Stochastic Capacitated Lot Sizing Subject to Maximum Acceptable Risk Level of Overutilization

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Abstract

Stochastic capacitated lot sizing problem is considered in the presence of probabilistic processing times and demand in this paper. A two-step hierarchical methodology is developed. First, stochastic capacity requirements are determined with statistical analysis & Monte-Carlo simulation. Second, a stochastic nonlinear mixed integer mathematical model is developed to solve the problem.

Keywords: lot sizing, stochastic programming, capacity requirements

Introduction

The goal of the lot sizing models is to determine the optimal timing and level of production (Süer et al. 2011). Production planning problems are often classified according to the hierarchical framework of strategic, tactical and operational decision making activities (Bitran and Tirupati 1993). Depending on the decision horizon and level of aggregation, lot sizing models are usually classified as; (1) tactical models (yearly master production schedule), (2) operational models (sequencing and loading), and (3) models between operational and tactical models (monthly lot sizing).

The simplest problem in lot sizing is the single item uncapacitated lot sizing problem. In this model only one item is considered for lot sizing for manufacturing with limitless capacity. However, this does not reflect the exact situation in almost all manufacturing facilities. Companies have limited capacities and mostly they produce more than one product. Any realistic model has to take this into account.

Capacitated Multi-Item Lot Sizing Problem considers production of multi items with limited production capacity (Süer et al. 2008). In Capacitated Multi-Item Lot Sizing Problem, generally, the objective is to minimize the total cost of production, set up and inventory. Demand is met from the production in the current period or the inventory left over from the previous period. Any excess is carried over as inventory to the next period. In each period, set up is required if anything will be produced.

Background and research motivation

Capacitated lot sizing is one of the widely studied problems in production planning and control literature. Researchers work on the models that solve one part of the lot sizing area. Many of the studies in the literature focus on the impact set up times (Armentano et al. 1999, Diaby et al. 1992, Gopalakrishnan et al. 2001, Manne 1958, Newson 1975, Trigeiro et al. 1989). Additionally, some works focus on the inventory management aspects of lot sizing (Erenguc and Aksoy 1990, Gutiérrez et al. 2003, Jaruphongsa et al. 2004, Sandbothe and Thompson 1993). Furthermore, demand aspect of the lot sizing is another area that has been widely studied. (Aksen et al. 2003, Federgruen and Tzur 1993, Jans and Degraeve 2004, Hsu and Lowe 2001, Hsu et al. 2005). In fact, the literature is abundant with works that address various aspects of the lot sizing area including the production and time horizon.

To reflect the inherent uncertainty exists in many production environments, stochastic lot sizing problem has been studied by many researchers and it is getting more attention recently. To name a few, Tarim and Kingsman (2004) proposed a mixed integer model with the objective of minimizing total cost of production, inventory holding and ordering costs for stochastic multi-period single item inventory lot sizing problem under the "static-uncertainty" strategy of Bookbinder and Tan (1988) where lot sizes and production periods are fixed in advance. Koca et al. (2015) studied lot sizing problem where demand is uncertain and processing times can be reduced by adjusting machine speed, number of operators assigned to jobs, outsourcing and some other factors. Tempelmeier and Hilger (2015) developed linear programming models and proposed a modified Fix-and-Optimize heuristic to solve the stochastic dynamic lot sizing problem according to "static-uncertainty" strategy of Bookbinder and Tan (1988). A review of studies in this area is presented by Glock et al. (2014). In this study, a stochastic capacitated lot sizing problem is considered in the presence of probabilistic processing times and demand. The goals of this study are;

- To visualize the impact of stochasticity on the optimal lot size values by comparing the results with those of deterministic model (total cost, shortages, on hand inventory)
- To enable production planners capture the variability in manufacturing system as a result of uncertainty in demand and processing times
- To enable decision makers decide their own risk level of overutilization which can be determined prior to planning

Methodology

Mathematical modeling and statistical analysis is used in a two-step hierarchical methodology in order to handle the uncertainty in processing times and demand. First, stochastic capacity requirements are determined with statistical analysis and Monte-Carlo simulation. In the second phase, a proposed stochastic nonlinear mathematical model is developed to solve lot sizing problem subject to a risk level. In this regard, risk level is introduced as a decision making parameter for production planner, integrated into the model as the maximum threshold for overutilization probability. Various risk levels and problem sizes are experimented. The proposed approach is compared with the deterministic model's results to visualize the impact of stochasticity on the optimal lot size values.

