

A heuristic approach for disassemble-to-order problem under binomial yields

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Abstract

In disassemble-to-order (DTO) systems randomness of recoverable parts gained from used products creates a major challenge for appropriate planning. Typically, it is assumed that yields from disassembly are either stochastically proportional (*SP*) or follow a binomial (*BI*) process. In the case of yield misspecification, it can be shown that the *BI* yield assumption usually results in a lower penalty than the *SP* yield assumption. For *BI* yield, however, a suitable, powerful heuristic is needed in order to facilitate DTO problem solving for complex real-world product structures. We present a heuristic approach that is based on a decomposition procedure for the underlying non-linear stochastic optimization problem and that can be applied to problems of arbitrary size. A numerical performance study reveals that this heuristic yields close-to-optimal results.

Keywords: Disassemble-to-order problem, Binomial yields, Heuristics

Introduction

Disassemble-to-order (DTO) systems are characterized by the determination of disassembly lot sizes in a way that the demand for remanufactured products can be met by the outcome of the disassembling and remanufacturing process. DTO systems require an adequate characterization of the yield loss process and, in particular, the distribution of the yield loss. The two most common forms of modeling yield uncertainty are stochastically proportional (*SP*) yield where the yield rate distribution is independent from the lot size and binomial (*BI*) yield where the outcome of a non-defective unit follows a Bernoulli process. Both modeling forms have their advantages and disadvantages. While *SP* yield allows to specify both mean and variance of the yield rate, the downside is the assumption that the yield rate is independent of the lot size. On the other hand, *BI*

yield only needs to specify a single parameter, however, it does not allow to specify the variance of the yield rate independently of the lot size. Yano and Lee (1995) provide a comprehensive review of stochastic yield formulations in lot sizing problems.

DTO problems with stochastic yields are in general complicated and difficult to solve to optimality. Hence, several heuristic approaches have been developed. Inderfurth and Langella (2003) provide the first contribution to DTO systems under *SP* yields. This work was motivated by the automotive industry where disassembling and remanufacturing of engines has gained particular importance as car manufacturers try to support their customers with spare parts as long as possible. Inderfurth and Langella (2006) extend the prior work by providing improved single period heuristics of varying complexity which perform very well. In this context, they have developed different procedures to decompose the complex original problem into smaller subproblems that are easier to handle. In the original DTO problem ``many" different parts can be obtained from disassembling ``many" different engine types (Many-to-Many product structure, MTM) where product part commonality plays an important role. The decomposition of the original problem leads first to One-to-One (OTO) relationships between parts engines and second to One-to-Many relationships (OTM), where many parts can be obtained from one engine. An extensive numerical study underlines the promising performance of the developed heuristics. Langella (2007) extends this analysis to multiple planning periods by providing and testing heuristics for the multi-period problem in addition to developing heuristics for deterministic yield cases with and without constraints on returned items. Inderfurth and Langella (2008) provide recourse model formulations for both the single and multiple period problem with stochastic yields and provide a glimpse into the computational complexity of models.

Preceding contributions have focused on DTO problems with complete disassembly under stochastically proportional yield assumption. To extend the existing work, Vogelgesang et al. (2012) just recently have addressed the case of binomial yield. Motivated by a data set from car engine disassembly, Vogelgesang et al. (2012) have analyzed which yield type results in a best fit to this data set. Thereby, it turned out that neither of the two yield types, *BI* and *SP* could definitely be rejected from explaining the empirical yield observations. Additionally, they have examined the impact of a misspecification of the underlying yield type and found that if the yield type is unknown, one should prefer the *BI* yield assumption. Since a full enumeration is necessary to determine the disassembly lot sizes for the specific product structure with many engines, parts and part commonality under *BI* yield, it is highly desirable to develop a well performing heuristic to accelerate the computation.

This paper proposes such a heuristic approach that is based on a decomposition procedure for the underlying non-linear stochastic optimization problem and that can be applied to problems of arbitrary size. A numerical performance study reveals that this heuristic yields close-to-optimal results.

