Are the theoretical parameters of *(s,Q)* model enough robustness to be used in real world? An empirical analysis using simulation.

Carlos A. Castro-Zuluaga
Production Engineer Department, Universidad Eafit
Medellin, Colombia
ccastro@eafit.edu.co

Jaime A. Castro-Urrego
Production Engineer Department, Universidad Eafit
Medellín, Colombia
jcastrou@eafit.edu.co

Abstract
This paper shows a comparison between the expected results of costs and service levels when are used any values of the parameters of the inventory control policy *(ROP,Q)* and the "real" results that could be achieved with these values. The real world is represented by a simulation model.

Keywords: Inventory control, parameters definition, simulation

Introduction

Many companies use some inventory control system to planning production, to purchase products or raw materials and to manage inventory, both in manufacturing environments such as in service environments. One of these systems is reorder point (ROP), in which a fixed or variable quantity is ordered every time the inventory position drops to the reorder point or lower. The reorder point falls in the category of continuous review systems, so the inventory position must be known on real time. In this inventory policy, two issues should be resolved: (1) When should a replenishment order be placed? and (2) How large should the replenishment order be? (Silver et al. 1998). Figure 1 shows the operation of two well-known continuous review systems.

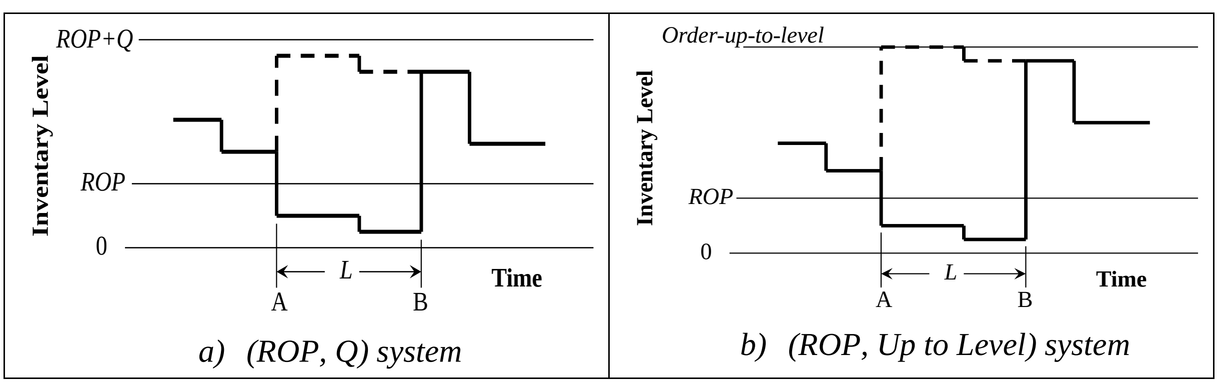

Figure 1 - Continuous Review Systems (Adapted from Silver et al., 1998)

In the system showed in Figure 1a, always is ordered a fixed quantity Q, while in the system illustrated in Figure 1b the quantity is variable because the inventory position must reach the order-up-to-level value, so the order quantity change every time that is necessary make an order. This paper is devoted only to the case 1a.

Often many inventory managers intuitively define the parameters of both systems, while some others estimated these parameters using statistical analysis, mathematical models and decision rules. It does not matter if the parameters were defined intuitive or technically, always exist the uncertainty about if with these values it will possible to achieve the service level and expected costs.

This paper try to find out to the case of an item with demand normally distributed if the expected values of costs and service levels are achieved when the parameters of the inventory policy *(ROP,Q)* are found technically from a required Cycle Service Level. The article is composed by 4 sections. This introduction is followed by the notation and the main assumptions used to define the parameters of the model. In section 3 there is a numerical example, which shows how calculate the parameters of the inventory policy *(ROP,Q)* as well as the expected values of the main inventory costs and service levels. A computer simulation is carried out to compare the expected performance with the "real" performance of the inventory policy. Finally, in section 4 there are some conclusions of the work done.

