# Vehicle routing problem with all-time connectivity constraints for the operations of mobile ad hoc networks

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**Abstract**
Vehicle Routing Problem with All-Time Connectivity Constraints (Connected VRP) has been examined as part of operations of mobile ad hoc networks. Connected VRP aims to find efficient routes to visit targets by ensuring network connectivity in all time periods. A local search algorithm and a mathematical model formulation were developed.

**Keywords:** Vehicle Routing Problem, Connectivity Constraints, Mixed Integer Programming, Local Search.

## INTRODUCTION

There are extensive applications of mobile ad hoc networks with wireless sensors including: manufacturing monitoring and control, military aerial surveillance, commercial air traffic control, delivery and distribution systems design, vehicle tracking and detection, buildings and structures monitoring, forest fire detection and many other applications (Akyildiz et al., 2002; García-Hernández et al., 2007). As an example, capabilities of UAVs can be employed for disaster response operations. For a variety of reasons, such as ability to collect data, and ease of transport, UAVs are better option in humanitarian response. A team of UAVs, which can perform as a mobile ad hoc network, provides greater flexibility through dynamic team coordination, greater efficiency through parallel task execution, and greater reliability through resource redundancy (Mosteo et al., 2008).

Sensor vehicles, which are equipped with suitable modules, use a wireless connection to communicate with the other team members. Therefore, it is assumed that the communication range of each vehicle extends to a circular area around its current location up to a certain communication radius ($R_c$). The binary disc model (Zhu et al., 2012) in this study is shown in Fig. 1. The external and internal discs represent communication $R_c$ and sensing $R_s$ ranges of vehicle $V_1$, respectively. Vehicle $V_1$ is able to establish communicated (connected) with vehicle $V_2$ but not connected to vehicle $V_3$. Moreover, Vehicle $V_1$ can cover the target point $T_1$ but not target point $T_2$. In this study, connectivity depends on only communication range. Besides, we assume that there is no service time for collecting data (service time equals zero). Connectivity of the ad hoc network is affected by factors that include transmitter power, environmental conditions, obstacles, and mobility.

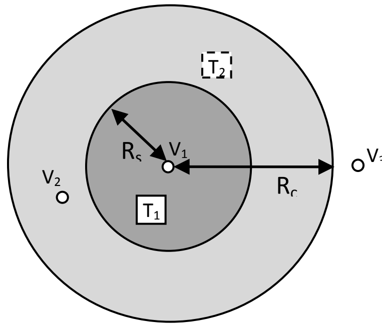


*Figure 1- Binary Disc Model (Zhu et al., 2012).*

All-time connectivity means that that all vehicles in the network communicate each other for all time periods during motion to efficiently transfer the collected data to the central station. All-time connectivity is crucial in applications where continuous real-time data sharing, situational awareness of well-functioning, collaboration for mission efficiency, motion coordination for obstacle avoidance, or on time information is desired (Sugiyama et al., 2009). Moreover, limitations to communicate sensor data to a sink in real time may cause ineffectiveness probably leading to fail in completing the mission (Ponda et al., 2012). By constantly exchanging information with others, vehicles are available to obtain global knowledge about the mission and the map.

In this study, Vehicle Routing Problem with All-Time Connectivity Constraints (Connected VRP) has been defined for mobile ad hoc networks. Connected VRP aims to find efficient routes to visit targets by ensuring network connectivity in all time periods. Problem definition, solution methodology and experimental results will be presented in the following sections.

## PROBLEM DEFINITION

Consider a set of vehicles $V = \{V_1, V_2, . . . , V_m\}$ with sensors randomly deployed to retrieve information from a set of targets $W= \{W_1, W_2, . . . , W_n\}$ given an area on the 2D space without obstacles. It is assumed that all target locations are known exactly beforehand. Real-time data which will be collected from target points or sensed environmental data, is transferred simultaneously to the center.

