

# Two-dimensional warranty for an End-of-Life derived products

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## Abstract

This paper presents an approach to determine a two-dimensional warranty period for the components, materials and products derived from End-of-Life products to meet components, materials and products demands while minimizing the cost associated with warranty and maximizing the manufacturer's profits.

**Keywords:** Reverse Supply Chain, Simulation, Non-Renewable Warranty Policies, Remanufacturing

## INTRODUCTION

Recently, the number of studies dealing with the End-Of-Life (EOL) stage of a product has gained much attention from researchers (Gungor and Gupta 1999), (Ilgin, and Gupta 2010b). This is due, on one hand, to environmental factors, government regulations and public demands, and on the other hand, to potential economical profits that could be obtained by implementing reverse logistics and product recycling resolutions. Manufacturers try to cope with consumer awareness towards environmental issues and stricter environmental legislation by setting up facilities that involve the minimization of the amount of waste sent to landfills by recovering materials and components from returned or EOL products (Gungor and Gupta 2002).

In product recovery, the disassembly process plays an important role since it allows for selective separation of desired parts and materials. EOL products containing missing and/or nonfunctional components increase the uncertainty associated with the disassembly yield. Sensor-Embedded Products (SEPs) eliminate a majority of uncertainties involved with EOL management by providing life-cycle information (Gupta and Lambert 2008), (Vadde et al. 2008). This includes information about the content of each product and component conditions that enables the estimation of remaining useful life of the components. Once the data about the product is captured, it is possible to make optimal EOL decisions without any preliminary disassembly or inspection operations (Ilgin, and Gupta 2010a, 2010b, 2010c, 2011). After the components are retrieved, the products can be remanufactured.

The quality of a remanufactured product is uncertain for consumers. Therefore, the consumers are unsure if the remanufactured products will render the expected performance. This ambiguity about a remanufactured product could lead a consumer to decide against buying it. With such apprehension held by consumers, remanufacturers could seek market mechanisms that

provide assurance about the durability of the products. One strategy that the manufacturers often use is to offer warranties on their products (Murthy and Blischke 2006). This concept could also be extended to remanufactured products.

Product warranties have three key roles. The first role is insurance and protection, allowing consumers to transfer the risk of product failure to sellers (Heal 1977). Next, product warranties can also signal product reliability to customers (Balachander 2001), (Gal-Or 1989), (Soberman 2003), (Spence 1977). Lastly, the sellers use warranties to extract additional profitability (Lutz and Padmanabhan 1995). There are a few references that consider warranty policies for new products' supply chain management (Blischke 1993, 1995, 2011). However, there are only a handful of papers that consider the warranty for the remanufactured products in the context of reverse and closed-loop supply chain management. Base and extended one-dimensional warranty can be offered for remanufactured products using Free Replacement Warrante (FRW) and Pro-Rata Warranty (PRW) policies (Alqahtani and Gupta 2015a, 2015b, 2015c). The warranty policy and its effect on consumer behavior from the prospective of consumers has been studied by Liao et al. 2015. Yazdian, et al. 2014 proposed a mathematical-statistical model where decisions involve pricing of returned used products (cores), the degree of their remanufacturing, selling price and the warranty period for the final remanufactured products to investigate the joint optimization of remanufacturing, pricing and warranty decision-making for end-of-life products. Kuik et al. 2015 presented mathematical models to examine two types of proposed extended warranty policies for manufacturers to make the comparisons of their possible gained profit of remanufactured products by manufacturers.

## SYSTEM DESCRIPTION

The Advanced Repair-To-Order, Disassembly-To-Order, and Remanufacturing-To-Order (ARTODTORTO) system deliberated in this study is a product recovery system. A sensor embedded air conditioner (AC) is considered here as an example product. Based on the condition of EOL AC, it goes through a series of recovery operations similar to the one shown in Figure 1. Refurbishing and Repairing processes may require reusable components to meet the demand of the product. This requirement satisfies the internal and the external component demand. Both will be satisfied using disassembly of recovered components.