Deterministic capacity requirements are calculated by simply multiplying the demand and the processing time. Stochastic capacity requirements, however, require statistical analysis to find probability distribution functions of the capacity requirements since demand and processing times are probabilistic, which are assumed to follow normal distribution. Arena Input Analyzer software is used to statistically analyze and find the fitted distributions of the product of these two probabilistic variables. (See Egilmez et al., 2012 for detailed information about the statistical analysis).

Deterministic and stochastic capacitated multi-item lot sizing models

In a deterministic case, annual demand, processing times and therefore capacity requirements are known exactly. The notation, i.e., the parameters, the decision variables and indices provided below are used for both deterministic and stochastic capacitated multi-item lot sizing models.

Indices:
i: product index
j: period index

Parameters:
M: Big number
N: Number of products
T: Number of periods
A: Set up cost
p: Shortage cost
k: inventory carrying cost
b: production cost
C_j: System capacity on period j
R: Risk level
$PrOU_j$: Probability of overutilization on period j
mPT_i: Expected processing time of product i
σ_{PT_i}: Standard deviation of processing time for product i

Decision Variables:
Y_{ij}: 1 if an order placed for product i on period j, 0 otherwise
X_{ij}: Optimal lot size for product i on period j
S_{ij}: Inventory level for product i on period j
ST_{ij}: Amount of shortage for product i on period j

Objective function:

$$\min Z = A * Y_{ij} + b * X_{ij} + k * S_{ij} + p * ST_{ij} \tag{1}$$

Subject to:

$$S_{i(j-1)} + X_{ij} + ST_{ij} = D_{ij} + S_{ij} \qquad \forall i \in N \;\; \forall j \in T \qquad (2)$$

$$X_{ij} \leq M * Y_{ij} \qquad \forall i \in N \;\; \forall j \in T \qquad (3)$$

$$\sum_{i=1}^{P} (X_{ij} * PT_i) \;\leq\; C_j \qquad (4)$$

$$X_{ij}, S_{ij} \geq 0, Y_{ij} \in \{0,1\} \qquad \forall i \in N \;\; \forall j \in T \qquad (5)$$

The objective of the model is to minimize the total of setup, production, inventory holding and shortage costs (Equation (1)). Equation (2) ensures that the demand is satisfied for each period and each product. Demand can be met from production in the current period and/or inventory left over from the previous period, where shortage is allowed. If the amount of products is more than the amount of demand in the current period, it is carried over as excess inventory to the next period. Equation (3) guarantees to have setup if any production is required for that period. Equation (4) limits the production up to the capacity. Equation (5) determines whether an order is placed for a product or not. It also guarantees that the production and inventory levels cannot be negative.

Deterministic capacity constraint, Equation (4), in CLSP model is converted to stochastic constraint since demand and processing times are probabilistic in the stochastic model. The objective function of the deterministic problem remains the same along with the equations (2), (3), and (5). Equations (6) and (7) are the stochastic constraints that prevent the probability of overutilization of the manufacturing facility to exceed the predetermined risk level. Various risk levels are investigated.

$$PrOU_j = p\left(Z_j \leq \frac{\left(\sum_{i=1}^{P} (X_{ij} * mPT_i) - C_j \right)}{\sqrt{\sum_{i=1}^{P} {X_{ij}}^2 * {\sigma_{PT_i}}^2}} \right) \qquad \forall i \in N \;\; \forall j \in T \qquad (6)$$

$$PrOU_j \leq R \qquad \forall j \in T \qquad (7)$$

Experimentation and results

In this section, data generation, experiments and the results of the experiments are explained. Data generation is explained in the first phase. Then parameters of the experiments are described. Finally, the results of the experiments are presented and analyzed in detail.

The mean of weekly demand data is generated from uniform distribution for various demand ranges of (10, 70), (10, 75) and (10, 80) for each product. The standard deviation of demand is assumed to be 25% of the mean.

Similarly, means of the processing times are generated via uniform distribution (1, 5). The standard deviation of processing times is assumed to randomly vary between 10% and 50% of the mean processing times for each product due to various difficulty levels of the products. Table 1 shows the randomly generated demand data and mean processing times between (10, 70) for five products and five days.

