Investment decision for supply chain resilience based on Evolutionary Game theory

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Abstract

We study the factors and improvement methods of supply chain resilience, then analyze the investment decisions between supply chain members based on the Evolutionary Game theory. We deduce that the government should use punitive and subsidy methods to control supply chain members' behaviors and enhance the resilience of supply chain.
Key Words: supply chain resilience, supply chain disruption, Evolutionary Game theory

Introduction

Recently, supply chain disruption has been studied by experts and scholars in every vocation. Due to supply chain disruptions, operating cost rising and brand influence declining often occur. When some unexpected disasters events just like 911 happen, immeasurable economic loss and casualties will occur. In 2000, the Philips chip factory in US fired, causing the procurement of Ericsson interrupted, resulting in a direct loss of $400 million, and its impact force Ericsson's market share dropped from 12% to 9%. It is increasingly important to improve the supply chain resilience to reduce disruption losses.

In this paper, we first study the factors and improvement methods of supply chain resilience based on previous studies by other scholars. In order to improve supply chain resilience, companies need to invest in many aspects and only all members of supply chain invest, the resilience of the whole supply chain can be enhanced. Then we build the model of investment decision for resilience to analyze investment decision between suppliers and manufacturers based on the evolutionary game. After that, we use regulatory mechanisms to prompt all members of supply chain to invest for supply chain resilience.

For factors of supply chain resilience, Petti et al. (2010) considered that the factors of resilience can be classified as capability and vulnerability factors. If capabilities increase and vulnerabilities decrease, supply chain resilience will be improved. For the improvement methods of resilience, Lee et al. (2005) learned from total quality management, found that companies can achieve a high level of safety through proper management measures and process reengineering. Christopher and Peck proposed a few important principles which can help companies to enhance resilience. Sheffi (2001) found that there are 3 ways to enhance supply chain resilience: increase redundancy, enhance agility, and change corporate culture.

Evolutionary game theory combines game theory and dynamic evolution, studies the trends and stability of the fraction of members in the game. Recent years, many scholars use evolutionary game to study supply chain problems. Xiao et al. (2006) studied supply chain disruption and coordination based on evolutionary game. Zhu et al. (2007) studied the evolutionary game between core companies and governments in green supply chain. Because supply chain members have bounded rationality, they can not necessarily make the optimal decisions. So it is more practical to use evolutionary game to analyze this topic.

The factors and improvement methods of Supply chain resilience

As mentioned above, after getting elicitation from Petti's study, we consider that firms should consider the vulnerability and capability factors of supply chain, and build the more balanced resilience. In our study, via researching previous studies of other scholars, we propose the specific methods to enhance resilience in three aspects. The classification is internal to the firm, external to the firm and virtual supply chain， shown as Table 1. It should be noted that, the virtual supply chain refers to information systems and information transmission system on the Internet which the actual supply chain relies on.

Table 1-The improvement methods of supply chain resilience

Classification	The improvement methods	Meanings
Internal to the firm	Appropriate redundancy	Build the safety stock of raw material and final product; Keep extra production ability and workers etc.
	Supply chain flexibility	Redesign and standardize product and procedures; Reduce the types of component; Make flexible contract etc.
	Supply chain agility	Reduce reaction time and total time of materials moving on the supply chain; Control of inventories and production schedules.
	Visibility of the firm	Business intelligence; Information gathering; Technology upgrades and other means.
	Dispersibility of the firm	Decentralize decision-making, production capabilities, personnel and other critical resources.
	Risk analysis and prevention	Establish some relevant departments; Develop disruption reaction mechanisms; Reserve relevant resources etc.
	Disruption response measures	Establish emergency response departments; Develop interrupt response mechanism; Deploy stuff and resources effectively etc.
	Risk management culture	Risk assessment and continuous communication among employees; Knowledge sharing and learning.
External to the firm	Design resilience in supply chain	Design resilience in bottle neck and critical path; Balance the cost, efficiency and risk based on company strategy etc.
	Collaboration between firms	Exchange information with suppliers and customers; Collaborative planning, forecasting and replenishment etc.
	Fitness to the environment	Fitness to the external environment such as natural, political, economic, legal and cultural environment.
Virtual supply chain	Design resilience in supply chain	Design resilience in bottle neck and critical nodes of the software system; Reserve interfaces to expand functions and

