Analysis of measurement of productivity and competitiveness through Leontief and Cobb Douglas mathematical intersection function

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Abstract
Global productivity and competitiveness indicators base their measurements on macro indicators making impossible to achieve a measurement between sectors. Therefore, this research tries to establish an option to measure productivity and competitiveness over the intersection of Leontief and Cobb Douglas mathematical function, allowing a micro level rating (e.g. organizations).

Keywords: Productivity, Competitiveness, Leontief equation, Cobb Douglas Function, Latin American markets

Introduction

The purpose of this research is to evaluate the productivity and competitiveness measurement through the interception of mathematical function that allows developing an indicator based on the variables of two study objects. Nowadays, there are available different methodologies for measuring the productivity and competitiveness, such as for nations and companies as well. Among them, the ones that are used by organizations like the World Economic Forum, the World Bank and the World Business Center stand out from the other methodologies that are considered empirical generalizations.

Productivity and competitiveness can be measured in two dimensions; macro and micro, from the macro level, is studied the comparative advantages of countries according to their resources: labor force and capital, human capital formation and innovation efforts, etc. (World Bank Group, 2010) At the micro level, competitiveness and productivity can be directly linked with the ability of organizations to operate with profitably in a given market (ECLAC, 2006).

However, in order to strengthen and to be sustainable in the meso competitiveness, it is necessary to study the micro competitiveness, with the aim to establish linkages that in its summation enable to set up a cohesive industry, and to encourage the development of a

competitive economy, through basic elements such as an improved market, innovation, human capital investment, enhancement in the functioning of the labor market, and reforms to the tax systems. These elements turn out to be primary pillars to measure and boost productivity and competitiveness of markets and nations (Gurría, 2012). Nevertheless, in order to achieve the stated above it is necessary to conduct a comprehensive measure of each organization, both on the internally and sectorial level, where the measurement of variables such as profitability, productivity, value added, innovation, technology, cost, market share, exports, product quality , distribution, and positioning of companies, allows in their conjunction, an indicator that propitiates continuous improvement and comparison between sectors and economies (McFetridge, 1995).

Therefore, the research is founded on the study of mathematical concepts that planned to execute two important functions, with the objective of enhance the productivity measurement at the micro level: (1) the Cobb Douglas´ productivity role, and (2) the equation of Leontief. Thus, it is their interception the one that allows a mathematical measurement of productivity and competitiveness, assuming that the Cobb Douglas function measures the relationship between product and variations in technology inputs, labor and capital (Sancho, 2012), and the equation of Leontief is the economic model that measures the interdependence of industries in an economy (Ramirez, 1992)

Methodology of Competitiveness and Productivity measurement

The entities responsible for measuring the global economic performance base their measure throughout the following methodologies

IMD world competitiveness center; measure competitiveness based on the national environment analysis, that is divided in four main factors; 1) Economic Performance, 2) Government Efficiency, 3) Business Efficiency and 4) Infrastructure. Subsequently, each of these factors are divided into 5 sub-factors which highlight every face of the analyzed areas. Altogether, the World Competitiveness Yearbook features 20 such sub-factors. These 20 sub-factors comprise more than 300 criteria, although each sub-factor does not necessarily have the same number of criteria (for example, it takes more criteria to assess Education than to evaluate Prices). Each sub-factor, independently of the number of criteria that contains, has the same weight in the overall consolidation of results, which is 5% (20x5 =100).

Criteria can be both, hard data, which analyzes competitiveness as it can be measured (e.g. GDP) or soft data, which analyzes competitiveness as it can be perceived (e.g. Availability of competent managers). Hard criteria represent a weight of 2/3 in the overall ranking, whereas the survey data represent a weight of 1/3. In addition, some criteria are intended for background information only, which means that they are not used in calculating the overall competitiveness ranking (e.g. Population under 15). Finally, aggregating the results of the 20 sub-factors makes the total consolidation, which leads to the overall ranking of the World Competitiveness Yearbook (IMD World Competitiveness Center, 2014).

The World Economic Forum defines competitiveness as the set of institutions, policies, and factors that determine the level of productivity of a country. The level of

productivity, sequentially, sets the level of prosperity that can be reached by an economy. The components are grouped into 12 pillar of competitiveness. This open-endedness is captured within the GCI by including a weighted average of many different components; each one measures a different aspect of competitiveness. The first pillar is Institutions; the institutional environment is determined by the legal and administrative framework within which individuals, firms, and governments interact to generate wealth. The second pillar is Infrastructure; that expressed extensive and efficient infrastructure are critical for ensuring the effective functioning of the economy, as in the same way, it is considered an important factor to determinate the location of economic activity and the kinds of activities or sectors that can develop within a country. The third pillar is Macroeconomic environment; Overall, the stability of the macroeconomic environment is truly important for business, therefore, it is significant for the general competitiveness of a country.