Product Structure and Problem Description

DTO problems in practice often refer to situations with a two-digit number of cores (e.g. engine types) and leaves (e.g. parts of engines) showing a high degree of commonality. For sake of

simplicity, in this paper we consider a basic simple problem structure for which it is easy to explain our heuristic and to test its performance.

We consider a simplified remanufacturing structure where two returned products, 1 and 2 (for example engine types), are disassembled into three parts, A , B , and C . As the parts cannot be further disassembled, they are referred to as *leaves* while the returned products are referred to as *cores*. This problem structure is visualized in Figure 1. It is the simplest structure that contains all relevant aspects, i.e., multiplicity of cores as well as commonality and singularity of parts.

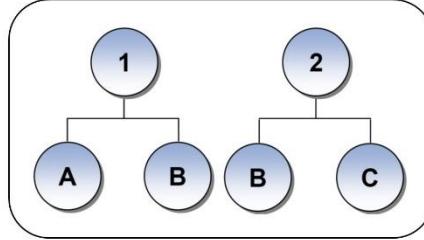


Figure 1: Product Structure (Many-to-Many Relationship)

One can see that part A and C are unique to cores 1 and 2, respectively, while part B is common to both cores.

For formal problem description we use the following notation:

- I : set of cores i ($i \in \{1, 2\}$)
- K : set of parts k ($k \in \{A, B, C\}$)
- D_k : total demand of part k
- c_i^z : disassembling cost per core type i
- c_k^p : external procurement cost for part k
- c_k^d : disposal cost for part k
- \hat{p}_{ik} : success probability for obtaining a reusable part k from engine type i
- Q_i : disassembly lot size of core type i
- $Y_{ik}(Q_i)$: random yield of part k that results from disassembling of Q_i
- C : total cost

We consider a single-period problem and assume that the demands of the three parts are known and that there is no restriction to the amount of engines that should be disassembled. These assumptions are driven by our experience from the automotive industry. Customers announce a demand for remanufactured parts and, if the outcome of the disassembling process is less than the demand, an external procurement of the missing parts is necessary. This results in a two-stage-decision process. At first, one has to decide how many of each engine type should be disassembled, taking into account that the disassembly process is characterized by uncertain yield due to different rates of wear. That results in costs c_i^z per engine type i . Second, after the total yield is known,

missing parts can be procured externally at cost c_k^p or an excess of parts has to be disposed of at costs c_k^d .

The firms objective is to minimize the expected total cost consisting of the disassembly cost of both cores and the expected external procurement and disposal cost in case of a possible lack or excess of parts after disassembly. The cost function can be formalized as follows:

$$C(Q_1, Q_2) = \sum_{i \in I} c_i^z Q_i + E_{Y_{ik}} \left[\sum_{k \in K} c_k^p \max \left\{ D_k - \sum_{i \in I} Y_{ik}(Q_i), 0 \right\} \right] + E_{Y_{ik}} \left[\sum_{k \in K} c_k^d \max \left\{ \sum_{i \in I} Y_{ik}(Q_i) - D_k, 0 \right\} \right] \quad (1)$$

The only restriction that needs to be considered in this problem formulation is the non-negativity of the decision variables ($Q_i \geq 0$). The problem of minimizing cost function (1) is a non-linear optimization problem under general yield functions $Y_{ik}(Q_i)$ does not necessarily possess an advantageous solution property like convexity. In particular, if yield is assumed to be of the *BI* type, it can be shown that the objective function is not necessarily convex. Under *BI* yield, the yield term $Y_{ik}(Q_i)$ is binomially distributed with success probability \hat{p}_{ik} . Thus, mean and variance of $Y_{ik}(Q_i)$ is \hat{p}_{ik} and $\hat{p}_{ik}(1 - \hat{p}_{ik})Q_i$, respectively. Using the probabilities for binomially distributed random variables, $Pr(Y_{ik}(Q_i) = x) = \binom{Q_i}{x} \hat{p}_{ik}^x (1 - \hat{p}_{ik})^{Q_i - x}$, the cost function in (1) can be reformulated as:

$$\begin{aligned} C(Q_1, Q_2) &= c_1^z Q_1 + c_2^z Q_2 \\ &+ c_A^p \sum_{x=0}^{\min\{D_A-1, Q_1\}} (D_A - x) \binom{Q_1}{x} \hat{p}_{1A}^x (1 - \hat{p}_{1A})^{Q_1 - x} + c_A^d \sum_{x=D_A+1}^{Q_1} (x - D_A) \binom{Q_1}{x} \hat{p}_{1A}^x (1 - \hat{p}_{1A})^{Q_1 - x} \\ &+ c_C^p \sum_{x=0}^{\min\{D_C-1, Q_2\}} (D_C - x) \binom{Q_2}{x} \hat{p}_{2C}^x (1 - \hat{p}_{2C})^{Q_2 - x} + c_A^d \sum_{x=D_C+1}^{Q_2} (x - D_C) \binom{Q_2}{x} \hat{p}_{2C}^x (1 - \hat{p}_{2C})^{Q_2 - x} \\ &+ c_B^p \sum_{x=0}^{\min\{D_B-1, Q_1+Q_2\}} (D_B - x) \sum_{j=\max\{x-Q_2, 0\}}^{\min\{x, Q_1\}} \binom{Q_1}{j} \hat{p}_{1B}^j (1 - \hat{p}_{1B})^{Q_1 - j} \binom{Q_2}{x-j} \hat{p}_{2B}^{x-j} (1 - \hat{p}_{2B})^{Q_2 - x+j} \\ &+ c_B^d \sum_{x=D_B-1}^{Q_1+Q_2} (x - D_B) \sum_{j=\max\{x-Q_2, 0\}}^{\min\{x, Q_1\}} \binom{Q_1}{j} \hat{p}_{1B}^j (1 - \hat{p}_{1B})^{Q_1 - j} \binom{Q_2}{x-j} \hat{p}_{2B}^{x-j} (1 - \hat{p}_{2B})^{Q_2 - x+j} \end{aligned} \quad (2)$$

A determination of values Q_1^* and Q_2^* that minimize $C(Q_1, Q_2)$ via full enumeration needs a very high computational effort, especially if demands for the parts *A* *B* and *C* are very high. Moreover, an extension of the product structure (more engines, more common parts) will increase the effort considerably so that it is essential to develop a simple, but powerful heuristic.

OTM-heuristic

We decompose the Many-to-Many (MTM) relationship in the above product structure into two sub-problems which results in two One-to-Many (OTM) relationships of engines and parts (see Figure 2).

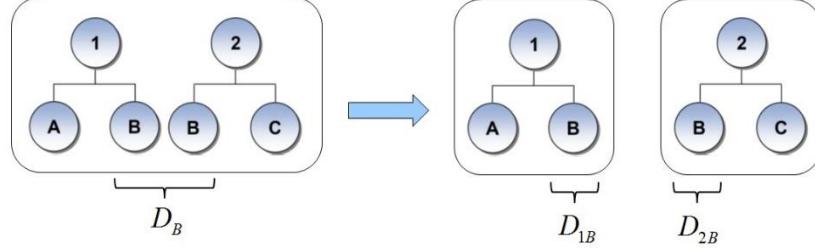


Figure 2: Core Separation

For solving these sub-problems independently, a split of the total demand for part B , which can be obtained from disassembling engine type 1 as well as engine type 2 into D_{1B} and D_{2B} is necessary (with $D_{1B} + D_{2B} = D_B$). In the following, we present a very effective procedure to split the demand and describe the steps that have to be performed to find near-optimal solutions for the DTO problem under BI yields as given in (2). The basic idea of the demand splitting is to reduce the MTM problem to a deterministic one by treating the BI success probabilities \hat{p}_{ik} as fixed yield rates \bar{p}_{ik} so that the yield functions become deterministic: $Y_{ik}(Q_i) = \bar{p}_{ik} \cdot Q_i$. Under these conditions we carry out a marginal cost analysis in order to decide how demand volume D_B can be gained from disassembling both cores in a most cost-effective way.