EOL ACs arrive at the ARTODTORTO system for information retrieval using radio frequency data reader that are stored in the facility's database. Then the ACs go through a six-station disassembly line. Complete disassembly is performed to extract every single component. There are nine components in an AC consisting of, evaporator, control box, blower, air guide, motor, condenser, fan, protector, and compressor. Exponential distributions are used to generate the disassembly times at each station, interarrival times of each component's demand, and interarrival times of EOL AC. All EOLPs after retrieval of the information are shipped either to station 1 for disassembly or, if EOLP needs only repair for a specific component, to the corresponding station. Two different types of disassembly operations, viz., destructive or nondestructive, are used depending on the component's condition. If the disassembled component is nonfunctional (broken, zero remaining life), then destructive disassembly is used such that the other components' functionality is not damaged. Therefore, unit disassembly cost

for a functional component is higher than nonfunctional component. After disassembly, there is no need for component testing due to the availability of information on components' conditions from sensors. It is assumed that the demands and life cycle information for EOLPs are known. It is also assumed that retrieval of information from sensors costs less than actual inspection and testing.

Recovery operations differ for each SEP based on its condition and estimated remaining life. Recovered components are used to meet components and spare parts demands, while recovered or refurbished products are used for products demands. Also, materials demands are met using recycled products and components. Recovered products and components are characterized based on their remaining lifetimes and are placed in different life-bins (e.g. one year, two years, etc.) waiting to be retrieved via a customer demand. Underutilization of any product or component could happen when it is qualified for a higher life-bin and is placed in a lower life bin because the higher life bin is full. Any product, component or material inventory that is greater than the maximum inventory allowed is assumed to be extra and is used for material demand or disposed of.

