

# Optimal Pricing for Reusable and Recyclable Products Using Nonlinear Physical Programming

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## Abstract

This research uses Physical Programming to solve a multi-criteria decision making problem to optimize the pricing policy of reusable and recyclable products to maximize their total profit and minimize their product recovery costs - including disposal cost, preparation cost, holding cost, disassembly cost, acquisition cost, and sorting cost.

**Keywords:** Pricing models, Multi-criteria decision making, and Product recovery

## Introduction and Related Work

With the rise of consumer awareness of environmental consciousness, the quantities of products discarded by customers are massively growing; they have both led to legislations that hold the original equipment manufacturers (OEM) responsible for their end-of-life products (Vadde *et al.*, 2006; Ilgin and Gupta, 2010). President Obama in his 2015 State of the Union address said "The best scientists in the world are all telling us that our activities are changing the climate" (Park, 2015). This has encouraged third-party firms to enter the market to exploit the economic opportunity in discarded products, which allows them to compete against the OEMs brand new products. The third-party firms, known as product recovery facilities (PRFs), are involved in collecting discarded products, implementing product recovery operations, and profiting through the sale of the recovered reusable and recyclable components in secondary markets (Vadde *et al.*, 2006). The challenges faced by PRFs are: (a) competition between OEMs and other PRFs; (b) requirement of costly, skilled workers for product recovery operations; (c) changes in environmental regulations; (d) uncertainty of the arrival time and the quantity for the discarded products; (e) recovered components inventory levels; and (e) promotional, markdowns, sales, and clearance price discounts to clear inventory (Vadde *et al.*, 2006; Vadde *et al.*, 2010)

Guide and Jayaraman (2000) conducted a survey and found that, to reduce the uncertainty of return quantity and quality, many remanufacturing firms in the United States have adopted what is called market-driven product acquisition management approach to collect used products. For example, Green Citizen Company buys back apple laptops, desktops, iPhones, & iPads. ecoATM is the world's first automated eWaste recycling station with instant cash for the responsible recycling of old cell phones, MP3 players and tablets. As of July 2014, ecoATM had approximately 1,100 kiosks located in shopping malls and select large retailers nationwide (Anonymous, 2014). Kodak controls the return quantity of used cameras by using cash

incentives to motivate people (Ayres and Ayres 2002). Dell Computers dynamically adjusts the price over time based on inventory levels of the exact same product on their website over time to improve their supply chain efficiencies (Agrawal and Kambil 2000).

Pricing of remanufacturing products was not given any importance in the past but the scale and unique processes of transforming used and recycled products into “like-new” conditions, and recapturing the missing values that added to the products during manufacturing stage, have made the pricing an important subject of research (Atasu, Guide, & Wassenhove, 2010; Atasu, Sarvary, & Wassenhove, 2008).

According to Gray & Charter (2008), remanufacturing products can reduce the use of energy, materials, and cut production cost. Guide *et al.* (2003) consider several quality classes of used products for the acquisition price of discarded products and the prices of remanufactured products. Pricing in a duopoly between the OEM and the PEF was addressed by Majumder and Groenevelt (2001), and Ferrer and Swaminathan (2006).

An affective pricing model of reusable and recyclable products can address these challenges by managing the inventory levels under the said conditions (Vaddeet *et al.*, 2006). The present work determines a pricing policy for reusable and recyclable products in a multi-criteria environment when PRFs passively accept discarded products and proactively acquire as needed where the goal is, to maximize the revenue and minimize the product recovery costs. An empirical study is performed to investigate the effect of disposal cost, preparation cost, holding cost, disassembly cost, acquisition cost, and sorting cost. Although Vaddeet *et al.*, (2006) have addressed issues in a multi-criteria environment for PRFs with the pricing aspects using genetic algorithms, this work is probably the first of its kind in the literature to provide a pricing model using Nonlinear Physical Programming (NPP) to satisfy the multiple criteria.

Generally, optimization problems fit in one of two classifications: Blind or Physical optimization, where blind optimization situation happens when we wish to minimize (or maximize) a function subject to constraints, and we do not have any knowledge about the physical meaning of the objective function, constraints, or the decision variable. On the other hand, physical optimization takes place within the context of human beings making decisions that leads to the most satisfactory outcome (Messac *et al.*, 1996).

NPP is a multi-criteria decision making tool that allows the decision maker (DM) to express ideas in a more realistic fashion for each criteria of interest (Ilgin and Gupta, 2012; Messac, 2006; and Messac *et al.*, 1996). In NPP, the following four different soft classes are used to help express the DM preferences: smaller is better (1S), larger is better (2S), value is better (3S), and range is better (4S). The most significant advantage of using NPP is that no weights are needed to be specified for the criteria of evaluation. The DM only needs to specify a preference structure for each criterion, which has a more physical meaning to the DM than weight that is randomly assigned to the criterion (Ilgin and Gupta, 2012a; Ilgin and Gupta, 2012b; and Messac *et al.*, 1996).