In Fig. 2, four vehicles are initially deployed at a center (D), traverse three targets ($W_1$, $W_2$, $W_3$), and then must return to the center in a minimum time. Target points in the order of $W_2$-$W_3$-$W_1$ are visited by vehicles $V_3$-$V_2$-$V_4$ respectively. However, each vehicle should communicate with other vehicle within distance $R_c$ directly, and with those outside its communication range via relay vehicles in all time periods. Thus, aim of the Connected VRP is minimizing total completion time (final visiting time of the center) under visiting target points by keeping all-time network connectivity, starting and ending at the center point. As seen in Fig. 2, each vehicle takes dynamically one of two roles within the network at different time periods. A vehicle is called as *source* node if it collects information from the targets and transfer information to other nodes. In addition to its own collected data, a source may also transfer data sent by other vehicles. A vehicle is a *relay* node if it is not sensing and is used only to re-transmit information collected by source nodes to other sensors.

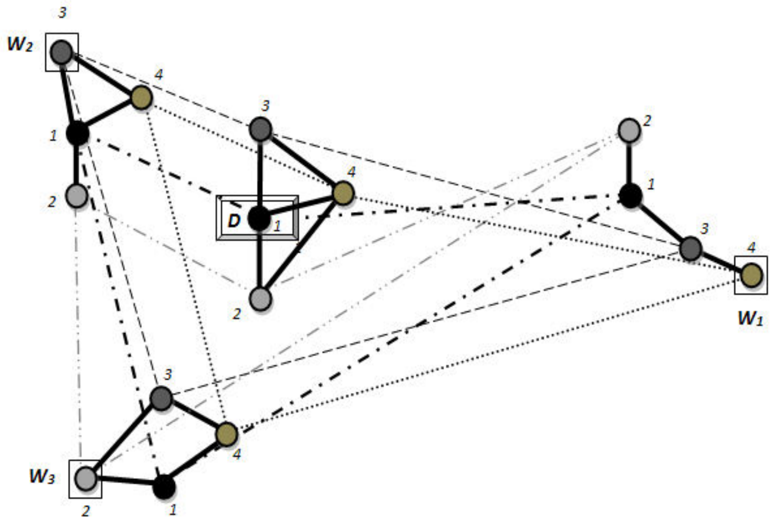


*Figure 2- Connected VRP*

## SOLUTION METHODOLOGY

A Mixed Integer Programming (MIP) model has been formulated to find lower bound solutions and a local search algorithm has been developed for approximate solutions. Before mathematical model formulation, some theoretical arguments are presented to clarify our approach to the "all-time connectivity" constraints.

First, all vehicles are assumed moving in a group ensuring connectivity on their own straight line from one target position to the other. Arrival times of vehicles with different speeds are equal between given two target points. Second, there exists at least one feasible path from target to the other by ensuring all-time connectivity. Thus, it is meaningful to relax connectivity constraints for middle time periods between consecutive visits for finding a lower bound. Thus, only positions of each vehicle at visiting times are considered. Third, as minimum cost topology for a connected graph is spanning tree which has minimum number of edges to connect all vertices, availability of any spanning tree structure is sufficient for the connectivity as shown in Fig. 3.

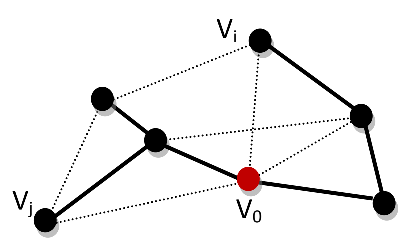


*Figure 3- Connected Graph*

By using this fact, the connectivity constraints are formulated by flow balancing constraints to check path availability from each node to the selected sink among the identical vehicles.

Mixed integer linear programming model formulation for the Connected VRP is given below. Mathematical model formulation for the Connected VRP is based on VRP formulation with flow balancing constraints for connectivity. Points indexed 0 and N represent the depot where there are N-1 target points.

## MILP Model for the Connected VRP

*Indices*

| | |
|---|---|
| Vehicles with Sink | : V = {*0,1,2,…,K*} |
| Sensor Vehicles | : S = {*1,2,3,…,K*} |
| Points | : W = {*0,1,2,…,N*} |
| Targets | : R = {*1,2,3,…,N*} |

*Decision Variables*

$V_i$ : visit time of the target i

$X_{ij}^{k}$ : 1, if target j is succeeding target i by vehicle k; 0, otherwise

$(a_k^i, b_k^i)$ : coordinates of vehicle k at the target i visit time.