	software system	coverage; Consider firm's strategies when designing etc.
	Redundancy of supply chain system	Keep redundancy of hardware and software; Ensure appropriate speed and bandwidth of network; Deploy relatively staff to administrate the virtual supply chain system etc.
	Protect virtual supply chain against risks	Purchase anti-virus software to prevent computer viruses; Increase investment to prevent network disruptions, and information distortion in transmission etc.

Model Description

In order to enhance supply chain resilience, firms should invest in many aspects such as increasing redundancy, improving flexibility and agility of supply chain and so on, all of which require capital investment. This paper differs from prior papers, study supply chain members' investment decision for supply chain resilience based on evolutionary game (Webull 1998), and propose some mechanisms to control members' behavior.

Assumptions

In this paper, we study the supply chain constituted by suppliers and manufacturers (denoted by *S* and *M*). In our model, an individual of supplier population (for short individual *S*) randomly plays a one-shot game with a matched individual of manufacturer population (for short individual *M*) every time. Every individual has two kinds of investment strategies for resilience: invest (for short *I*) or not invest (for short *N*). By invest for resilience, firms can reduce supply chain disruption losses, and improve revenues of them. Assumptions are:

(1) If both individuals don't invest for resilience, their revenues when disruptions occur are $(1-a_1)R_s, (1-b_1)R_m$; $R_s(R_s>0)$ and $R_m(R_m>0)$ are respectively normal revenue of individual *S* and individual *M* when disruptions do not occur. $a_1(a_1>0)$ and $b_1(b_1>0)$ are respectively revenue loss rate of individual *S* and individual *M* when disruptions occur.

(2) If both individuals invest and improve the resilience, their profitability can be improved, then the disruption losses of them can be reduced. Their revenues are $R_s - a_1R_s + a_2R_s - C_s$ and $R_m - b_1R_m + b_2R_m - C_m$, respectively; $a_2R_s(a_2R_s>0)$ and $b_2R_m(b_2R_m>0)$ are compensation value of resilience for individual *S* and individual *M* after they invest, respectively. $a_2(a_2>0)$ and $b_2(b_2>0)$ are compensation rate of individual *S* and individual *M*, respectively. $C_s(C_s>0)$ and $C_m(C_m>0)$ are investment cost of individual *S* and individual *M*, respectively.

(3) If individual *S* invests and individual *M* does not invest, the revenue of individual *S* is $R_s - a_1R_s + a_2R_s - C_s$ when disruption occurs. Because individual *M* can benefit from individual *S*' invest, for example manufacturers' procurement can be guaranteed in quality, and more prices can be selected when purchasing, the cost of manufacturers can be reduced. Due to individual *M*' free-riding, the revenue of individual *M* when disruption occurs is $T_m(T_m > R_m - b_1R_m)$.

(4) Similarly, if individual *M* invests and individual *S* does not invest, the revenue of individual *M* is $R_m - b_1R_m + b_2R_m - C_m$. Due to individual *S*' free-riding, the revenue of it is $T_s(T_s > R_s - a_1R_s)$.

For the convenience of demonstration, we denote R_s^a and R_m^a as individual *S*' and individual *M*' revenue when all of them don't invest, in other words, replace $(1-a_1)R_s$ with

R_s^a and $(1-b_1)R_m$ with R_m^a, respectively; denote R_s^b and R_m^b as individual *S*' and individual *M*' revenue when all of them invest, in other words, replace $(1-a_1+a_2)R_s-C_s$ with R_s^b and $\left(1-b_1+b_2\right)R_m-C_m$ with R_m^b, respectively.

According to the above assumptions, we establish the payoff matrix of individual *S* and individual *M*, which is shown in Table 2.