## Nomenclature

These notations are quoted from (Vaddeet *et al.*, 2006), to work with the same example below data with this new unique a pricing model.

$C_D$  Total disposal cost.

$C_P$  Total preparation cost.

$C_H$  Total holding cost.

$C_A$  Total disassembly cost.

$C_Q$  Total acquisition cost.

$C_S$  Total sorting cost.

$P_N$  Net profit.

$n_r$  Number of unique reusable components in a discarded product.

$n'_r$  Number of unique recyclable components in a discarded product.

$n_d$  Number of unique disposable components in a discarded product.

$m_{ri}$  Multiplicity of reusable component  $i$ .

$m'_{ri}$  Multiplicity of recyclable component  $i$ .

$m_{di}$  Multiplicity of disposable component  $i$ .

$w_{ri}$  Weight of reusable component  $i$ .

$w'_{ri}$  Weight of recyclable component  $i$ .

$w_{di}$  Weight of disposable component  $i$ .

$w_p$  Weight of discarded product.

$p_{ri}$  Selling price of remanufactured component  $i$  (\$/unit).

$p'_{ri}$  Selling price of high quality recyclable component  $i$  (\$/lb).

$p_{ai}$  Selling price of high grade as-is reusable component  $i$  (\$/unit).

$p_{si}$  Price of scrap grade reusable component  $i$  (\$/lb).

$p'_{si}$  Price of scrap quality recyclable component  $i$  (\$/lb).

$p_p$  Price of discarded product (\$/lb).

$\lambda_{ri}$  Demand for remanufactured component  $i$ .

$\lambda'_{ri}$  Demand for high quality recyclable component  $i$ .

$\lambda_{ai}$  Demand for high grade as-is reusable component  $i$ .

$\lambda_{si}$  Demand for scrap grade reusable component  $i$ .

$\lambda'_{si}$  Demand for scrap quality recyclable component  $i$ .

$\lambda_p$  Demand for damaged discarded products.

$B$  Yield of sorting process.

$\gamma_{ri}$  Yield of high grade reusable component  $i$ .

$\gamma'_{ri}$  Yield of high quality recyclable component  $i$ .

$\theta_{ri}$  Yield of remanufacturable quality reusable component  $i$ .

$R_q$  Quantity of proactively acquired returns.

$R_p$  Quantity of passively accepted returns.

$C_s$  Cost to sort a discarded product.

$C_r$  Cost to disassemble a product.

$C_q$  Cost to acquire a discarded product (acquisition price) (\$/unit).

$C_{pi}$  Cost to remanufacture high grade reusable component  $i$ .

$C'_{pi}$  Cost to prepare (such as crushing) high quality recyclable component  $i$ .

$C_{ai}$  Cost to prepare high grade reusable component  $i$  for as-is sale.

$C_{hi}$  Holding cost for high grade reusable component  $i$ .

$C'_{hi}$  Holding cost for high quality recyclable component  $i$ .

$C_{di}$  Cost to dispose reusable component  $i$ .

$C'_{di}$  Cost to dispose recyclable component  $i$ .

$C_{ddi}$  Cost to dispose the disposable component  $i$ .

$C_{dp}$  Cost to dispose the discarded product.

$C_{oi}$  Penalty cost to dispose reusable component  $i$ .

$C'_{oi}$  Penalty cost to dispose recyclable component  $i$ .

$C_{odi}$  Penalty cost to dispose the disposable component  $i$ .

$C_{op}$  Penalty cost to dispose the discarded product.

$D_{ri}$  Disposal limit for reusable component  $i$ .

$D'_{ri}$  Disposal limit for recyclable component  $i$ .

$D_{di}$  Disposal limit for disposable component  $i$ .

$D_p$  Disposal limit for damaged discarded products.

## Problem formulation

The following four different classes are used to help in categorizing the reusable and recyclable components (Vaddeet al., 2006):

1. *High grade reusable components:*
  - a. First grade which have more economic value,
  - b. Second grade which sold in as-is condition.
2. *Scrap grade reusable components*
3. *High quality recyclable components*
4. *Scrap quality recyclable components*

To obtain an effective economic model the PRF prefers to sell the remanufactured components by the order of (Vaddeet al., 2006):

1. high grade as-is reusable,
2. high quality recyclable,
3. scrap grade reusable,
4. scrap quality recyclable.

After the selling period, all the reusable and recyclable leftover scrap components are disposed. The disposal regulation assigns penalty if the quantity exceeds the restricted limit.