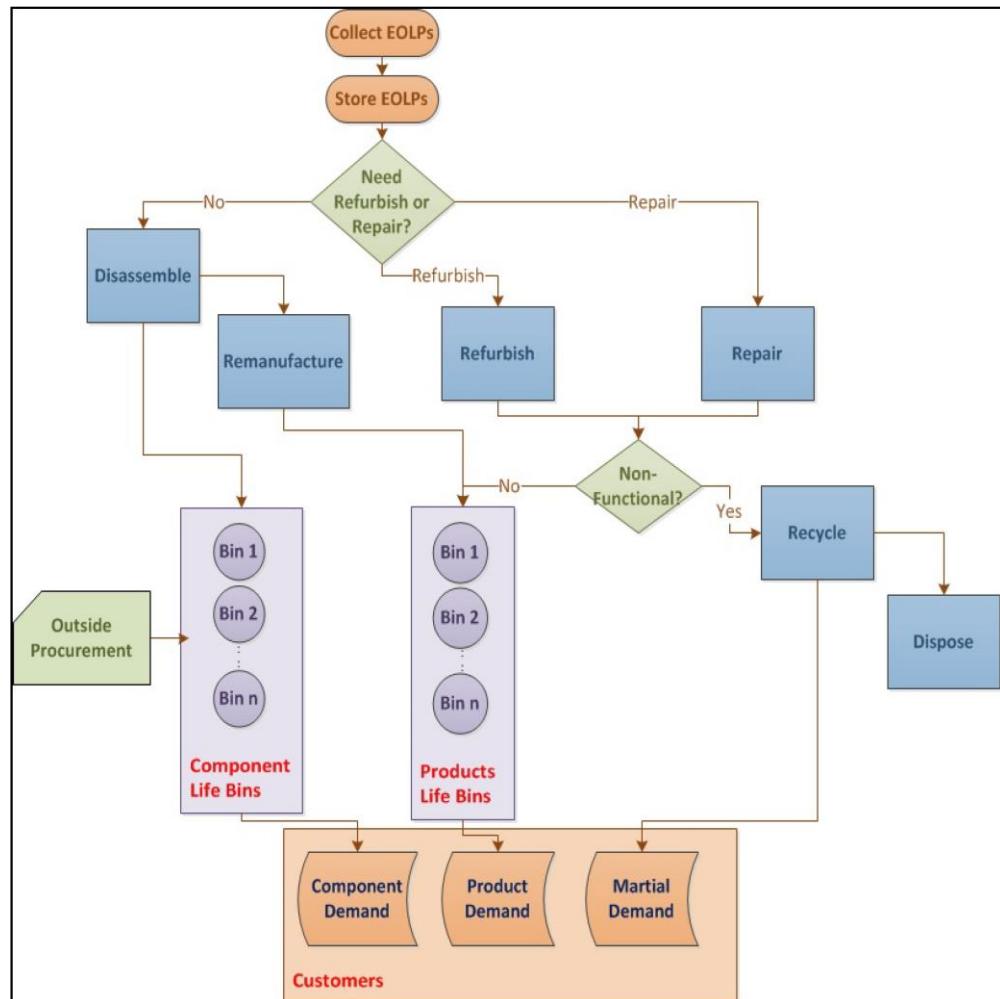


Figure 1: ARTODTORTO System's recovery processes

In order to meet the product demand, repair and refurbish options could also be chosen. EOLP may have missing or nonfunctional (broken, zero remaining life) components that need to be replaced or replenished during the repairing or refurbishing process to meet certain remaining life requirement. EOLP may also consist of components having lesser remaining lives than desired, and for that reason might have to be replaced.

## **TWO-DIMENSIONAL WARRANTY**

In the one-dimensional warranty, a policy is defined by an interval called the warranty period, which is defined with respect of a signal variable such as age or usage. On the other hand, two-dimensional warranties are defined by a region in a two-dimensional plane, typically with one axis representing time or age and the other axis representing the usage. Therefore, many different types of warranties may be defined based on the shape of the warranty coverage region.

Three common possibilities about the region of coverage are shown in Figure 2. Figure 2(a), shows the usual warranty region with independent limits on time and usage. This region has a tendency to favor the manufacturer because it bounds both the maximum time and the maximum usage for the buyer. For a buyer who is a heavy user, the warranty expires before the warranty period  $W$  ends because usage has reached its maximum limit  $U$ . Likewise, for a buyer who is a light user, the warrant expires at time  $W$  with total usage below the limit  $U$ . In contrast, Figure 2(b) shows the warrant coverage such that the buyer is assured with a minimum time of coverage and a minimum usage. This warranty region is in buyer favor. In this case, a heavy user is covered for a time period  $W$ , by which the usage may have exceeded the limit  $U$ , and a light user is covered well beyond the time period  $W$ , for the policy expires only when the total usage reaches the limit usage  $U$ .

The region shown in Figure 2(c) is a compromise between (a) and (b). Here, the buyer is provided warranty coverage for a minimum time period  $W_1$  and for a minimum usage  $U_1$ . At the same time, the manufacturer is obliged to cover the item for a maximum time period  $W_2$  and for a maximum usage  $U_2$ .

## **WARRANTY COST ANALYSIS**

In the process of deciding to purchase merchandise, the buyers usually compare features of a product with other competing brands that are selling the same product. In some cases, the competing brands make similar products to each other with similar features such as cost, special characteristics, quality and credibility of the product and even insurance from the provider. In these cases, after sale factors come into effect such as discount, warranty, availability of parts, repairs, and other additional services. These factors will be very significant to the buyer in such situation and specially the warranty since it further assures the buyer of the reliability of the product.

A warranty is an agreement that requires the manufacturer to correct any product failures or compensate the buyer for any problems that occurs with the product during the warranty

period in relevance to its sale. The objective of the warranty is to promote the products quality and guarantee its performance to assure production for both the manufacturer and the buyer.

There are many different available two-dimensional warranty policies that most products are sold with. The most famous used consumer warranty is the basic Free Replacement Warrantee (FRW). The basic FRW is when the seller agrees to repair or provide a replacement for failed items free of charge up to a time  $W$  or up to a usage  $U$ , whichever occurs first, from the time of the initial purchase. The warranty region is the one shown in Figure 2(a).