Table 1 – Sample demand and processing time data for demand data range (10, 70)

Demand data (10,70)	**Periods**					**mPT (min)**
Products	**1**	**2**	**3**	**4**	**5**	
1	12	30	51	46	11	3
2	17	34	21	49	17	1
3	50	30	29	15	27	3
4	22	24	45	21	66	4
5	70	68	54	38	63	1

The manufacturing facility is assumed to work 8 hours/day, e.g., 480 minutes/day. Small data set is used due to computational limitations in Lingo software. The experimentation is performed for 5 periods and 5 products. Three different levels of average utilizations of all periods are considered for both deterministic model and stochastic models in order to capture the behavior of the system in various capacity scenarios. In the first case, we made sure that the weekly capacity is, on average, more than enough to cover the weekly demand. In the second case, weekly capacity is barely enough to cover the demand on average. In the third case, on average, there is no way that weekly capacity can cover all the demand without outsourcing or shortage. The scenarios are listed below. The utilization levels are calculated using mean demand and mean processing times of the products. Table 2 shows daily and average utilization values along with daily capacity requirements.

- ✓ Demand Range (10, 70) → capacity requirement = 85 % (underutilized)
- ✓ Demand Range (10, 75) → capacity requirement = 97 %(~fully utilized)
- ✓ Demand Range (10, 80) → capacity requirement = 106 % (over utilized)

Table 2 – Daily capacity requirements and average utilization values

(10, 70)	Daily capacity requirements					
Products	Periods					Average
	1	2	3	4	5	
1	36	90	153	138	33	90
2	17	34	21	49	17	27.6
3	150	90	87	45	81	90.6
4	88	96	180	84	264	142.4
5	70	68	54	38	63	58.6
Total	361	378	495	354	458	409.2
Capacity Utilization	75%	79%	103%	74%	95%	**85%**

The maximum allowed probability of overutilization is called the risk level of overutilization. Six different risk levels, between 0.1% to 50%, are evaluated for the stochastic model. The risk levels evaluated are listed below;

- ✓ Risk level 1 = 0.1% (almost no risk for overutilization)
- ✓ Risk level 2 = 10% overutilization risk
- ✓ Risk level 3 = 20% overutilization risk
- ✓ Risk level 4 = 30% overutilization risk
- ✓ Risk level 5 = 40% overutilization risk
- ✓ Risk level 6 = 50% overutilization risk

Results

The results of the experiments performed are provided and discussed in detail in this section. Both deterministic capacitated multi-item lot sizing and the stochastic version of the model are run for the same data set with respect to three capacity requirements level. The results obtained from the models are provided and compared with respect to total cost, on-hand inventory, shortage, and over-utilization risk levels.

Table 3 – Total costs for deterministic and stochastic models for capacity requirement scenarios

Capacity requirement	**85%**	**97%**	**106%**
Risk Level	**Under-utilization ($)**	**Fully-utilization ($)**	**Over-utilization ($)**
Deterministic	4811.25	6820.75	7271.75
Risk = 0.001	4839	7828	9045
Risk = 0.10	4821	7252	7915
Risk = 0.20	4818	7123	7687
Risk = 0.30	4815	7030	7510
Risk = 0.40	4815	6946	7408
Risk = 0.50	4812	6856	7273

Table 3 summarizes the results of the deterministic and stochastic models. At the 85% requirement level, total cost stays almost flat since no shortage occurs in any scenario due to more than enough capacity to meet the demand. When the demand increases, thus capacity requirement increases, the total cost increases at all scenarios as expected. The model increasingly uses shortage as a tool to meet the increasing capacity requirements. Deterministic model produces better results at all cases since it does not consider variability in processing times, thus capacity requirements. At 50% overutilization risk, the stochastic models produce total cost values which are very close to the deterministic model total cost values. This means that the model acts like the deterministic model at 50% risk of overutilization.

When the capacity available is more than the capacity required, such as it is at 85% capacity requirement level, decreasing probability of overutilization has very little effect on the total cost values. However, when the capacity available is very close to or less than the capacity required, such as it is at 97% and 106% capacity requirement levels, decreasing probability of overutilization increases the total cost values significantly. The stochastic model does not allow the manufacturing system to pass its capacity beyond the allowed risk level. At 0.001 risk level, for example, there is 0.1% risk that the capacity required for any period can pass the capacity available. This minimized overutilization risk increases the cost significantly, but it also gives the decision maker a safer production plan for the planning periods.