Therefore, we substitute the uncertain yield rates by deterministic ones and concentrate on the common part B . At first, one has to determine the demand-based disassembly lot sizes \bar{Q}_{ik} assuming deterministic yield rates \bar{p}_{ik} for each engine type i and each part k and a One-to-One-relationship between parts and engines (e.g. engine type 1 - part A) $\left(\bar{Q}_{1A} = \frac{D_A}{\bar{p}_{1A}}, \bar{Q}_{1B} = \frac{D_B}{\bar{p}_{1B}}, \bar{Q}_{2B} = \frac{D_B}{\bar{p}_{2B}}, \bar{Q}_{2C} = \frac{D_C}{\bar{p}_{2C}} \right)$. Afterwards, we analyze each engine type separately and start with a distinction of cases.

We illustrate this procedure in detail for engine type 1 with part A and common part B . At first we analyze the case $\bar{Q}_{1A} < \bar{Q}_{1B}$. If the disassembly lot size Q_1 is in the range between 0 and \bar{Q}_{1A} the question arises which savings from avoiding external procurement of part A can be achieved if the disassembly decision is to obtain one “good” B -part. The marginal cost is given as $\delta_{1B}^{(1)} = \frac{c_1^z}{\bar{p}_{1B}} - \frac{c_A^p \cdot \bar{p}_{1A}}{\bar{p}_{1B}}$. On the other hand, if $\bar{Q}_{1A} < Q_1 \leq \bar{Q}_{1B}$, the marginal cost $\delta_{1B}^{(2)} = \frac{c_1^z}{\bar{p}_{1B}} + \frac{c_A^d \cdot \bar{p}_{1A}}{\bar{p}_{1B}}$ reflects the fact that additional cost for the disposal of part A would occur if the goal is to obtain a good B -part from disassembling engine type 1. In the second case, $\bar{Q}_{1A} \geq \bar{Q}_{1B}$, the relevant lot size

is in between 0 and \bar{Q}_{1B} which leads to marginal cost $\delta_{1B}^{(3)}$ that is equal to $\delta_{1B}^{(1)}$. This can be done analogously for $\delta_{2B}^{(1)}$, $\delta_{2B}^{(2)}$ and $\delta_{2B}^{(3)}$.

To decide which part of the demand D_B should be allocated to subproblem 1 or subproblem 2, we have to create an ascending sequence of the marginal costs $\delta_{iB}^{(1)}$ to identify the profitable lot-size intervals. Following this sequence of marginal costs we decide in which sequence and to which extent both cores should be disassembled.

A general formulation for the above described procedure is given in the following pseudo code:

1. Demand split:

- (a) Determine disassembly lot sizes for each engine type i and each part k assuming deterministic yield rates \bar{p}_{ik} : $\bar{Q}_{1A} = \frac{D_A}{\bar{p}_{1A}}$, $\bar{Q}_{1B} = \frac{D_B}{\bar{p}_{1B}}$, $\bar{Q}_{2B} = \frac{D_B}{\bar{p}_{2B}}$, $\bar{Q}_{2C} = \frac{D_C}{\bar{p}_{2C}}$.
- (b) Determine the marginal costs of producing one unit of B , $\delta_{iB}^{(1)}$, of each engine type i . (B is the common part, k is either A if $i=1$ or C if $i=2$).

For $i = 1$ to 2

If $\bar{Q}_{ik} < \bar{Q}_{iB}$ then

If $0 < Q_i < \bar{Q}_{ik}$ then

$$\delta_{iB}^{(1)} = \frac{c_i^z}{\bar{p}_{iB}} - \frac{c_k^p \cdot \bar{p}_{ik}}{\bar{p}_{iB}}$$

Else if $\bar{Q}_{ik} < Q_i \leq \bar{Q}_{iB}$ then

$$\delta_{iB}^{(2)} = \frac{c_i^z}{\bar{p}_{iB}} + \frac{c_k^d \cdot \bar{p}_{ik}}{\bar{p}_{iB}}$$

End if.

Else if $\bar{Q}_{ik} \geq \bar{Q}_{iB}$ then

$$\delta_{iB}^{(3)} = \delta_{iB}^{(1)}$$

End if.

Next i .

- (c) Create ascending sequence of marginal costs $\delta_{iB}^{(1)}$.

- (d) Create demand split $(D_{1B} + D_{2B} = D_B)$.