Table 2- Payoff matrix of individual S and individual M

S	*M*	
	I	*N*
I	$(1-a_1+a_2)R_s-C_s$, $\left(1-b_1+b_2\right)R_m-C_m$	$(1-a_1+a_2)R_s-C_s$, T_m
N	T_s, $\left(1-b_1+b_2\right)R_m-C_m$	$(1-a_1)R_s$, $(1-b_1)R_m$

Replicator dynamic system

According to the payoff matrix, we can get the replicator dynamic system of supplier population (for short population *S*) and manufacturer population (for short population *M*). When the game begin, we let the fraction of individuals *S* using strategy *I* be *p*, so the fraction of individuals *S* using strategy *N* is *1-p*; Similarly we let the fraction of individuals *M* using strategy *I* be *q*, so the fraction of individuals *M* using strategy *N* is *1-q*.

The revenue functions when individual *S* chooses strategy *I*, strategy *N*, and the average revenue function of population *S* respectively are:

$$U_1=q[(1-a_1+a_2)R_s-C_s]+(1-q)[(1-a_1+a_2)R_s-C_s] \tag{1}$$

$$U_2=qT_s+(1-q)(R_s-a_1R_s) \tag{2}$$

$$\bar{U}=pU_1+(1-p)U_2 \tag{3}$$

Similarly, the revenue functions when individual *M* chooses strategy *I*, strategy *N*, and the average revenue function of population *M* respectively are:

$$V_1=p[(1-b_1+b_2)R_m-C_m]+(1-p)[(1-b_1+b_2)R_m-C_m] \tag{4}$$

$$V_2=pT_m+(1-p)(R_m-b_1R_m) \tag{5}$$

$$\bar{V}=qV_1+(1-q)V_2 \tag{6}$$

Because individuals have bounded rationality, after a period of evolution they will re-choose strategies based on the revenues they get, the individuals who have less revenues will change their strategies. Then the fraction *p* and *q* will change over time. According to Malthusian dynamic system (Friedman. 1991), we can obtain that the replicator dynamic system for population *S* and population *M* (denoted by system1) is

$$\begin{cases} \dfrac{dp}{dt} = p(U_1 - \bar{U}) = p(1-p)[q(1-a_1)R_s + a_2 R_s - C_s - qT_s] \\ \dfrac{dq}{dt} = q(V_1 - \bar{V}) = q(1-q)[p(1-b_1)R_m + b_2 R_m - C_m - pT_m] \end{cases} \tag{7}$$

Proposition 1. For the system 1given by Eq. (7), we have the following results:

(1) (0, 0), (0, 1), (1, 0) and (1, 1) are its equilibriums.

(2) If $C_s < a_2 R_s < T_s - (1-a_1)R_s + C_s$, $C_m < b_2 R_m < T_m - (1-b_1)R_m + C_m$, (p_0, q_0) is also an equilibrium of system1, $p_0 = \dfrac{b_2 R_m - C_m}{T_m - (1-b_1)R_m}$, $q_0 = \dfrac{a_2 R_s - C_s}{T_s - (1-a_1)R_s}$.

Proof. For system1, when $dp/dt = 0$, $dq/dt = 0$, obviously we get that (0,0), (0,1), (1,0) and (1,1) are its equilibriums. If $a_2 R_s$ and $b_2 R_m$ satisfy $C_s < a_2 R_s < T_s - (1-a_1)R_s + C_s$, $C_m < b_2 R_m < T_m - (1-b_1)R_m + C_m$, we can easily know that $0 < p_0 < 1, 0 < q_0 < 1$, thus (p_0, q_0) is also an equilibrium.

