCLUSION

With the downfall of virgin resources, environmentally conscious manufacturing has gained a lot of importance. Sensors are embedded in the products to eliminate the uncertainties related to the conditions of the received EOL products. These sensors record the dynamic life cycle data of the products which can be used to determine extra information such as the remaining useful lives of components which is used to define the quality level of components.

In this paper, an ARTODTO system was considered which received two design alternatives of a refrigerator. The objective of the model was to determine an optimal strategy in order to fulfill the demands for products, components, and materials. The model was formulated using linear physical programming and was solved using LINGO 13.0. The total cost was in the desirable range, number of disposed items in the undesirable range, material value in the tolerable range and quality level in the highly undesirable range.

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