- i. Start with the minimum value in the sequence of marginal costs $\{\delta_{iB}^{(1)}\}$ for allocating as much as possible of the total demand of part B (D_B) to core 1 (D_{1B}) or core 2 (D_{2B}). The allocation decision is limited

by the upper bound of the order size interval (relevant order size is at most \underline{Q}_{iB}) where the $\text{MIN}\{\delta_{iB}^{(c)}\}$ refers to.

- ii. If $D_{1B} + D_{2B} = D_B$ then go to (e).
- iii. If $D_{1B} + D_{2B} < D_B$ then delete $\text{MIN}\{\delta_{iB}^{(c)}\}$ from the set of marginal costs $\{\delta_{iB}^{(c)}\}$ and repeat step (d)

(e) end.

2. Disassembly lot size determination: Determination of disassembly lot-sizes Q_i^+ ($i=1,2$) via full enumeration of the respective single-core problems so that the respective cost function is minimized using the demand split from step 1.

$$\begin{aligned}
 Q_i^+ \text{ from } \text{MIN } C(Q_i) = & c_i^z \cdot Q_i + c_k^p \cdot \sum_{x=0}^{\min\{D_k-1, Q_i\}} \left[(D_k - x) \binom{Q_i}{x} \cdot (\hat{p}_{ik})^x \cdot (1 - \hat{p}_{ik})^{Q_i-x} \right] \\
 & + c_B^p \cdot \sum_{x=0}^{\min\{D_{iB}-1, Q_i\}} \left[(D_{iB} - x) \binom{Q_i}{x} \cdot (\hat{p}_{iB})^x \cdot (1 - \hat{p}_{iB})^{Q_i-x} \right] \\
 & + c_k^d \cdot \sum_{x=D_k+1}^{Q_i} \left[(x - D_k) \binom{Q_i}{x} \cdot (\hat{p}_{ik})^x \cdot (1 - \hat{p}_{ik})^{Q_i-x} \right] \\
 & + c_B^d \cdot \sum_{x=D_{iB}+1}^{Q_i} \left[(x - D_{iB}) \binom{Q_i}{x} \cdot (\hat{p}_{iB})^x \cdot (1 - \hat{p}_{iB})^{Q_i-x} \right]
 \end{aligned}$$

3. Adapt demand split

$$D_{1B} = \text{Round} \left(\frac{Q_1^+ \cdot \bar{p}_{1B}}{Q_1^+ \cdot \bar{p}_{1B} + Q_2^+ \cdot \bar{p}_{2B}} \cdot D_B \right)$$

$$D_{2B} = D_B - D_{1B}$$

- 4. Repeat disassembly lot size determination, resulting in \hat{Q}_1 and \hat{Q}_2
- 5. Determine corresponding costs $C(\hat{Q}_1, \hat{Q}_2)$
- 6. END

This heuristic OTM approach can also be applied to more general product structures. To this end the demand split procedure must simply be extended to the case of multiple common parts from multiple cores.

Performance Analysis

To examine the performance of the OTM-Heuristic we did run a numerical study where we chose the problem parameters randomly from given parameter ranges. We evaluated the performance using the relative cost deviation in percent which is calculated as:

$$\Delta = 100 \cdot \frac{C(\hat{Q}_1, \hat{Q}_2) - C(Q_1^*, Q_2^*)}{C(Q_1^*, Q_2^*)} \quad (3)$$

As parameter ranges we defined:

- Procurement cost: $c_k^p \in \{1; 2; \dots; 10\} \quad \forall k$
- Disposal cost: $c_k^d \in \{0; 1; \dots; 6\}$ with $c_k^d \leq c_k^p \quad \forall k$
- Success probability: $\hat{p}_{ik} \in \{0.5; 0.6; 0.7; 0.8; 0.9\} \quad \forall i, k$
- Disassembly cost: $c_i^z \in \{0; 1; 2; \dots; 15\} \quad \forall i$ and with $c_i^z < \hat{p}_{il} \cdot c_l^p + \hat{p}_{im} \cdot c_m^p \quad \forall i$ we assure, that disassembling is profitable (if parts l and m are contained in engine type i)
- Demand: $D_k \in \{4; 5; \dots; 30\} \quad \forall k$

Assuming a discrete uniform distribution for each of these parameters, we randomly created 200 different parameter combinations. We then computed the respective minimal costs and the corresponding costs from applying the OTM-Heuristic.