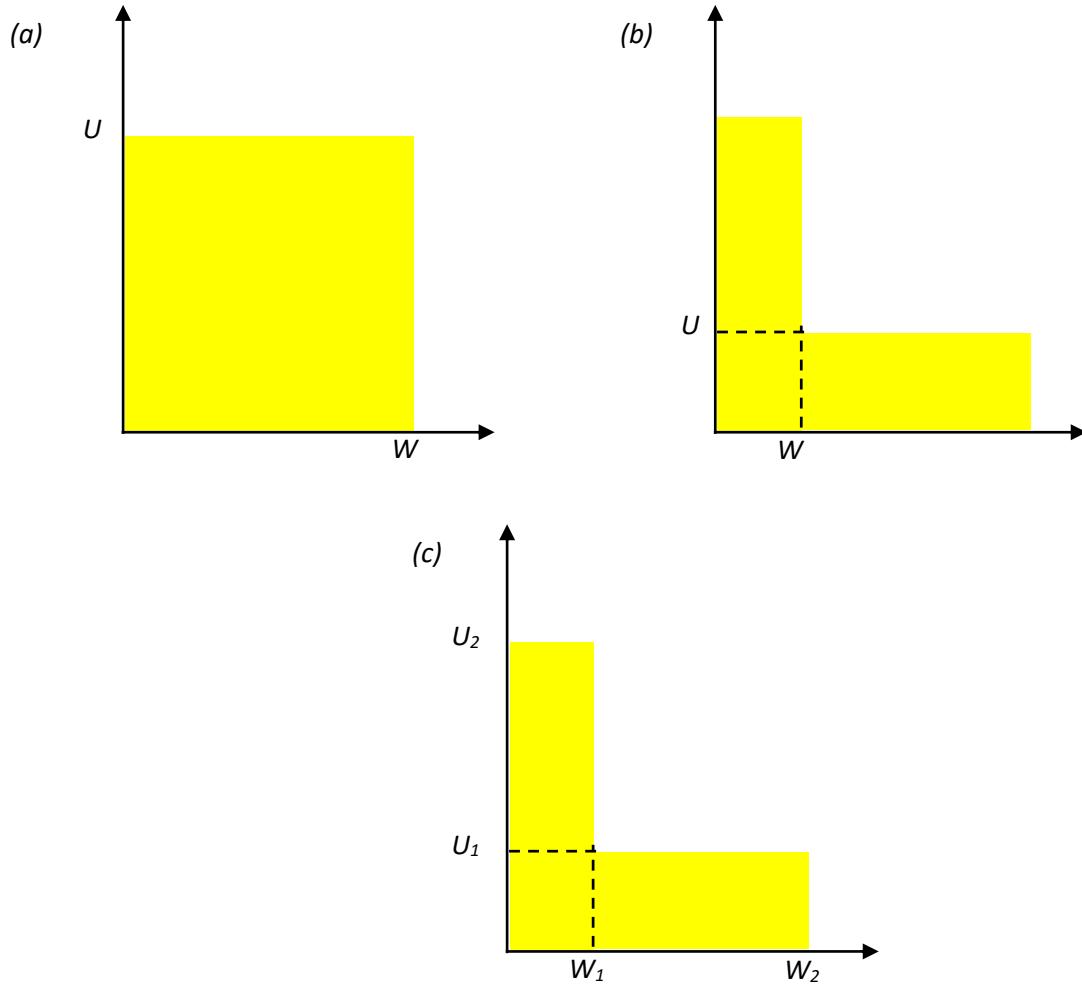


Figure 2: Two-dimensional warranty regions for usage versus time

As noted, under this policy, the buyer is provided warranty coverage for a maximum time period  $W$  and a maximum usage  $U$ . if the usage is heavy, the warranty can expire well before  $W$ , and if the usage is very light, then the warranty can expire well before the limit  $U$  is reached. Should a failure occur at age  $X$  with usage  $Y$ , it is covered by warranty only if  $X$  is less than  $W$  and  $Y$  is less than  $U$ . if the failed item is replaced by a new item, the replacement item is warrantied for a time period  $W - X$  and for usage  $U - Y$ .

## NOMENCLATURE

Variables	Definition
$W$	Warranty period;
$U$	Usage;
$C_p$	Sales price;
$C_s$	Cost of each replacement;
$C_r$	Cost of each repair;
$L$	Remaining Life;
$r$	Usage rate;
$G(r)$	Density function of a random variable $r$ ;
$X_r$	Time of warranty expire
$\gamma_1, \gamma_2$	Uniform distribution parameters
$Z_r$	Time at which the first failure outside the warranty period occurs;
$N(W, U)$	Number of failures under warranty;
$N(W, U r)$	Number of failures under warranty conditional on Usage rate = $r$ ;
$\tilde{N}(t r)$	Number of failures over the interval $[0, t]$ ;
$\lambda(t r)$	Intensity function for failures occur according to Poisson process;
$E[C_s(W, U)]$	Expected warranty service cost per unit sale;

