Demand and cost disruptions and coordination of the supply chain under advertising level dependent demand

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Abstract
We investigate advertising level, pricing and production decision problems in a supply chain when demand and cost disruptions occur simultaneously. The channel consists of one manufacturer and one retailer where demand depends on retail price and advertisement expenditure. We examine the case in both cooperative game and non-cooperative Stackelberg game.

Keywords: Coordination mechanism, Disruption management, Advertising level

Introduction

Supply chain coordination has been the hotspot of research on supply chain management for a long time. Supply chain coordination simultaneously focused on maximizing total profits of the supply chain and individual profits of channel members. When each channel member makes decisions from their own perspective to maximize their own, the total supply chain profits have not been maximized which generates the well-known phenomenon: “double marginalization”. To avoid that problem, channel members choose to determine their decisions variables cooperatively to reach an optimal decision. On the one hand, coordination can expand the market to achieve the maximum supply chain profit. On the other hand, channel members get higher individual profits. In this paper, we introduce advertising level into customer demand and investigate how to coordinate a one-manufacturer-one-retailer supply chain after disruptions of demand and cost occur.

In today’s customer-oriented market, firms must provide products with high performance to attain and retain enough market demand, while advertising is the most efficient marketing method for managers to strengthen customer loyalty, maintain market share and boost sales. The impact of advertising level on the profits of the supply chain is direct, surplus advertising generates excessive costs, and insufficient advertising fails to earn an adequate profit, it is practical and significant to investigate the optimal advertising level in supply chain disruption management. Vertical cooperative (co-op) advertising is an arrangement whereby a manufacturer agrees to pay for a portion costs, referred as the “participation rate”, of advertising undertaken by a retailer (Bergen and John, 1997). Co-op advertising plays an important role in firms’ marketing programs.

Co-op advertising has attracted much attention in the academic field. Huang and Li (2001) examined the role of co-op advertising efficiency in three models including Nash, Stackelberg-manufacturer and cooperative game, Xie and Wei (2009) sought optimal cooperative advertising strategies and equilibrium pricing under cooperative and Stackelberg-manufacturer game, SeyedEsfahani et al. (2011) and Xie and Neyret (2009) discussed how to coordination pricing and advertising in Nash, Stackelberg-manufacturer and Stackelberg-retailer, and cooperative game. Nevertheless, they all assume the supply chain runs under a deterministic environment.

As the economic integration and globalization is deepening, the unexpected risks are more common as never before. The market demand and production cost are often disrupted unexpectedly due to the change of market environment, such as terrorist attack, economic policies adjustment, natural disasters, transportation delays, application of new technology, and so on. Therefore, the supply chain needs to be re-coordinated under the case of disruptions since the coordination scheme designed under the static case may become invalid. We focus on how to revise the original production plan under the disrupted environment in this paper.

Supply chain disruption management, first introduced by Clausen et al. (2001) in OR/MS Today, has gained much attention in the communities of academics and practitioners. Xu et al. (2003) showed how to effectively handle demand disruption variations under nonlinear demand functions. Qi et al. (2004) investigated an one-supplier–one-retailer supply chain with demand disruptions considering two kinds of deviation costs. Xu et al. (2006) studied the supply coordination problem with production cost disruptions. Chen and Xiao (2009) examined the supply chain with a dominant retailer after demand disruptions.

This paper is inspired by the scarcity of models that simultaneously considering co-op advertising and disruptions of both cost and demand in supply chain management. In the field of supply chain disruption management, we can only find a handful of studies extended the model to the case that demand and cost are disrupted simultaneously, e.g. Xiao and Qi (2008), Lei et al. (2012) and Cao et al. (2013). Furthermore, the optimal advertising level and production quantity can be greatly influenced by the demand and cost disruptions, and those decisions should raise managers' attention. However, co-op advertising has not been introduced into the study of the coordination problem of a disrupted supply chain in the existing literature. Only Chen and Zhuang (2011) studied the coordination mechanism combining demand-stimulating service with demand disruption. Differ from Chen and Zhuang (2011), we concern on co-op advertising, which is not widely defined as service level in Chen and Zhuang (2011), and also both demand and cost disruptions.