Evolutionarily stable strategies

The equilibriums of system 1 which we have got are not necessarily ESS. So we should use Jacobian method (Hofbauer et al. 1998) to judge them. The Jacobian matrix of system 1 is

$$J = \begin{bmatrix} [q(1-a_1+a_2)R_s - C_s - qT_s](1-2p) & p(1-p)[(1-a_1)R_s - T_s] \\ q(1-q)[(1-b_1)R_m - T_m] & [p(1-b_1+b_2)R_m - C_m - pT_m](1-2q) \end{bmatrix} \tag{8}$$

The local stability of equilibriums is determined by both determinant and trace. The determinant of the Jacobian matrix is

$$\begin{aligned} detJ = &(1-2p)[q(1-a_1+a_2)R_s - C_s - qT_s](1-2q)[p(1-b_1+b_2)R_m - C_m - pT_m] \\ &- p(1-p)[(1-a_1)R_s - T_s]q(1-q)[(1-b_1)R_m - T_m] \end{aligned} \tag{9}$$

The trace of the Jacobian matrix is

$$trJ = [q(1-a_1+a_2)R_s - C_s - qT_s](1-2p) + [p(1-b_1+b_2)R_m - C_m - pT_m](1-2q) \tag{10}$$

Furthermore, we can derive the following.

Proposition 2.

(1) If $0 < a_2 R_s < C_s, 0 < b_2 R_m < C_m$, the equilibrium (0, 0) is the ESS of system1.

(2)If $C_m < b_2 R_m < T_m - (1-b_1)R_m + C_m$, $0 < a_2 R_s < C_s$, (0, 1) is the ESS of system 1.

(3)If $C_s < a_2 R_s < T_s - (1-a_1)R_s + C_s, 0 < b_2 R_m < C_m$, (1, 0) is the ESS of system 1.

(4)If $C_s < a_2 R_s < T_s - (1-a_1)R_s + C_s$, $C_m < b_2 R_m < T_m - (1-b_1)R_m + C_m$, (p_0, q_0) exists, (0, 1) and (1, 0) are the ESS of system 1.

(5)If $a_2 R_s > T_s - (1-a_1)R_s + C_s, b_2 R_m > T_m - (1-b_1)R_m + C_m$, (1, 1) is the ESS.

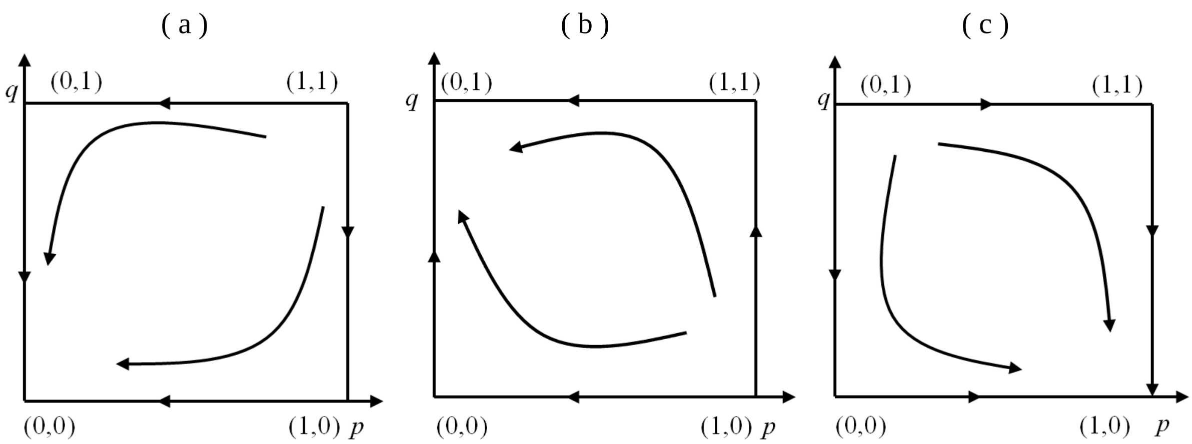
Proof. According to the Jacobian method, we know that if *trJ<0, detJ>0,* the equilibrium of the replicator system is ESS. We can get the values of *trJ* and *det* on the equilibriums of system 1, then judge the stability of equilibriums and get the ESS. The analysis of ESS in every case is shown in Table 3.