Soft Classes:

**Class-1S: Smaller-Is-Better – Min:**

*These are basically the cost criteria*

Total Disposal Costs (g1):

$g1$  of product  $i$  is calculated by multiplying the component disposal cost by the number of component units disposed times the penalty cost to dispose the component.

$$g1 = \sum_{i=1}^{nr} C_{di} + \sum_{i=1}^{n'r} d'_{di} + \sum_{i=1}^{nd} c_{ddi} \quad (1)$$

Total Preparation Costs (g2):

$g2$  of product  $i$  is the summation of the cost of:

1. remanufacture high grade reusable component  $i$ ,
2. prepare high quality recyclable component  $i$ ,
3. prepare high grade reusable component  $i$  for as-is sale.

$$g2 = \sum_{i=1}^{nr} C_{pi} + \sum_{i=1}^{n'r} C'_{pi} + \sum_{i=1}^{nd} C_{xi} \quad (2)$$

Total Holding Costs (g3):

g3 of product i is the summation of both holding cost for high grade reusable component and high quality recyclable component.

$$g3 = \sum_{i=1}^{nr} Chi + \sum_{i=1}^{n'r} C'hi \quad (3)$$

Total Disassembly Costs (g4):

g4 of product i is calculated by multiplying the cost to disassemble a product by the sum of quantity of proactively acquired returns and the quantity of passively accepted returns.

$$g4 = Cr(\beta Rp + Rq) \quad (4)$$

Total Acquisition Costs (g5):

g5 of product i is calculated by multiplying the cost to acquire a discarded product by the quantity of proactively acquired returns.

$$g5 = CqRq \quad (5)$$

Total Sorting Costs (g6):

g6 of product i is calculated by multiplying the cost to sort a discarded product by the quantity of passively accepted returns.

$$g6 = CsRp \quad (6)$$

## Class-2S: Larger-Is-Better – Max:

Total Revenue (g7)

*These are basically the revenue criteria*

g7 is the summation of: the selling price of remanufactured component i (\$/unit), selling price of high quality recyclable component i (\$/lb), selling price of high grade as-is reusable component i (\$/unit), price of scrap grade reusable component i (\$/lb), price of scrap quality recyclable component i (\$/lb), and the price of discarded product (\$/lb).

$$g7 = \sum_{i=1}^{nr} Pri Qri + \sum_{i=1}^{n'r} P'ri Q'ri + \sum_{i=1}^{nr} Pxi Ari + \sum_{i=1}^{nr} Psi Fri + \sum_{i=1}^{n'r} P'si F'ri + PpJ \quad (7)$$

## Goal constraints

$$h_1 \leq RETMAX \quad (\text{Retrieval cost is not more than maximum allowed value}) \quad (8)$$

$$g_p - d^+_{pr} \leq t^+_{p(r-1)} \quad (\text{Deviation is measured from corresponding target value}) \quad (9)$$

$$g_p \leq t^+_{p5} \quad (\text{Criterion value is in acceptable range}) \quad (10)$$

$$d^+_{pr} \geq 0 \quad (\text{Deviation is nonnegative number}) \quad (11)$$

## System Constraints

Hard Classes:

**Class-1H Must be smaller, i.e.,  $g_p \leq t_{p,max}$**

It has six constraints: the quantity of proactively acquired returns and quantity of passively accepted returns larger or equal to the six types of demands:

1. Demand for remanufactured component i.
2. Demand for high quality recyclable component i.
3. Demand for high grade as-is reusable component i.
4. Demand for scrap grade reusable component i.
5. Demand for scrap quality recyclable component i.
6. Demand for damaged discarded products.

**Class-2H Must be larger, i.e.,  $g_p \geq t_{p,min}$**

It has one constrain, which is, all the demands for discarded products are positive.

## NPP Problem Model

MINIMIZE  $J = \log_{10} \{ 1/n_{sc} \sum z_i [\mu_i(x)] \}$  (for soft classes)

Subject to

$\mu_i(x) \leq t_{i,5}^+$  (for class 1S objectives)

$\mu_i(x) \geq t_{i,5}^-$  (for class 2S objective)

$\mu_j(x) \leq t_{j,max}$  (for class 1H objectives)

$\mu_j(x) \geq t_{j,min}$  (for class 2H objectives)

$x_{j,min} \leq x_j \leq x_{j,max}$  (for design variables), where  $t_{i,max}$ ,  $t_{i,min}$ , and  $t_{i,val}$  = specified

preferences values for the  $i$ th hard objective;  $t_{i,min}$  and  $t_{i,max}$  = minimum and maximum values, respectively, for  $x_j$ ; ranges of desirability,  $t_{i,5}^+$  and  $t_{i,5}^-$ , are provided by the designer; and  $n_{sc}$  = number of soft objectives. In the above formulation, hard classes are treated as constraints and soft classes are part of the objective function (Messac, 2006).