In this paper, we investigate the pricing and advertising decisions in a supply chain when the market scale and the production cost experience disruptions. The channel consists of one manufacturer and one retailer where demand depends on retail price and advertisement expenditure. We examine this case in both cooperative game and non-cooperative Stackelberg game. Our particular interest in this paper is the effects of disruptions of both demand and cost on the optimal production quantity, retail price, advertising level and participate rate. Our aim is to develop a supply chain coordination scheme for revising the initial production plan after disruptions occur, dealing with uncertainties of demand and cost in the planning stage.

The paper proceeds as follows: the next section presents the basic game-theoretic model structure and the baseline cases. Section 3 studies the coordination mechanism of a supply chain under disruptions. Finally, Section 4 provides conclusions.

Model Framework and Baseline Cases

Model framework

We consider a single-manufacturer-single-retailer supply chain in which the manufacturer sells his products through a single retailer and the retailer sells the manufacturer's product only. To illustrate the timeline of the model, we depict Figure 1. First, the channel members obtain the certain estimation of production cost and demand. Based on the estimation, an initial production plan is made. Then, the actual market demand and production cost are realized. Taking disruptions into consideration, the initial production plan has to be revised accordingly.

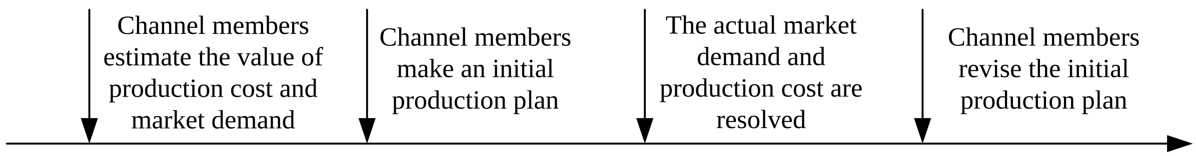

Figure 1 - Timeline of the model

Decision variables are retailer's retail price, retailer's advertising expenditure, manufacturer's wholesale price, and the manufacturer participation rate. The variable a denote the retailer's advertising expenditure, and p denote the retail price. The consumer demand $D(p,a)$ depends on the retail price p and the advertising level a as

$$D(p,a)=\gamma\sqrt{a}(\alpha-\beta p). \tag{1}$$

Similar with Xie and Wei (2009), we use a multiplicative effect by price and advertising to model consumer demand. $\gamma\sqrt{a}$ represents the impact of advertising on demand, where γ is an positive constant reflecting the effectiveness of advertising in generating sales. It has the property that additional advertising spending generates continuously diminishing returns, which is consistent with the "advertising saturation effect". Variables α and β are positive constants, α is the market scale without advertising and β is the sensitivity of price.

Denote by t , $0 \le t \le 1$, the manufacturer participation rate, and by w the wholesale price. c represents the manufacturer's unit production cost and is assumed to be a positive constant. The profits of the manufacturer, the retailer and the whole channel are as follows, respectively:

$$\Pi_m=\gamma\sqrt{a}(\alpha-\beta p)(w-c)-ta, \tag{2}$$

$$\Pi_r=\gamma\sqrt{a}(\alpha-\beta p)(p-w)-(1-t)a, \tag{3}$$

$$\Pi_t=\gamma\sqrt{a}(\alpha-\beta p)(p-c)-a. \tag{4}$$

Throughout this paper, subscripts "m", "r", and "t" represent the manufacturer, the retailer and the whole channel. To avoid the negativity of profit, we assume $\alpha-\beta c>0$.

Baseline cases

In this part, we analyze the initial production plan under no disruption, as a benchmark to study the impact of disruptions on the optimal decisions. First, we model the manufacturer-retailer relationship as a cooperative game structure in which both channel members agree to make decisions cooperatively. The decision problem of the supply chain is

$$Max \quad \prod_t (p,a) = \gamma\sqrt{a}(\alpha-\beta p)(p-c)-a. \tag{5}$$

Eq. (5) is a joint concave function in p and a , so we can get the optimal retail price, advertising level and production quantity, as well as the total profit of supply chain, as follows (the superscript "co" means the variables corresponding to the equilibrium under cooperation):

$$p^{co}=\frac{\alpha+\beta c}{2\beta}, a^{co}=\frac{\gamma^2(\alpha-\beta c)^4}{64\beta^2}, Q^{co}=\frac{\gamma^2(\alpha-\beta c)^3}{16\beta}, \prod_t^{co}=\frac{\gamma^2(\alpha-\beta c)^4}{64\beta^2}. \tag{6}$$

Then, we study the scenario in a Stackelberg-manufacturer supply chain when no disruption occurs, where the channel members make decisions independently to maximize their own profits. The manufacturer, as the leader, first decides the wholesale price and the manufacturer participation rate, while the retailer, as the follower, then sets its advertising level and retail price.