## PROBLEM FORMULATION

Based on Blischke and Murthy (1994) Two-dimensional warranty formulas, the warranty ceases at time  $X_r$ , which is given by:

$$X_r = \frac{U}{r} \quad (1)$$

Where  $\gamma_1$  is given by:

$$\gamma_1 = \frac{U}{W} \quad (2)$$

The relationship between  $N(W, U|r)$  AND  $\tilde{N}(t|r)$  is given by:

$$N(W, U|r) = \begin{cases} \tilde{N}(W|r), & \text{if } r < \gamma_1 \\ \tilde{N}(U/r|r), & \text{if } r \geq \gamma_1 \end{cases} \quad (3)$$

The expected value of  $N(W, U|r)$  is given by:

$$E[N(W, U|r)] = \begin{cases} \int_0^W \lambda(t|r) dt, & \text{if } r < \gamma_1 \\ \int_0^{X_r} \lambda(t|r) dt, & \text{if } r \geq \gamma_1 \end{cases} \quad (4)$$

The expected number of failures over the warranty region given by:

$$\begin{aligned} E[N(W, U)] &= E\{E[N(W, U|r)]\} \\ &= \int_0^{\gamma_1} \int_0^W \lambda(t|r) dt dG(r) + \int_{\gamma_1}^{\infty} \int_0^{X_r} \lambda(t|r) dt dG(r) \end{aligned} \quad (5)$$

With  $X_r$  and  $\gamma_1$  given by (1) and (2) respectively, the expected warranty service cost per unit sale is given by:

$$E[C_s(W, U)] = C_r E[N(W, U)] \quad (6)$$

## NUMERICAL EXAMPLE

The example here considers a non-renewing two-dimensional Free Replacement Warrantee (FRW) policy for the remanufactured AC's components and product with three different remaining lives (1 year, 2 years and 3 years) with three different warranty periods (1 month, 2 months and 3 months) and usage (100 hours, 200 hours and 300 hours). Under this warranty, all failed items are repaired or replaced at no cost to the buyer as long as the failure occurs within the warranty period and usage. The warranty expires when either the age or usage limit is reached. The usage could be tracked and recorded using an embedded monitoring device. The mean time to failure of the AC is 20 days ( $MTTF = 20$  days). The other data used for implementation of the model are shown in Table 1:

**Table 1: Operation Costs (disassembly, assembly), Sale Price and Repair Cost for AC Components**

Components	$C_s$ = Operation costs (\$/unit)	$C_p$ = Sale Price (\$/unit)			$C_r$ = Repair costs (\$/unit)
		$L = 1$ Year	$L = 2$ Years	$L = 3$ Years	
Evaporator	\$4.00	\$10	\$15	\$35	\$8.00
Control Box	\$4.00	\$20	\$30	\$15	\$8.00
Blower	\$2.80	\$5	\$12	\$15	\$5.60
Air Guide	\$1.20	\$5	\$12	\$60	\$2.40
Motor	\$4.00	\$45	\$55	\$25	\$8.00
Condenser	\$1.66	\$15	\$18	\$20	\$3.32
Fan	\$2.34	\$15	\$18	\$20	\$4.68
Protector	\$0.60	\$15	\$20	\$65	\$1.20
Compressor	\$3.40	\$50	\$60	\$35	\$6.80
AC	\$55.00	\$180	\$240	\$310	\$85.00

## RESULTS

Digital simulation was used to model the ARTODTORTO system by defining three scenarios as 1, 2, and 3, corresponding to warranties of 1 month/100 hours per month, 2 months/200 hours per month and 3 months/300 hours per month, respectively. The following three cases were considered:

**Case 1** [light usage]: the usage rate is uniformly distributed over the interval of zero to one. This implied that the average usage rate is 0.5 or an average usage of 50 hours per month. Here all the consumers are light users with usage varying from 0 to 100 hours per month.

**Case 2** [light and medium usage]: the usage rate uniformly distributed over the interval of zero to two. This corresponds to half of the consumers being light users with average usage of 50 hours per month, and the other half being medium users, with usage varying from 100 to 200 hours per month and an average usage rate of 150 hours per month.