Notation and main assumptions

The notation to be used includes:

D	Expected annual demand
v	Unit variable cost
A	Fixed ordering cost
r	Annual Inventory carrying charge
B_1	Fixed cost per stockout occasion
EOQ	Economic Order Quantity
Q'	Any order quantity different to EOQ
ROP	Reorder Point
k	Safety factor
L	Replenishment Lead time
ss	Safety Stock
σ_l	Standard deviation of demand per unit of time
x_l	Expected demand per unit time
P_1	Cycle Service Level *(CSL)*
$p_{u\geq}(k)$	Probability that a unit normal variable takes a value greater or equal to k
P_2	Fill Rate
$G_u(k)$	The partial loss function of unit normal distribution
C_m	Material cost per unit time
C_o	Order cost per unit time

C_c	Carrying cost per unit time
C_s	Stock out cost per unit time
Q_i'	Quantity ordered at any period i in the simulation
m	Number of periods
n'	Number of orders in the horizon of the simulation
h	Number of cycles with negative stock
I_{aver}	Average inventory over the horizon of simulation
sou	Number of units in stock out in the horizon of simulation

It supposed that the demand is normally distributed, the lead-time is known and discrete and there is not variability on it (Zipkin, 2000). Both values of demand and standard deviation are known in advance. Top managers of the company define the desired value of CSL and the parameters of the inventory policy are established technically, this means that the quantity to order is an economic lot (*EOQ*) and the safe stock is found by a decision rule. Finally, we assume that the initial inventory for the simulation is the maximum quantity allowed by the model, that is the reorder point plus the economic order quantity.

The next section describes how to find technically the parameters of *(ROP, Q)* model for a CSL given. Likewise, the expected performance of the inventory policy is measure in terms of costs and fill rate. These values later will be compared with the results obtained of the "real world" through a simulation model.

Numerical example and results

With the purpose of show the robustness or not of the model, let us suppose an item with the information shown in Table 1

Table 1 – Basic data of the numerical example

	Value	**Unit of measure**
D	35000	units/year
v	25	$/unit
A	250	$/order
r	0,36	$/$/year
B_1	15000	$
L	30	days
σ_l	650	units/month
P_1	0,95	

The first step is to define the quantity to order every time position inventory drops under some value. Because the company wants to find an optimal quantity to the order, it is necessary use the following expression (Erlenkotter 1990).

$$EOQ = \sqrt{\frac{2AD}{vr}} \qquad (1)$$

Using the data of Table 1 in equation (1), the economic order quantity EOQ is 1394,43 units. Now, assuming there is not variability on lead-time, the standard deviation of errors of forecasting over a replenishment lead-time is 650 units, which is calculated using equation 2.

$$\sigma_L = \sigma_l \sqrt{L} \tag{2}$$

The safety factor should be calculated using the function available in Excel ®, which is shown in equation (3) (Chopra and Menindl, 2007)

$$k = NORM.S.INV(P_1) \tag{3}$$

For this case, for a CSL of 95%, the safety factor k is 1,644853627. The safety stock is defined as the average level of the net stock just before a replenishment arrives and it is calculated using the equation 4.

$$ss = k\sigma_L \tag{4}$$

Using the values calculated previously with equations 2 and 3, the appropriate value of safety stock for the numerical example is 1069 units. Finally, the reorder point must be set in 3986 units according with equation (5). This point is the expected demand during the lead-time (X_L) plus the safety stock.

$$ROP = X_L + ss \tag{5}$$

So, for an item with the data shown in Table 1, using an inventory control policy of reorder point with fixed order quantity, the theoretical inventory control parameters are (3986; 1394) in order to obtain An expected CSL of 95%. That is, each time the inventory position drops to 3986 or under, a fixed quantity of 1394 (or a multiple of this value) is ordered with aim to increase the inventory position above 3986 units.

To measure the expected performance of the model, formulae (6) to (9) were used to calculate the theoretical annual costs involved in inventory management of the item and formula (10) was used to calculate the fill rate estimated for this sku with an expected CSL of 95% (Silver et al. 1998). These results for the numerical example are show in the first row of Table 2.

$$C_m = Dv \tag{6}$$

$$C_o = \frac{D}{Q} A \tag{7}$$

$$C_c = \left(\frac{Q}{2} + ss\right) vr \tag{8}$$

$$C_s = \frac{D}{Q} p_{u\geq}(k) B_1 \tag{9}$$

$$P_2 = 1 - \frac{\sigma_L G_u(k)}{EOQ + \sigma_L G_u(k)} \tag{10}$$

The simulation was made in IMSS (Inventory Management Support System) a tool developed in VBA (Visual Basic for Applications) which generate normal demand randomly according to historical data of the demand and the standard deviation the sku to analyze. The "real" data of costs and service levels achieved with the implementation of the inventory management policy were calculated with formulae (11) to (16).

$$C_m = \sum_{i=1}^{m} Q_i' \, v \tag{11}$$

$$C_o = n'A \tag{12}$$

$$C_c = I_{aver} v r \tag{13}$$

$$C_s = h \, B_1 \tag{14}$$

$$P_1 = \frac{n' - h}{n'} \tag{15}$$

$$P_2 = \frac{sou}{\sum_{i=1}^{m} Q_i} \tag{16}$$

In order to define if theoretical parameters of the inventory model can obtain in practice the performance expected, we carry out three simulations. The first simulation was made using periods of one month and a time horizon of a year. Were made five runs, each one with a number statistically significant of iterations. The results of the runs made and the percentage of variation of the costs and service levels versus the expected values of the inventory policy is shown in Table 2.