In 1781 of the analyzed instances we observed cost deviations of less than 0.5%. This corresponds to approximately 89% of all instances and underlines the promising performance of the OTM-heuristic. From figure Figure 3 we can additionally observe that there is only a very small amount of instances where the cost deviations exceed the 1%-level. On average, we experienced a relative cost deviation of only 0.19% (median: .00 %, worst case: 6.21%).

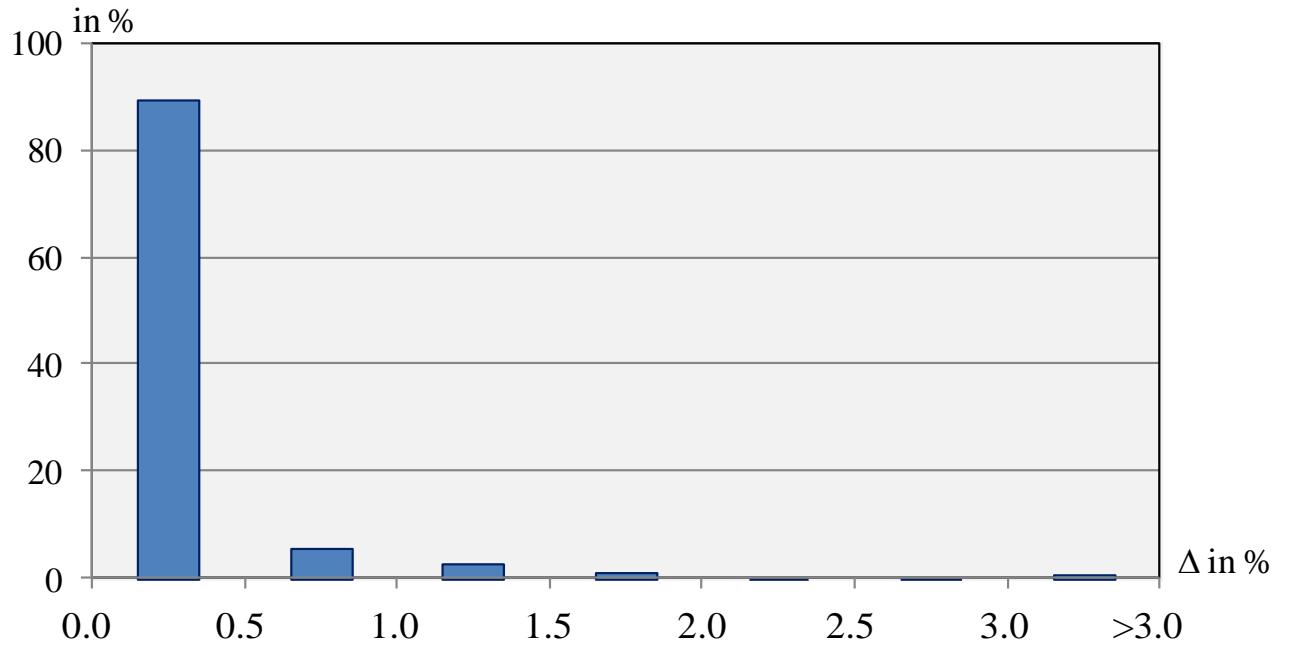


Figure 3: Frequency of cost deviations

Conclusion and outlook

There is a need to develop effective heuristics to solve the stochastic DTO problem for instances of arbitrary size. In this paper such a heuristic is presented which basically relies on separating the stochastic multiple-core problem into multiple single-core ones. To this end a demand split for common parts is necessary for which a very effective procedure has been developed. A first numerical study reveals that the total heuristic solution procedure leads to close-to-optimal results.

It is matter of future research to carry out more comprehensive numerical tests which also should give more insights for which parameter constellations our heuristic may need to be refined. An additional interesting point for further research would be to extend the OTM-heuristic to the multi-period case.

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