**Case 3** [light, medium and heavy usage]: the usage rate is uniformly distributed over the interval of zero to three. This corresponds to 1/3 of the consumers being light users, with average usage 50 hours per month, and 1/3 of consumers are medium users with an average usage rate of 150 hours per month. The remaining 1/3 are heavy users, with usage varying from 200 to 300 hours per month and an average usage rate of 250 hours per month.

Table 2 gives the expected number of warranty claims under warranty for the three cases for scenarios 1, 2 and 3. This can be used to predict warranty costs to the remanufacturers as shown in Table 2. Consider, for example, the situation of a three years remaining life AC (sold at \$310) with a 2 months/200 hours warranty (scenario 2). The average repair cost is  $\$85(0.503) = \$42.77$  or 13.79% of the sale price of the AC for Case 1. This changes to  $\$85(1.167) = \$99.15$  or 31.99% of the sales price of the AC for Case 2. If the warranty is increased to 3 months/300 hours (scenario 3), the average warranty cost for Case 3 increases to  $\$85(1.802) = \$153.17$  or 49.41% of the sales price.

## CONCLUSION

The non-renewing two-dimensional Free Replacement Warranty (FRW) cost for remanufactured products and components was evaluated in this paper for different periods and usages. The main objective was to introduce the idea of providing a two-dimensional warranty policy for an end-of-life (EOL) derived product and how to predict a warranty period for the disassembled components using the sensor information about the age and usage of each and every EOL product on hand to meet products, components and recycled materials demands while minimizing the cost associated with warranty and maximizing manufacturer's profit. A simulation model was used to predict the warranty period.

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**Table 2 Expected Warranty Costs for AC Components and Remanufactured AC**

Components	Scenario	Expected number of failures			Expected cost to remanufacturer		
		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Evaporator	1	0.021	0.048	0.096	\$0.16	\$0.38	\$0.77
	2	0.229	0.530	1.066	\$1.83	\$4.24	\$8.53
	3	0.462	1.070	1.541	\$3.69	\$8.56	\$12.33
Control Box	1	0.314	0.727	1.464	\$2.51	\$5.82	\$11.71
	2	0.519	1.204	1.651	\$4.16	\$9.63	\$13.21
	3	0.740	1.715	1.947	\$5.92	\$13.72	\$15.58
Blower	1	0.088	0.205	0.412	\$0.49	\$1.15	\$2.31
	2	0.253	0.586	0.593	\$1.42	\$3.28	\$3.32
	3	0.686	1.590	1.610	\$3.84	\$8.91	\$9.02
Air Guide	1	0.150	0.347	0.698	\$0.36	\$0.83	\$1.68
	2	0.427	0.990	1.002	\$1.02	\$2.38	\$2.40
	3	0.755	1.752	1.773	\$1.81	\$4.20	\$4.26
Motor	1	0.005	0.012	0.024	\$0.04	\$0.09	\$0.19
	2	0.587	1.361	1.377	\$4.69	\$10.88	\$11.02
	3	0.890	2.064	2.090	\$7.12	\$16.51	\$16.72
Condenser	1	0.059	0.136	0.273	\$0.19	\$0.45	\$0.91
	2	0.823	1.907	1.931	\$2.73	\$6.33	\$6.41
	3	0.953	2.210	2.237	\$3.16	\$7.34	\$7.43
Fan	1	0.249	0.577	1.160	\$1.16	\$2.70	\$5.43
	2	0.621	1.441	1.458	\$2.91	\$6.74	\$6.82
	3	0.847	1.963	1.987	\$3.96	\$9.19	\$9.30
Protector	1	0.017	0.038	0.077	\$0.02	\$0.05	\$0.09
	2	0.162	0.375	0.380	\$0.19	\$0.45	\$0.46
	3	0.242	0.562	0.569	\$0.29	\$0.67	\$0.68
Compressor	1	0.134	0.312	0.627	\$0.91	\$2.12	\$4.26
	2	0.368	0.852	0.863	\$2.50	\$5.80	\$5.87
	3	0.709	1.645	1.665	\$4.82	\$11.19	\$11.32
AC	1	0.455	1.054	1.067	\$38.65	\$89.60	\$90.70
	2	0.503	1.167	1.414	\$42.77	\$99.15	\$120.21
	3	0.853	1.377	1.802	\$72.49	\$117.05	\$153.17