The fourth pillar is Health and primary Education; a healthy workforce is vital to a country's competitiveness and productivity. Workers who are ill or with poor health cannot function to their highest potential and in consequence will be less productive. There is no doubt, poor health leads to significant costs to business, as sick workers are often absent or operate at lower levels of efficiency. The fifth pillar is Higher education and training; Quality higher education and training are crucial for economies that want to move up the value chain beyond simple production processes and products. The sixth pillar is Goods market efficiency; countries with efficient goods markets are well positioned to produce the right mix of products and services given their particular supply-and-demand conditions, as well as to ensure that these goods can be most effectively traded in the economy. The seventh pillar is labor market efficiency, The efficiency and flexibility of the labor market are critical for ensuring that workers are allocated to their most effective use in the economy and are provided with incentives to give their best effort into their jobs. The eighth pillar is Financial Market development; financial and economic crisis have highlighted the central role of a sound and well-functioning financial sector for economic activities. The ninth pillar is Technological readiness; The technological readiness pillar measures the agility on which an economy adopts existing technologies to enhance the productivity of its industries, with specific emphasis on its capacity to fully leverage information and communication technologies (ICTs) in daily activities and production processes for increased efficiency and enabling innovation for competitiveness.

The tenth pillar is Market size; the size of the market affects productivity since large markets allow firms to exploit economies of scale. Traditionally, the markets that are available to firms have been constrained by national borders. The eleventh pillar is Business sophistication; in the analysis of this pillar there is no doubt that sophisticated business practices are conducive to higher efficiency in the production of goods and services. Business sophistication concerns two elements that are intricately linked: the quality of a country's overall business networks and the quality of individual firms' operations and strategies. The twelfth pillar is Innovation; Innovation can emerge from new technological and non-technological knowledge. Non-technological innovations are closely related to the know-how, skills, and working conditions that are embedded in organizations and are therefore largely covered by the eleventh pillar of the GCI.

These entire twelve pillars are measure through the interrelation among them because they are not independent; they tend to reinforce each other. At the same time, each pillar is aggregated into a single index. The WEF considers that the economies will be affect in many different ways, for example, the best way to improve in competitiveness in Asia is not the same as the best way for all Europe to do so, due to the fact ,both continents are settle in very different stages of development. In this meaning, is recommended to attribute a weight to each sub index as it is shown below (World Economic Forum , 2014).

Table 1- Subindex weights and income thresholds for stages of development

	STAGES OF DEVELOPMENT				
	Stage 1: Factor-driven	Transition from stage 1 to stage 2	Stage 2: Efficiency-driven	Transition from stage 2 to stage 3	Stage 3: Innovation-driven
GDP per capita (US$) thresholds*	<2,000	2,000–2,999	3,000–8,999	9,000–17,000	>17,000
Weight for basic requirements subindex	60%	40–60%	40%	20–40%	20%
Weight for efficiency enhancers subindex	35%	35–50%	50%	50%	50%
Weight for innovation and sophistication factors	5%	5–10%	10%	10–30%	30%

Souce: The Global Competitiveness Report 2013-2014.

On the other hand, **the World Bank measure competitiveness and productivity based on the Doing Business project** that provides objective measurements of business regulation for local firms in 189 economies and selected cities at the subnational level (Doing Business, 2014), The Doing Business project based their measure on the following indicators; 1) Starting a Business, 2) Dealing with construction permits, 3) Getting electricity, 4) Registering property, 5) Protecting minority investors, 6) Paying taxes, 7) Trading across borders, 8) Enforcing contracts, 9) Resolving insolvency and have an additional topic to measure labor market regulation. The aim of this annual report is to measure the regulations that enhance business activity and those that constrain it. Doing Business presents quantitative indicators on business regulations and the protection of property rights that can be compared across 189 economies around the world (Doing Business, 2015).