Table 1- Preference ranges for  $\mu$

Highly Desirable	< 89
Desirable	89-70
Tolerable	70-51
Undesirable	51-30
Highly undesirable	30-19
Unacceptable	> 19

## Numerical example

For lack of access to real-world DM data, this paper uses data used by Vaddeet *et al.* (2006) trying to simulate the real-world environments as closely as possible, and this data were solved by Genetic Algorithms. However, letting the DM involves in the decision with the Physical

Programming (PP) mechanism, the results will be more realistic and desirable to the DM. Consider that a PRF is processing discarded PCs with configuration and data cost shown in Table 2 and Table 3. The weights were obtained by using Matlab Code for PP algorithm (Messac, 2006), based on the given DM preference ranges for  $\mu$  on Table 1. Let the data for the PC be,  $wp = 5.95$  lb,  $\beta = 0.8$ ,  $Cs = \$9$ ,  $Cr = \$15$ ,  $Cdp = \$15$ ,  $Cop = \$12$ ,  $Dp = 300$  lb,  $Rp = 20$ ,  $Rq = 7Cq$ ,  $\lambda p = 30 - 2.4pp$ . Linear demand functions are assumed for the case example:  $\lambda r1 = 125 - 1.2pr1$ ,  $\lambda r2 = 120 - 2.4pr2$ ,  $\lambda a1 = 70 - 2.5pa1$ ,  $\lambda a2 = 65 - 3.2pa2$ ,  $\lambda s1 = 18 - 5.2ps1$ ,  $\lambda s2 = 19 - 4.1ps2$ ,  $\lambda' r1 = 80 - 4.6p'r1$ ,  $\lambda' r2 = 90 - 2.1p'r2$ ,  $\lambda' r3 = 125 - 5.3 p'r3$ ,  $\lambda' r4 = 110 - 8.5p'r4$ ,  $\lambda' r5 = 105 - 3.5p'r5$ ,  $\lambda' x1 = 18 - 2.2 p's1$ ,  $\lambda' x2 = 12 - 1.9 p's2$ ,  $\lambda' x3 = 19 - 3.7 p's3$ ,  $\lambda' x4 = 11 - 4.5 p's4$ ,  $\lambda' x5 = 17 - 3.5 p's5$ ,  $\lambda p = 30 - 2.5 Pp$  (Vadde *et al.*, 2006).

Table 2-Product configuration

Index (i)	Component	Multiplicity	Weight	Yield	Yield	Disposal
<b>(Recycle)</b>						<b>Limit (lb)</b>
1	LCD 12.1"	1	1.10	0.85	n/a	26
2	Chassis	1	0.68	0.95	n/a	38
3	128MB RAM	1	0.05	0.70	n/a	25
4	64MB RAM	1	0.02	0.80	n/a	20
5	1.44MB FD	1	0.68	0.75	n/a	19
<b>(Reuse)</b>						
1	24x CD-ROM	1	0.90	0.90	0.50	50
2	10GB HD	2	1.30	0.70	0.60	90
<b>(Dispose)</b>						
1	150MHz Processor	1	0.40	n/a	n/a	120

Table 3 - Cost data

Preparation		As-Is	Holding		Disposal	Disposal
Cost		Cost	Cost		Cost	Penalty
7	n/a		1.02		8	9
9	n/a		1.01		9	6
8	n/a		0.95		7	4
9	n/a		1.03		7	6
8	n/a		1.04		7	7
12	3		1.05		6	8
8	5		1.04		9	6
n/a	n/a		n/a		10	14

## Results

The overall profit is \$ 1187.89; price to purchase a PC (g5) = \$3.58, number of proactively acquired returns (Rq) = 25.09 units, other results obtained from executing the NPP algorithm are listed in Table 4.

Table 4- Results obtained from executing the NPP algorithm

Component	Price			Inventory		
	High grade/ quality(\$/lb)	Scrap grade/ Quality(\$/lb)	As-Is (\$)	High grade/ quality(lb)	As-Is (units)	Disposed (lb)
LCD 12.1"	9.52	6.15	n/a	2.10	n/a	0.96
Chassis	31.52	6.23	n/a	0.70	n/a	0.57
128MB RAM	24.08	5.94	n/a	0.32	n/a	0.62
64MB RAM	12.57	3.02	n/a	0.73	n/a	0.30
1.44MB FD	24.81	4.07	n/a	0.21	n/a	3.72
24x CD-ROM	90.42	3.13	n/a	0.42	0	0.14
10GB HD	40.03	0.76	21.36	2.71	2.10	14.50
150MHz Processor	n/a	n/a	14.09	n/a	n/a	17.04
Computer	n/a	4.02	n/a	n/a	n/a	0.30

## Conclusion

The NPP model was presented for identifying the return flow of discarded products and the demand for reusable and recyclable components. The multi-criteria decision making problem is solved to optimize the pricing policy of reusable and recyclable products to maximize their total profit and minimize their product recovery costs.

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