$d_{ij}^{\max}$ : maximum travel time from the target i to the target j

$A_{kl}^{i}$ : 1, if vehicle *k* is adjacent to vehicle *l* at the target *i*; 0, otherwise

$P_{kl}^{i}$ : 1, if a directed flow from vehicle *k* to vehicle *l* exists at the target *i*; 0, otherwise

$F_{kl}^{i}$ : flow amount from vehicle k to vehicle l at the target *i*

*Parameters*

$(a_i, b_i)$: coordinates of target *i*
*R* : communication range radius
*M*: large integer

$$min \;\; Z = V_N \tag{1}$$

s.t

$$\sum_{i \in W: i \neq j} \sum_{k \in V} X_{ij}^{k} = 1 \quad \forall j \in R: j \neq N \tag{2a}$$

$$\sum_{j \in R: i \neq j} \sum_{k \in V} X_{ij}^{k} = 1 \quad \forall i \in R: i \neq N \tag{2b}$$

$$\sum_{j \in R: j \neq N} \sum_{k \in V} X_{0j}^{k} = 1 \tag{2c}$$

$$\sum_{i \in R: i \neq N} \sum_{k \in V} X_{iN}^{k} = 1 \tag{2d}$$

$$\sum_{i\in W}\sum_{k\in V} X_{i0}^{k} + \sum_{j\in W}\sum_{k\in V} X_{Nj}^{k} = 0 \tag{3}$$

$$V_j \geq d_{0j}^{\max} - M.(1 - \sum_{k\in V} X_{0j}^{k}) \qquad \forall j \in R : j \neq N \tag{4a}$$

$$V_j \geq V_i + d_{ij}^{\max} - M.(1 - \sum_{k\in V} X_{ij}^{k}) \qquad \forall i, j \in R : i \neq j \tag{4b}$$

$$d_{ij}^{\max} \geq d_{ij}^{k} - M.(1 - \sum_{k\in V} X_{ij}^{k}) \quad \forall i \in W, j \in R : i \neq j, \forall k \in V \tag{5}$$

$$d_j^k \leq M.(1 - \sum_{i\in W: i\neq j} X_{ij}^{k}) \qquad \forall j \in R, \forall k \in V \tag{6a}$$

$$d_j^k \geq 1 - \sum_{i\in W: i\neq j} X_{ij}^{k} \tag{6b}$$

$$d_{kl}^{i} \leq R\sqrt{2} + M.(1 - A_{kl}^{i}) \qquad \forall k, l \in V : k \neq l, \forall i \in R \tag{7a}$$

$$P_{kl}^{i} \leq A_{kl}^{i} \qquad \forall k, l \in V : k \neq l, \forall i \in R \tag{7b}$$

$$F_{kl}^{i} \leq K.P_{kl}^{i} \qquad \forall k, l \in V : k \neq l, \forall i \in R \tag{7c}$$

$$F_{kl}^{i} + M.(1 - P_{kl}^{i}) \geq 1 \qquad \forall k, l \in V : k \neq l, \forall i \in R \tag{7d}$$

$$\sum_{l\in V: k\neq l} P_{kl}^{i} = 1 \quad \forall k \in S, \forall i \in R \tag{8a}$$

$$P_{kl}^{i} + P_{lk}^{i} \leq 1 \qquad \forall k, l \in V : k \neq l, \forall i \in R \tag{8b}$$

$$\sum_{k\in S} F_{k0}^{i} = \mathrm{K} \quad \forall i \in R \tag{9a}$$

$$\sum_{k\in S} F_{0k}^{i} = 0 \quad \forall i \in R \tag{9b}$$

$$\sum_{l\in V: k\neq l} F_{kl}^{i} = 1 + \sum_{l\in V: k\neq l} F_{lk}^{i} \quad \forall k \in S, \forall i \in R \tag{9c}$$

$$\begin{aligned} & X_{ij}^{k} : \{0,1\} \quad \forall i, j \in W, k \in V \\ & A_{kl}^{i}, P_{kl}^{i} : \{0,1\} \quad \forall i \in R; k, l \in V \\ & F_{kl}^{i} : \mathbb{R}^{+} \quad \forall i \in R; k, l \in V \\ & a_i^k, b_i^k, V_i, d_{ij}^{\max} : \mathbb{R}^{+} \quad \forall i, j \in W, k \in V \end{aligned} \tag{10}$$