In addition, **the heritage foundation based on the measurement of productivity** and competitiveness in the index of Economic Freedom focuses in four key aspects of the economic environment, over which governments typically exercise policy control; 1) Rule of Law; that refers to the property rights component, like a quality assessment of the extent to which a country's legal framework allows individuals to freely accumulate private property, secured by clear laws that are enforced effectively by the government. 2) Freedom from corruption; Corruption erodes economic freedom by introducing insecurity and uncertainty into economic relations. It also reduces economic vitality by increasing costs and shifting resources into unproductive activities. The score for this component is directly derived from Transparency International's Corruption Perceptions Index (CPI), which measures the level of perceived corruption in 177 countries. 3) Government size; the analysis focuses in three quantitative sub-factors: a) The top marginal tax rate on individual income, b) The top marginal tax rate on corporate income, and c) The total tax burden as a percentage of GDP. 4) Regulatory Efficiency; enclose the Business freedom such an overall indicator of the efficiency of government regulation of business. The quantitative score is derived from an array of measurements of the ease of starting, operating, and closing a business. 5) Open Markets; means the Trade freedom as

a composite measure of the extent of tariff and non-tariff barriers that affect imports and exports of goods and services. The trade freedom score is based on two inputs; a) The trade-weighted average tariff rate and b) Non-tariff barriers (NTBs) (The Heritage Foundation, 2015)

By analyzing the previous content, we can identify that the measurement of competitiveness and productivity, in most cases is concentrated on measure countries competitiveness and not competitiveness between companies or business to compare the performance of each business and to drive continuous improvement that brings and promotes the development of the nations. It is necessary to analyze all the existing methods to create new methodology for measuring micro-level companies (e.g. organizations). The responsible entities for the worldwide economic performance measurement based their analysis the following methodologies.

Mathematics and economic methods to competitiveness and productivity measurement

On the mathematical and economical fields, there are different ways to measure the competitiveness and productivity. Although, completely for this research, we considered two mathematical equations. The first one corresponds to Cobb Douglas´ function. This mathematical equation represents one of the most used functions in economy, considered such neoclassic production function for excellence. The function is composed of:

$$(1) Y_t = AX_t^{\alpha} Y_t^{\beta};\ \ where\ \ 0 < \alpha, \beta < 1$$

$Y_t = Production$

$A = Exogenous\ technical\ \ progress$

$X_t = Capital\ Stock$

$Y_t = Number\ of\ \ employees\ in\ an\ economy$

Where Y, K, L and A, are real GDP, labor input, capital input and the total factor productivity level respectively (HÁJCOVÁ, 2007). The variables that have a direct relation to the equation Leontief are; Yt, Lt and Kt. On the other hand, the competitiveness and productivity can be measured over the prospect of mono and multifactorial productivity. From this theory, the competitiveness and the productivity permit to create a feedback cycle (Inputs/ quality inputs – transformation process + value added – Outputs/World Class services) that propitiates the continuous improvement to continue developing the quality of inputs and hence outputs, positioning in ideal conditions to compete in the world. However, we must not forget that the inputs should be generating income for the organization, manifesting in actions such as the following ones; 1) Altering the physical or chemical properties of the inputs, 2) Making goods available at different points in time (storage activities) and 3) Changing goods instead.

At the same time, another linked aspect between the equations is that the production of one sector is considered such an input of another sector, and the outputs like a demand and organization competitiveness

Table 2- Matrix of Leontief equation to measure the productivity and competitiveness

Organization	***E1***	***E2***	***……***	***En***	***Competitiveness***	***Productivity***
E1	*X11*	*X12*	*……*	*X1n*	*D1*	*X1*
E2	*X21*	*X22*	*……*	*X2n*	*D2*	*X2*
……	*….*	*….*	*……*	*….*	*….*	*….*
En	*Xn1*	*Xn2*	*……*	*X11*	*Dn*	*Xn*

Where, (Xij) will be the amount of output / demand of sector (i) which is considered as inputs by industry j; Di is the quantity demanded by the sector (i), requested by the consumer and converted into market competitiveness. Meanwhile, (Xi) will be the sector's productivity (i). Of the foregoing the coefficient matrix A = (aij), where (aij) is the division between Xij and Xj, resulting on the number of outputs of the (i) sector used as input to produce one unit of output in (j) sector. Considering that productivity directly depends on the inputs received from one sector to obtain outputs that allow to be competitive in the market, and the operation of inequalities arising from the matrix quoted in Table 1, the matrix equation is obtained: Xn = an1X1 + annXn an2X2 + ... + Dn; X = AX + B, where X is the column matrix of productivity, the coefficient matrix A and matrix D competitiveness column from demand (Teresa Mediavilla, Unknown).

Reaching inputs and outputs with the highest standards of quality, rest in the administration practices of management and strategies emerge and will be crucial for existing goods and services that can be turned into products, goods and world class services. As well, it´s important to contextualize the degree application of world-class manufacturing practices of companies through the following graph.