Given the manufacturer determining the wholesale price and participation rate, we solve the retailer's decision problem first. The optimal responsive pricing strategy of the retailer is given by $p(w,t)=(\alpha+\beta w)/2\beta$, and the optimal responsive advertising strategy of the retailer is given by $a(w,t)=\gamma^2(\alpha-\beta w)^4/64\beta^2(1-t)^2$. Substituting $p(w,t)$ and $a(w,t)$ into Eq.(2), the decision problem of manufacturer can be formulated as

$$Max \quad \prod_m (w,t) = (w-c)\frac{\gamma^2(\alpha-\beta w)^3}{16\beta(1-t)} - t\frac{\gamma^2(\alpha-\beta w)^4}{64\beta^2(1-t)^2}, \tag{7}$$

The above function is joint concave function in w and t . Solving the above decision problem, we can obtain the manufacturer's optimal wholesale price, participation rate, the retailer's retail price and advertising level as follows (the superscript "sm" means the variables corresponding to the Stackelberg equilibrium):

$$p^{sm}=\frac{2\alpha+\beta c}{3\beta}, a^{sm}=\frac{\gamma^2(\alpha-\beta c)^4}{144\beta^2}, w=\frac{\alpha+2\beta c}{3\beta}, t=\frac{1}{3}. \tag{8}$$

Therefore, the optimal production quantity, the optimal profits of the manufacturer, the retailer and the whole channel can be calculated as

$$Q^{sm}=\frac{\gamma^2(\alpha-\beta c)^3}{36\beta}, \prod_m^{sm}=\frac{\gamma^2(\alpha-\beta c)^4}{144\beta^2}, \prod_r^{sm}=\frac{\gamma^2(\alpha-\beta c)^4}{216\beta^2}, \prod_t^{sm}=\frac{5\gamma^2(\alpha-\beta c)^4}{432\beta^2}. \tag{9}$$

Disruptions of Demand and Cost

Modeling of disruptions

The initial production plan may be partially implemented before some uncertainty is resolved, since unexpected risks exist from the external environment to internal operational conditions. When the actual demand and production cost are resolved in this period, they may differ from the estimated value, and thus bring some deviation costs to the supply chain. Therefore, the initial production plan should be revised to handle the disruption risk in the supply chain.

We assume the disruptions of demand and cost are observable for all channel members. We use the notation with "$\bar{\ }$" to denote the case of demand and cost disruptions. $\bar{c} = c + \Delta c$ and $\bar{D} = \gamma\sqrt{\bar{a}}(\alpha + \Delta\alpha - \beta\bar{p})$ represent the real market demand and production cost after disruptions, respectively. After disruptions, there may generate a deviation cost for either producing some unplanned products or disposing of some unsold items. When actual production is larger than anticipated, manufacturer should increase production to satisfy the customers' demand and λ_1 denote the unit production cost associated with the additional demand. When actual production is less than anticipated, the channel members should dispose excess inventory in the form of final product or work in process at the cost of λ_2 per unit. Generally speaking, the manufacturer fully bears the production deviation cost.