Rectilinear distance decision variables depend on the location variables:

$$d_{ij}^{k} = \left| a_i^k - a_j^k \right| + \left| b_i^k - b_j^k \right| \quad \forall i, j \in W : i \neq j, \forall k \in V \tag{11}$$

$$d_{kl}{}^{i} = \left|a_k^i - a_l^i\right| + \left|b_k^i - b_l^i\right| \qquad \forall i \in R;\ \ k,l \in V : k \neq l \tag{12}$$

$$d_j^k = \left|a_j^k - a_j\right| + \left|b_j^k - b_j\right| \ \ \forall j \in R, \forall k \in V \tag{13}$$

Eq.(1) minimizes traversing completion time. Constraints (2) guarantee that every target points are visited exactly once by one of the vehicles. Constraints (3) state that each vehicle must return to the depot. Constraints (4) and (5) are assignment constraints for target visiting times and center return times. Constraints (6) ensure the center is the last assigned target. Constraints (7) and (8) determine the visiting variables and adjacent vehicles. Connectivity Constraints (9)-(15) guarantee flow paths from each sensor vehicles to the sink vehicle. Constraints (9) - (11) provide relationship between topological and flow decision variables such that data can flow over selected flow arcs and flow arcs can be selected only among arcs of adjacent vertices. Constraints (12) convert undirected flows to only directed flows. In constraints (13), construct a spanning tree consisting of directed flows. Constraints (14) and (15) are flow formulations for the sink and the other vehicles respectively. Constraints (16) limit the motion of the vehicles in a period. Finally, binary and non-negativity are defined in Constraints (17).

Connected VRP combines two fundamental topics: routing the group of vehicles, and connectivity maintenance. The minimum total traveling time in the Connected VRP is obviously higher than or equal to that of standard VRP because of connectivity constraints. Communication range between two vehicles is typically less than possible distances between two nodes, $R < d_{ij}$. When the communication range is considerably high, Connected VRP reduces to the VRP. On the contrary, when the communication range is considerably small (close to zero), vehicles move together like a single vehicle and problem reduces to the TSP.

## Local Search

As it is not likely to reach good quality solutions for Connected VRP instances in reasonable computational times, heuristic methods are needed to obtain near optimal solutions. In this paper, a Local Search (LS) algorithm with a special feasibility test for all-time connectivity has been developed for the Connected VRP. The LS algorithm improves a random initial solution by generating feasible solutions. If generated solution is feasible that maintains all-time connectivity, then the objective function value of the new candidate solution is then compared to the objective function value of the current solution. If the new objective function value is better than the value of the current solution, the new solution is accepted and set as the current solution. Otherwise, the current solution does not change. Pseudocode for the LS Algorithm is given below.

A sample solution representation for the problem depicted in Fig. 2 is shown in Fig.4.

| Target Seq. | 2 | | | | 3 | | | | 1 | | | | 0 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Vehicle Seq. | 3 | 4 | 1 | 2 | 2 | 3 | 4 | 1 | 4 | 3 | 1 | 2 | 1 | 2 | 4 | 3 |
| Positioning Angle | | -50 | -150 | -92 | | 60 | -30 | -160 | | 150 | 120 | 90 | | -90 | 60 | 120 |

*Figure 4- A sample representation*

| | |
|---|---|
| $\theta$ | *: current solution* |
| $f(\theta)$ | *: objective function value of the current solution* |
| $t$ | *: iteration counter* |
| $t_{max}$ | *: max number of iterations* |
| $N(\theta)$ | *: neighborhood of* $\theta$ |
| $\theta'$ | *: neighbor feasible solution of the current solution* |
| $\theta^*$ | *: best solution* |