Table 2 – Comparison between expected theoretical values and 5 simulation runs of (ROP,Q) inventory management model. Periods of 1month

	Expected Values	95,00%	99,04%	875.000 $	6.275 $	15.897 $	18.825 $	915.997 $
	Iteration	P1	P2	Cm	Co	Cc	Cs	TC
1	205	99,27%	99,96%	$ 855.536	$ 3.000	$ 16.494	$ 1.317	876.347 $
	% variation	4,5%	0,9%	-2,2%	-52,2%	3,8%	-93,0%	-4,3%
2	205	98,94%	99,92%	$ 856.216	$ 2.996	$ 16.565	$ 1.829	877.607 $
	% variation	4,2%	0,9%	-2,1%	-52,2%	4,2%	-90,3%	-4,2%
3	246	99,05%	99,93%	$ 865.427	$ 2.997	$ 16.183	$ 1.646	886.254 $
	% variation	4,3%	0,9%	-1,1%	-52,2%	1,8%	-91,3%	-3,2%
4	205	98,21%	99,85%	$ 863.018	$ 2.998	$ 16.247	$ 3.073	885.336 $
	% variation	3,4%	0,8%	-1,4%	-52,2%	2,2%	-83,7%	-3,3%
5	205	98,70%	99,89%	$ 864.039	$ 2.995	$ 16.127	$ 2.195	885.356 $
	% variation	3,9%	0,9%	-1,3%	-52,3%	1,4%	-88,3%	-3,3%

According to the above results, it is important to stand out that both C_o as C_s are the costs that have greater difference between the values yielded by the simulation and expected values. The main reason for this difference is that to calculate the expected values of both terms, the formula includes the ratio D/Q (that is the number of expected orders in a year). For the numerical example the number of orders expected is 25,1 orders/year, while in the simulation only can be a maximum of 12 orders (because there are 12 months). Therefore, there is an error near of 60% only for this reason.

The CSL achieved in the simulation was always higher (about 3,5% above the goal) than the expected value of 95%, it is again because the maximum number of orders could be only 12, so many of the orders release were of 2, 3 and until 4 times the economic order quantity, improving this measure.

The values of C_m simulated were near 1,5% below the expected value. On the other hand, the values of C_c were a 2,5% above the expected value. Something that explain in part this mismatch is the initial inventory value used in the simulation. For our case, this value was set in the higher level, so the quantity to purchase is lower and the quantity in inventory is greater than the expected quantities.

The results of the second simulation are shown in Table 3. In this simulation, the horizon is one year, but the period is a day. Again, we made 5 simulations and compared the results of them with the theoretical expected values.

Table 2 – Comparison between expected theoretical values and 5 simulation runs of (ROP,Q) inventory management model. Periods of 1day

	Expected Values	95,00%	99,04%	875.000 $	6.275 $	15.897 $	18.825 $	915.997 $
	Iteration	P1	P2	Cm	Co	Cc	Cs	TC
1	412	90,18%	98,14%	$ 985.580	$ 7.068	$ 13.877	$ 42.451	1.048.977 $
	% variation	-5,1%	-0,9%	12,6%	12,6%	-12,7%	125,5%	14,5%
2	434	89,45%	97,99%	$ 986.064	$ 7.071	$ 13.878	$ 45.760	1.052.773 $
	% variation	-5,8%	-1,1%	12,7%	12,7%	-12,7%	143,1%	14,9%
3	417	90,25%	98,13%	$ 983.389	$ 7.052	$ 13.966	$ 42.063	1.046.471 $
	% variation	-5,0%	-0,9%	12,4%	12,4%	-12,2%	123,4%	14,2%
4	427	89,24%	97,88%	$ 987.043	$ 7.078	$ 13.844	$ 46.475	1.054.441 $
	% variation	-6,1%	-1,2%	12,8%	12,8%	-12,9%	146,9%	15,1%
5	425	89,61%	98,06%	$ 984.552	$ 7.061	$ 13.878	$ 44.965	1.050.455 $
	% variation	-5,7%	-1,0%	12,5%	12,5%	-12,7%	138,9%	14,7%

In this case, for both C_o and C_s, the values obtained were above his respective expected values, but is significantly higher in C_s (near of 135% higher). The explanation for this is that the number of orders in the simulation was a 12,5% more than the expected orders. Because B_1 is a fixed cost that normally is high, these two values can explain in part the reported difference.