The cooperative model under demand and cost disruptions

In this section, we analyze the cooperative relationship model under both demand and production cost disruptions. Given the optimal production quantity under the resolved cooperative relationship, the optimization problem can be described as follow:

$$Max\ \ \bar{\prod}_t(\bar{p},\bar{a}) = \gamma\sqrt{\bar{a}}(\alpha + \Delta\alpha - \beta\bar{p})(\bar{p} - c - \Delta c) - \bar{a} - \lambda_1(\bar{Q} - Q^{co})^+ - \lambda_2(Q^{co} - \bar{Q})^+. \tag{10}$$

The above decision problem is joint concave function in $\bar{p}$ and $\bar{a}$, so the problem has unique optimal solution. To obtain the optimal solution, we need the following lemma:

Lemma 1. *Let $\bar{Q}^{co}$ be the optimal production quantity of the cooperative relationship model with disruptions. Then $\bar{Q}^{co} \geq Q^{co}$ if $\Delta\alpha > \beta\Delta c$, and $\bar{Q}^{co} \leq Q^{co}$ if $\Delta\alpha < \beta\Delta c$.*

Lemma 1 implies that the change of production quantity depends on the relative distance between $\Delta\alpha$ and $\beta\Delta c$ when demand and cost are disrupted simultaneously. When $\Delta\alpha > \beta\Delta c$, the realized demand will exceed the original production quantity, the shortage cost will be induced by the disruptions. And when $\Delta\alpha < \beta\Delta c$, the realized demand will be less than the original production quantity, the disposal cost will be induced by the disruptions.

When $\Delta\alpha > \beta\Delta c$, from Lemma 1 we know $\bar{Q} \geq Q^{co}$ and there exists the shortage cost. The decision problem can be described as

$$\begin{aligned} &Max\ \ \overline{\prod}_t(\overline{p},\overline{a}) = \gamma\sqrt{\overline{a}}(\alpha+\Delta\alpha-\beta\overline{p})(\overline{p}-c-\Delta c)-\overline{a}-\lambda_1(\overline{Q}-Q^{co}) \\ &s.t.\quad \overline{Q}\ge Q^{co}. \end{aligned} \tag{11}$$

When $\Delta\alpha < \beta\Delta c$, from Lemma 1 we know $\overline{Q}\le Q^{co}$ and there exists the disposal cost. The decision problem can be described as

$$\begin{aligned} &Max\ \ \overline{\prod}_t(\overline{p},\overline{a}) = \gamma\sqrt{\overline{a}}(\alpha+\Delta\alpha-\beta\overline{p})(\overline{p}-c-\Delta c)-\overline{a}-\lambda_2(Q^{co}-\overline{Q}) \\ &s.t.\quad \overline{Q}\le Q^{co}. \end{aligned} \tag{12}$$

Combining the two scenarios above, we obtain Proposition 1.

Proposition 1. *In a cooperative model with disruptions* $\Delta\alpha$ *and* Δc *, the optimal retail price and advertising level are given by*

$$\overline{p}^{co} = \begin{cases} \dfrac{\alpha+\Delta\alpha+\beta(c+\Delta c+\lambda_1)}{2\beta} & \Delta\alpha-\beta\Delta c\ge\lambda_1\beta \\ \dfrac{\alpha+\beta c}{2\beta}+\dfrac{\Delta\alpha}{\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta \\ \dfrac{\alpha+\Delta\alpha+\beta(c+\Delta c-\lambda_2)}{2\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\le-\lambda_2\beta, \end{cases}$$

$$\overline{a}^{co} = \begin{cases} \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_1)]^4}{64\beta^2} & \Delta\alpha-\beta\Delta c\ge\lambda_1\beta \\ \dfrac{\gamma^2(\alpha-\beta c)^4}{64\beta^2} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta \\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_2)]^4}{64\beta^2} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\le-\lambda_2\beta, \end{cases}$$

the optimal production quantity and the supply chain's maximum profit are given by,

$$\overline{Q}^{co} = \begin{cases} \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_1)]^3}{16\beta} & \Delta\alpha-\beta\Delta c\ge\lambda_1\beta \\ \dfrac{\gamma^2(\alpha-\beta c)^3}{16\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta \\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_2)]^3}{16\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\le-\lambda_2\beta, \end{cases}$$

$$\overline{\Pi}_t^{co} = \begin{cases} \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_1)]^4}{64\beta^2}+\dfrac{\lambda_1\gamma^2(\alpha-\beta c)^3}{16\beta} & \Delta\alpha-\beta\Delta c\ge\lambda_1\beta \\ \dfrac{\gamma^2(\alpha-\beta c)^4}{64\beta^2}-\dfrac{\gamma^2(\alpha-\beta c)^3\Delta c}{16\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta \\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_2)]^4}{64\beta^2}-\dfrac{\lambda_2\gamma^2(\alpha-\beta c)^3}{16\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\le-\lambda_2\beta. \end{cases}$$