**LS Algorithm**

1. Generate a random feasible initial solution $\theta$
2. Calculate $f(\theta)$
3. Set iteration $t = 0$, *feasibility* = false
4. **while** $t < t_{MAX}$
5.     **while** *feasibility* = false
6.         Choose $\theta' \in N(\theta) \subseteq S$
7.         *Position* ( $\theta'$ )
8.         Feasibility = *Check Feasibility* ( $\theta'$ )
9. **end while**
10.     Calculate f($\theta'$)
11.     **if** $f(\theta') \leq f(\theta)$ **then** $M = \{(\theta', f(\theta'))\}$
12.     Set $t = t+1$ and *feasibility* = false
13. **end while**
14. Find $\theta^* = Improve\ (\theta)$
15. Output best solution $\theta^*$

*Neighbour Solution :* The LS algorithm generates a new solution by using specific operators for three hierarchical levels of a current solution. Perturbation level is selected randomly by a parameter (*pertrand*). In the first level, target sequence is perturbated by insertion operator, a new vehicle sequence is generated by swapping operator in the second level and angle of randomly selected vehicle at a random target is deviated by a random amount.

*Positioning :* In order to guarantee the network connectivity at the visiting times, vehicles are positioned constructively according to vehicle sequence and corresponding positioning angles by starting from the visitor vehicle. For larger coverage, nodes should be located as far as possible to each other by keeping connectivity. Relative position ($x^i_{k+1}$ , $y^i_{k+1}$) of the successive vehicle ordered by ($k+1$) can be calculated as follows:

$$x^i_{k+1} = x^i_k + R.\ cos(\delta_{k+1}) \tag{14}$$

$$y^i_{k+1} = y^i_k + R.\ sin(\delta_{k+1}) \tag{15}$$

where ($x^i_k$ , $y^i_k$) is position of preceeding vehicle at visiting time of the target $i$, $R$ is communication range radius, $\delta_{k+1}$ is randomly deviated angle for vehicle ordered by ($k+1$). As an example, Vehicle 3 is visitor vehicle of the first visited target (i = 2) in Fig. 4. While the

Vehicle 3 ($k$=0) is positioned exactly on the target location ($x^2_0$ , $y^2_0$), the second Vehicle 4 ($k$=1) is positioned at the location ($x^2_1$ , $y^2_1$) with maximum distance to the Vehicle 3 with $\delta_1 = -50$.

*Feasibility Check:* The two-side feasibility check is used to understand whether the resulting network solution satisfy connectivity constraints. Under assumption of straight line motion, it is meaningful to check connectivity for each edge of the initial graph only at the breakaway point (the earliest time stopping connectivity) before arriving the subsequent visiting point. Alternatively, in the two-side feasibility check, link failures are detected in the very early and very late part of the motion that do not allow compensations by relaying vehicles. The rationale for this option is that it is likely to compensate link failures because of traffic intensification at middle time periods. Thus, we have two timing for checking feasibility between two visits: 1) just after departure from the preceding visit point, 2) just before arrival at the succeeding point.

*$t_i$ : departure time of the group of vehicles from the preceding target*
*$t_j$ : arrival time of the group to the succeeding target*
*$t_{ij}$ : travel time of the group between consecutive points (i,j) where $t^k_{ij} = t^l_{ij}$ for all k and l.*
*$d^{\varepsilon}_{kl}$: distance between the vehicles (k,l) at the very immediate time $t_{\varepsilon} = t_i + \varepsilon.\ t_{ij}$, where $\varepsilon$ is a very small ratio, e.g. $0 < \varepsilon \leq 0.1$*
*$d^0_{kl}$ : initial distance at the departing time from the preceding target*
*$d^1_{kl}$ : final distance at the arrival time to the succeeding target.*
*$Seq_T$: Target Sequence*
*$Seq_V$: Vehicle Sequence at the specific target*
*$P^{\varepsilon}_{kl}$ : availability of a path from the vehicle k to vehicle l*