The cost of the shortage is way higher than the costs of production, setup and inventory holding. Therefore, if there is a big difference in total cost between scenarios, the main driver of that difference would be the shortage cost. The model balances the production planning by allowing more shortages, as seen in Table 4, if the capacity required is more than capacity available for than the capacity available. At 85% capacity requirement level, no shortages occur for both of the deterministic and the stochastic models. At 97% and 106% capacity requirement levels, for 0.1% risk level, stochastic model produces a production plan that requires more shortages than the deterministic level.

Table 4 – Period-wise shortages for deterministic and stochastic models for 0.1% risk level

Capacity utilization	85% Cap. requirement		97% Cap. requirement		106% Cap. requirement	
Periods	Deterministic Model	Stochastic Model	Deterministic Model	Stochastic Model	Deterministic Model	Stochastic Model
P1	0	0	0	0	15	26
P2	0	0	14	30	0	17
P3	0	0	14	20	26	32
P4	0	0	0	0	0	3
P5	0	0	0	0	0	1
Total	**0**	**0**	**28**	**50**	**41**	**79**

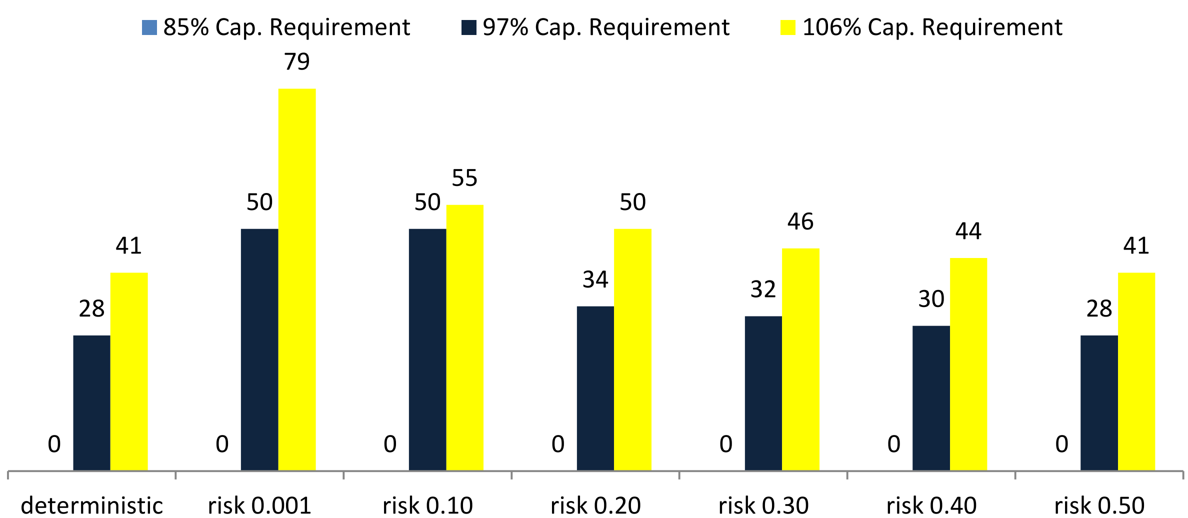

Figure 1 – Shortage levels for deterministic and stochastic models at various risk levels

Figure 1 shows the shortage levels for deterministic and stochastic models at various risk levels. No shortage occurs at 85% capacity requirement level for all of the models. As the risk of overutilization decreases, the shortage amounts increases at 97% and 106% capacity requirement levels. The deterministic model and the stochastic model with 50% risk level yield the same amount of shortage.