By comparing Proposition 1 with the cooperative equilibrium in the baseline case, we observe that when disruptions of both demand and cost occur, not only the original production quantity has some robustness, but also the original advertising level has some robustness. When the difference between the cost and the demand disruption is mildly, i.e. $-\lambda_2\beta < \Delta\alpha - \beta\Delta c < \lambda_1\beta$, the optimal production quantity and advertising level should be kept unchanged, the retail price need to be added an adjustment term $\Delta\alpha/\beta$ and has nothing with Δc. When the difference between the cost and the demand disruption is large enough, the original production quantity and advertising level should be changed accordingly. Specifically, the production quantity and advertising level should be increased if $\Delta\alpha - \beta\Delta c \geq \lambda_1\beta$, the production quantity and advertising level should be decreased if $-\lambda_2\beta - (\alpha - \beta c) < \Delta\alpha - \beta\Delta c \leq -\lambda_2\beta$.

The Stackelberg model under demand and cost disruptions

After discussing the cooperative relationship model, the question at this scenario is how to revise the original production plan in the leader-follower relationship model when disruptions of demand and cost occur. In a Stackelberg-manufacturer relationship model, we first solve the retailer's decision problem when the manufacturer's decision variables are given:

$$Max\ \overline{\prod}_r(\overline{p},\overline{a}) = \gamma\sqrt{\overline{a}}(\alpha + \Delta\alpha - \beta\overline{p})(\overline{p} - \overline{w}) - (1-\bar{t})\overline{a}. \tag{13}$$

By solving the two first order equations equal to zero, we can get the retailer's best response to the manufacturer's decision, as follows:

$$\overline{p}(\overline{w},\bar{t}) = \frac{\alpha + \Delta\alpha + \beta\overline{w}}{2\beta},\quad \overline{a}(\overline{w},\bar{t}) = \frac{\gamma^2(\alpha + \Delta\alpha - \beta\overline{w})^4}{64\beta^2(1-\bar{t})^2}. \tag{14}$$

Based on the retailer's optimal responsive strategy, we solve the manufacturer's decision problem which is described as:

$$\begin{aligned} &Max\ \overline{\prod}_m(\overline{w},\bar{t}) = \gamma\sqrt{\overline{a}}(\alpha + \Delta\alpha - \beta\overline{p})(\overline{w} - c - \Delta c) - \bar{t}\overline{a} - \lambda_1(\overline{Q} - Q^{sm})^+ - \lambda_2(Q^{sm} - \overline{Q})^+ \\ &s.t.\quad \overline{p} = \frac{\alpha + \Delta\alpha + \beta\overline{w}}{2\beta},\quad \overline{a} = \frac{\gamma^2(\alpha + \Delta\alpha - \beta\overline{w})^4}{64\beta^2(1-\bar{t})^2}. \end{aligned} \tag{15}$$

Substituting the constraints into the o function, we can simplify the manufacturer's optimization problem as

$$Max\ \overline{\prod}_m(\overline{w},\bar{t}) = (\overline{w} - c - \Delta c)\frac{\gamma^2(\alpha + \Delta\alpha - \beta\overline{w})^3}{16\beta(1-\bar{t})} - \frac{\gamma^2\bar{t}(\alpha + \Delta\alpha - \beta\overline{w})^4}{64\beta^2(1-\bar{t})^2} - \lambda_1(\overline{Q} - Q^{sm})^+ - \lambda_2(Q^{sm} - \overline{Q})^+. \tag{16}$$

It is easy to verify that the objective function above is joint concave in $\overline{w}$ and $\bar{t}$, so the problem above has unique optimal solution. Similar with the cooperative relationship model under disruptions, we can obtain the optimal solution of Eq. (16) and we describe it in the following proposition.