**Feasibility Check Procedure**

1. Set feasibility= True
2. For each consecutive points (i, j) in $Seq_T$
3.    For each adjacent vehicles (k,l) in $Seq_V$ at the target i.
4.       Calculate $d^0_{kl}$ and $d^1_{kl}$
5.       If $d^1_{kl} > R$
6.          If $d^{\varepsilon}_{kl} > d^0_{kl}$ and $P^{\varepsilon}_{kl} = 0$ then update feasibility = False
7.          else if $P^{1-\varepsilon}_{kl} = 0$ then update feasibility = False
8.       end
9.    end
10. end
11. Return Feasibility

*Improvement*: Best solution found so far can be improved by random perturbation of the positioning angles by preserving connectivity. For this aim, only new neighbor positioning angles are generated for selected targets and vehicles by keeping the current target sequence and vehicle sequences. The resulting solution with new vehicle positions is then compared to the best solution found so far.

## Experiment Results

In order to test effectiveness of the LS algorithm, 10 instances are generated randomly for both levels of number of targets 6 and 12. Minimum values of 10 replications are compared with

MIP solutions found by implementation in IBM ILOG CPLEX Optimization Studio 12.5.1 solver with 600 seconds of CPU time. LS algorithm is implemented in C++. Relative error is calculated by equation *Rel. Error = (LS Solution-MIP Solution)/ MIP Solution*. Each instance has been solved for two levels of two factors: number of vehicles (m= 2;4) and communication range (R= 5;20). Parameters of the LS algorithm are given in Table-1.

*Table 1- LS Algorithm Parameters*

| Parameter | Value |
|---|---|
| Max. # of iterations for LS | 10000 |
| Max. # of iterations Improvement | 1000 |
| Target perturbation percentage | 30% by insertion |
| Vehicle perturbation percentage | 30% by swapping 10 times |
| Angle perturbation percentage | 40% by deviation 10 times |
| Deviation range (Δ) | 30 |

Experiment results are summarized in Table 2 and Table 3.

*Table 2 - Results for n = 6*

| Number of Vehicles | Range | Avg. MIP | Avg. Gap % | Rel. Error % |
|---|---|---|---|---|
| 2 | 5 | 335,19 | 0,00 | 0,01 |
| 4 | 5 | 313,10 | 0,96 | 0,05 |
| 2 | 20 | 287,70 | 0,00 | 0,05 |
| 4 | 20 | 192,54 | 0,92 | 0,66 |

*Table 3- Results for n = 12*

| Number of Vehicles | Range | Avg. MIP | Avg. Gap % | Rel. Error % |
|---|---|---|---|---|
| 2 | 5 | 388,35 | 0,91 | 0,00 |
| 4 | 5 | 450,79 | 1,00 | -0,14 |
| 2 | 20 | 343,68 | 0,98 | 0,13 |
| 4 | 20 | 368,01 | 1,00 | 0,24 |

The LS algorithm presented good performance in terms of relative error for all instances, except instances with four vehicles and large communication range (R=20). Relative error increased by number of vehicles and communication range. Computational time of the algorithm for small size (n=6) problems is less than 1 sec. and is less than 3 secs for larger size (n=12) problems. As it was not possible to reach optimal solution within a limited running time, LS outperformed relative to MIP solutions for some instances (n=12, m= 4, R=20). Average objective values increases by number of targets and decreases by number of vehicles and communication range. It was observed that LS algorithm hardly generated feasible neighbor solutions.

## Conclusion

In this paper, Connected VRP as an original version of well known VRP is defined for mobile ad hoc networks. A mixed integer programming model (MIP) formulation and local search (LS) algorithm with special connectivity test are proposed to find efficient solutions. Despite obstacles of the LS algorithm, it performed well in most of instances relative to the lower bound solutions obtained by MIP model. However, in future works, Connected VRP needs advanced metaheuristic algorithms with efficient methods for testing feasibility, or new methods for generating feasible neighbor solutions, restart mechanisms to avoid getting stuck in local optima. Finally, extensive experiments are required to test effectiveness of proposed methods.

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