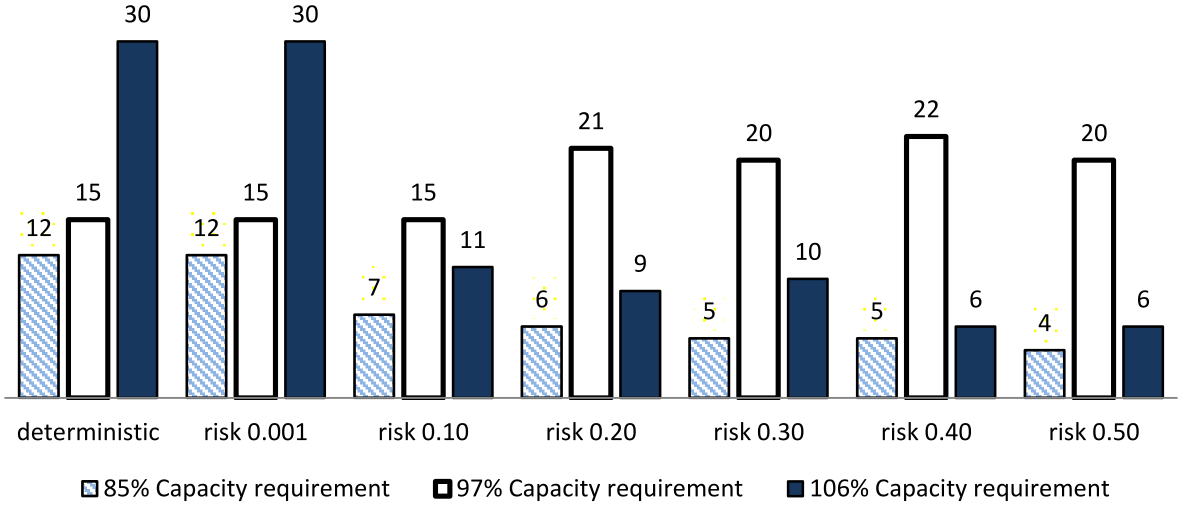

Figure 2 – Total on-hand inventory amounts

Figure 2 shows the total on-hand inventory levels for the investigated risk levels of stochastic model and the deterministic level. Deterministic inventory levels increase, as expected, with the increasing capacity requirement. However, stochastic model yields mixed results. At 85% capacity requirement level, total on-hand inventory amount increases as the risk of overutilization decreases. However, no trend is detected for 97% and 106% capacity requirement levels. The fluctuation is not an expected situation since the processing times are probabilistic, and more importantly the goal of the model is cost minimization and inventory holding cost is one of four cost components in the objective function.

Table 5 – Period-wise risk levels for stochastic model

Capacity requirement	85% Capacity requirement		97% Capacity requirement		106% Capacity requirement	
	Stoch. Risk	Stoch. Risk	Stoch. Risk	Stoch. Risk	Stoch. Risk	Stoch. Risk
Periods	50%	0.1%	50%	0.1%	50%	0.1%
P1	0%	0%	50%	0.09%	50%	0.09%
P2	0%	0%	50%	0.08%	50%	0.09%
P3	45%	0.06%	50%	0.10%	47%	0.08%
P4	0%	0%	0%	0%	0.37%	0.09%
P5	0.12%	0.02%	0%	0%	50%	0.10%

Table 5 shows risk levels of each period obtained from the results of stochastic lot sizing model. The results indicate that the model did not allow the overutilization risk level exceed the predetermined value. Other stochastic models reported similar results.

Conclusions

Deterministic lot sizing models do not catch real world variability as the models assume that the variables such as demand and processing times are exactly known and deterministic. Therefore deterministic models find the optimal solutions for perfect inputs. However, this is not usually the case in real world. Most of the variables that affect lot sizing decisions are uncertain. In this study, an attempt is made to capture this uncertainty by defining a new parameter called maximum allowable probability of overutilization of the capacity of the manufacturing system. A new stochastic multi-item lot sizing mathematical model is proposed in order to capture the variability in processing times and demand. Three capacity requirement levels are defined in order to evaluate the behavior of the model in loose, normal, and tight capacity levels. Six overutilization risk levels are proposed. The first maximum allowable probability of capacity overutilization is 0.1%, which means that almost no risk is taken. Then the overutilization risk increase incrementally to 50%.

Results show that when the capacity available is very close to or less than the capacity required, decreasing probability of overutilization increases the total cost values significantly. Similarly when the risk of overutilization decreases, the shortage amounts increases at 97% and 106% capacity requirement levels. The deterministic model and the stochastic model with 50% risk level yield the close total cost and amounts of shortage. Stochastic models allow the production planners to determine their own risk level of overutilization. Industry experts would know the variability in their production systems and in demand better.

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