Proposition 2. *In a* Stackelberg *model with disruptions* $\Delta\alpha$ *and* Δc, *the optimal retail price, advertising level, wholesale price and participation rate are given by*

$$\bar{p}^{sm}=\begin{cases}\dfrac{2(\alpha+\Delta\alpha)+\beta(c+\Delta c+\lambda_1)}{3\beta} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{2\alpha+\beta c}{3\beta}+\dfrac{\Delta\alpha}{\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{2(\alpha+\Delta\alpha)+\beta(c+\Delta c-\lambda_2)}{3\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta,\end{cases}$$

$$\bar{a}^{sm}=\begin{cases}\dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_1)]^4}{144\beta^2} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{\gamma^2(\alpha-\beta c)^4}{144\beta^2} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_2)]^4}{144\beta^2} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta,\end{cases}$$

$$\bar{w}=\begin{cases}\dfrac{\alpha+\Delta\alpha+2\beta(c+\Delta c+\lambda_1)}{3\beta} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{\alpha+2\beta c}{3\beta}+\dfrac{\Delta\alpha}{\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{\alpha+\Delta\alpha+2\beta(c+\Delta c-\lambda_2)}{3\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta,\end{cases}$$

$$t=\frac{1}{3}.$$

The optimal production quantity and the profits of manufacturer and retailer are given by

$$\bar{Q}^{sm}=\begin{cases}\dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_1)]^3}{36\beta} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{\gamma^2(\alpha-\beta c)^3}{36\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_2)]^3}{36\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta,\end{cases}$$

$$\bar{\Pi}_m^{sm}=\begin{cases}\dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_1)]^4}{144\beta^2}+\dfrac{\lambda_1\gamma^2(\alpha-\beta c)^3}{36\beta} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{\gamma^2(\alpha-\beta c)^4}{144\beta^2}-\dfrac{\gamma^2(\alpha-\beta c)^3\Delta c}{36\beta} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_2)]^4}{144\beta^2}-\dfrac{\lambda_2\gamma^2(\alpha-\beta c)^3}{36\beta} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta,\end{cases}$$

$$\bar{\Pi}_r^{sm}=\begin{cases}\dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c+\lambda_1)]^4}{216\beta^2} & \Delta\alpha-\beta\Delta c\geq\lambda_1\beta\\ \dfrac{\gamma^2(\alpha-\beta c)^4}{144\beta^2} & -\lambda_2\beta<\Delta\alpha-\beta\Delta c<\lambda_1\beta\\ \dfrac{\gamma^2[\alpha+\Delta\alpha-\beta(c+\Delta c-\lambda_2)]^4}{216\beta^2} & -\lambda_2\beta-(\alpha-\beta c)<\Delta\alpha-\beta\Delta c\leq-\lambda_2\beta.\end{cases}$$

Proposition 2 indicates a similar coordination scheme that can be used to coordinate the supply chain. We can observe that the original plan also has some robustness in a Stackelberg-manufacturer relationship model when demand and cost are disrupted. When the disruption is mildly, $\Delta\alpha - \beta\Delta c \geq \lambda_1\beta$, no change in production quantity, advertising level and participation rate is required. The manufacturer adjusts the wholesale price and the retailer adjusts the retail price, both the adjustment amounts are $\Delta\alpha / \beta$, which also show the independence between adjustment amount and amount of changes in production cost. Only when the disruption exceeds some given thresholds will the supply chain members take an overall adjustment to optimize the maximum profit. To be specific, when $\Delta\alpha - \beta\Delta c \geq \lambda_1\beta$, the production quantity and advertising level should be increased, the production quantity and advertising level should be decreased when $-\lambda_2\beta - (\alpha - \beta c) < \Delta\alpha - \beta\Delta c \leq -\lambda_2\beta$. Moreover, the manufacturer participation rate has absolute robustness under disruptions. The introduction of participation rate makes the supply chain achieve a higher level of performance.

Conclusions

This paper complements the literature by investigating how to coordinate the supply chain with co-op advertising and how the disruptions of demand and cost affect the coordination mechanism. We develop a coordination scheme to revise the original production plan in response to the demand and production cost disruptions. It is found that the production quantity and advertising level exhibit some robustness under disruptions in both cooperative and Stackelberg game, while the optimal retail price does not. Nevertheless, when the disruption is below the threshold, the production quantity and advertising level should be increased. Otherwise, the production quantity and advertising level should be decreased. Moreover, we believe that there are still many interesting problems to study in this field, for example, the scenarios of dual-channels and competitive multiple retailers can be further researched.

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