The number of orders also affect the CSL. An increase in the number of orders, make that exist more chance to have a stockout which affect the level service both in the cycle as in the fill rate. Now in simulation the number of orders increased because the demand also increases, from an expected value of 35,000 units/year, to near of 39,500 units/year (that is 12,5% plus).

Is important to notice that the standard deviation used in this simulation was estimated from the monthly deviation using equation (2). The monthly standard deviation is 650 units/month while daily standard deviation is 118,67 units/day (that means that one day represents a 18.25% of the monthly deviation). In this case when the standard deviation is higher than average demand, the performance of the model had a negative effect in our numerical example, because the demand increases in 12,5%.

Finally we made a third simulation with the same conditions than in simulation 2, but we change the value of the daily standard deviation from 118.67 to 97.22 (same as the average daily demand). As is shown in Table 3, the performance of the model presents a great improvement.

Table 3 – Comparison between expected theoretical values and 5 simulation runs of (ROP,Q) inventory management model. Periods of 1day. Standard deviation reduced

	Expected Values	95,00%	99,04%	875.000 $	6.275 $	15.897 $	18.825 $	915.997 $
	Iteration	P1	P2	Cm	Co	Cc	Cs	TC
1	520	96,45%	99,46%	$ 937.774	$ 6.725	$ 15.197	$ 14.701	$ 974.397
	% variation	1,5%	0,4%	7,2%	7,2%	-4,4%	-21,9%	6,4%
2	594	96,82%	99,55%	$ 931.031	$ 6.677	$ 15.435	$ 13.106	$ 966.248
	% variation	1,9%	0,5%	6,4%	6,4%	-2,9%	-30,4%	5,5%
3	347	96,99%	99,64%	$ 931.899	$ 6.683	$ 15.352	$ 12.363	$ 966.297
	% variation	2,1%	0,6%	6,5%	6,5%	-3,4%	-34,3%	5,5%
4	123	96,51%	99,51%	$ 934.440	$ 6.701	$ 15.293	$ 14.512	$ 970.947
	% variation	1,6%	0,5%	6,8%	6,8%	-3,8%	-22,9%	6,0%
5	323	96,36%	99,51%	$ 935.738	$ 6.711	$ 15.253	$ 15.046	$ 972.748
	% variation	1,4%	0,5%	6,9%	6,9%	-4,1%	-20,1%	6,2%

Conclusions and Future Work

Inventory control policies require of parameters to operate. For the inventory policy (*ROP*,*Q*) is necessary to define when to put an order and how much order any time than the inventory position drops under a determinate value.

One position almost generalized of inventory managers is that they do not believe in the theoretical models because these are based on unrealistic assumptions. Although this can be true, the other true is that these are remarkably robustness to deal with these “unrealistic assumptions”.

Of the three simulations made and its comparison of the values obtained in these with the expected theoretical values is important to stand out that the period of time used in the simulation have a significant effect in the percentage of variations, because the number of cycles can vary drastically, specifically when the relation $^{D}/_{Q}$ is higher than the number of periods in the time horizon. Two constraints must be considerate when the period used is one month. (1) the lead time must be greater than one month and (2) the lead times must be an integer.

Another factor that affect drastically the results is the standard deviation of the demand, mainly when the simulation uses periods of one day, mainly when his value exceeds the average daily demand. The consequence of that situation is that the number of units demanded and the number of orders increased considerably affecting the performance obtained by the model.

It is clear that inventory control does not work well when a high variability exist in data, specifically in the demand data. Most theoretical models assume that the demand is normally distributed, which is a strong assumption but according with the results of the simulations, an inventory manager can expect to have a good performance of a (*ROP*,*Q*) model in practice based on the expected values obtained by the theoretical model.

Although there is still a lot of resistance to the implementation of inventory management models based on calculated theoretical parameters, it is necessary to continue working in order to reduce the existing gap between theory and practice, in order to improve the competitiveness of the companies, in this case from inventory management.

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