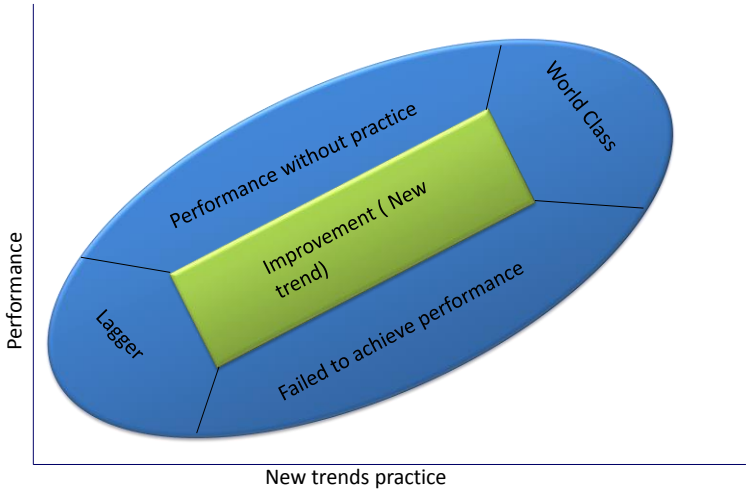

Figure 1- Five Items of Competitiveness Management, Schroeder, R., Flynn, 2001

Result

This research intends to propose a point where the equation of Leontief and Cobb Douglas functions will be intercepted.

$$(2) f(x,y) = \mathrm{A}X^{b}Y^{1-b}$$

$F = Production$

$A = Exogenous\ technical\ \ progress$

$X_t = Capital\ Stock$

$Y_t = Number\ of\ employees\ in\ an\ economy$

$b_t = Productivity$

Thus,

$\frac{\partial f}{\partial x} = Productividad\ Marginal\ de\ la\ Mano\ de\ Obra$

$\frac{\partial f}{\partial y} = Productividad\ Marginal\ del\ Capital$

$$(2)\ \frac{\partial f}{\partial x} = Ay^{(1-b)}bX^{(b-1)} = Aby^{(1-b)}X^{(b-1)}$$

$$(3)\ \frac{\partial f}{\partial y} = AX^{(b)}(1-b)Y^{(1-b)-1)} = A(1-b)X^{(b)}Y^{(1-b)-1)}$$

The variable A can be replaced by: $\frac{i}{j}\ value\ from\ leontief\ equation$

$$(4)\ \frac{\partial f}{\partial x} = Ay^{(1-b)}bX^{(b-1)} = \frac{i}{j}by^{(1-b)}X^{(b-1)}$$

$$(5)\ \frac{\partial f}{\partial y} = AX^{(b)}(1-b)Y^{(1-b)-1)} = \frac{i}{j}(1-b)X^{(b)}Y^{(1-b)-1)}$$

In each variable we need to introduce the values to middle demand, final demand, total production, capital, technology, values to productivity, etc. with this data (future research) we can test the value of competitiveness and productivity and compare with the scale of this values measured under macro approaches. In future research we try to test with real values of the different sector of the Latin American economies.

Conclusion

The function of the productivity and the equation of Leontief allow through different inputs and outputs to analyze the measurement of competitiveness on the micro level ,considering different variables such profitable, technology, investment, performing a measurement sector with high level approach to reality of the economic sector under study.

The analysis of measurement of productivity involves the incorporation of technology, factor presented in Cobb Douglas´ productivity function. In addition , the variables that are directly associated with the equation of Cobb Douglas and Leontief are exogenous technical progress, capital stock and number of employees in an economy.

The interception of the equation previously mentioned is present on the technological variable, that is required to productivity in all levels, showing with this concept, that the Total Factor Productivity is directly proportional to the productivity and focuses through the analysis of inputs and outputs.

To realize the measurement of competitiveness and productivity in the micro level we considered the scales of; 1) Adaptability, 2) Business Services, 3) Management of limitation, 4) Environmental Analysis, 5) Human Resource Management, 6) continuous Improvement, 7) Information System and finally, 8) Lean and JIT scale contained in the measurement of the outputs/demands, quantity demands, sectorial productivity, exogenous factor, capital and the number of employees in the economic.

Emerging markets need to strengthen their production processes through the implementation of practices of world class manufacturing over prior analysis environment that facilitates adaptability with profitable results for the organization, leading to competitiveness and increase the participation in world markets.

Developers of global manufacturing standards need to identify environment variables on each region, in which manner theories, models and practices can be replicated at all scales or business models, this will expand the sector whose transformation processes of matter are made with the highest quality standards, without being located in industrialized countries. This will give the possibility to improve competitive, flexibility, innovation, quality, efficiency, decrease cost and increase cohesion labor.

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