Table 3-Stability analysis of equilibriums for system 1

	Equilibrium	*TrJ*	*DetJ*	Stability
Case1	(0,0)	−	+	ESS
	(0,1)		−	Saddle point
	(1,0)		−	Saddle point
	(1,1)	+	+	Unstable
Case2	(0,0)		−	Saddle point
	(0,1)	−	+	ESS
	(1,0)		−	Saddle point
	(1,1)	+	+	Unstable
Case3	(0,0)		−	Saddle point
	(0,1)		−	Saddle point
	(1,0)	−	+	ESS
	(1,1)	+	+	Unstable
Case4	(0,0)	+	+	Unstable
	(0,1)	−	+	ESS
	(1,0)	−	+	ESS
	(1,1)	+	+	Unstable
	(p_0, q_0)		−	Saddle point
Case5	(0,0)	+	+	Unstable
	(0,1)		−	Saddle point
	(1,0)		−	Saddle point
	(1,1)	−	+	ESS

Evolutionary analysis results

Based on the analysis above, we get the evolutionary game process of population *S* and population *M* in five cases, shown in Figure 1, respectively. From the phase diagrams of dynamic evolution of system 1, we can obtain the following results.



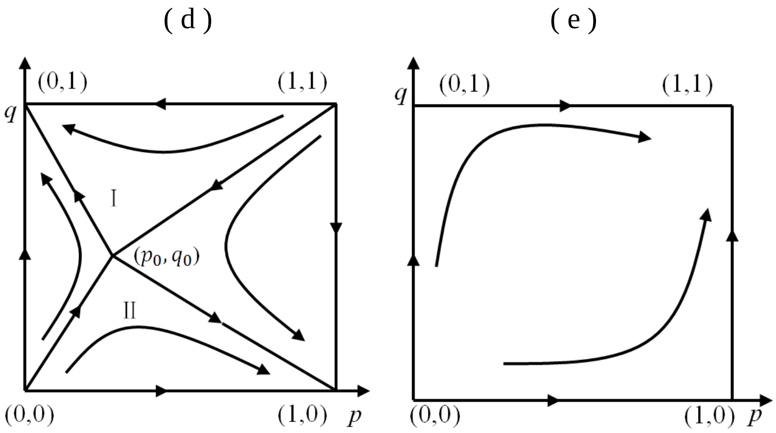


Figure 1-The phase diagrams of dynamic evolution of all cases for system1

(1) If $0 < a_2R_s < C_s$, $0 < b_2R_m < C_m$, a_2R_s and b_2R_m are all less than the invest cost C_s, C_m. So the revenue after individual *S* and individual *M* invest are less than the revenue when they don't invest. See Fig. 1(a), (1,1) is unstable, (0,1) and (1,0) are saddle points , (0,0) is an ESS of system 1. In other words, (*N*, *N*) is an ESS of system1, population *S* and population *M* all don't invest.

(2) If $C_m < b_2R_m < T_m - (1-b_1)R_m + C_m$, $0 < a_2R_s < C_s$, compensation value a_2R_s is less than invest cost C_s, so individual *S* won't invest. b_2R_m is larger than invest cost C_m, but R_m^b is less than free-riding revenue T_m. However, individual *S* does not invest so that the free-riding behavior of individual *M* cannot be implemented, investing for resilience is optimal for individual *M*. See Fig. 1(b), (1,1) is unstable, (0,0) and (1,0) are saddle points , (0,1)is an ESS of system 1. Therefore population *S* does not invest and population *M* invest.

(3) Similarly, if $C_s < a_2R_s < T_s - (1-a_1)R_s + C_s$, $0 < b_2R_m < C_m$, b_2R_m is less than C_m, so individual *M* won't invest. a_2R_s is larger than C_s, but R_s^b is less than T_s. See Fig.1 (c), (1, 1) is unstable, (0, 0) and (0, 1) are saddle points, (1, 0) is an ESS of system 1. In other words, population *M* does not invest, population *S* invest.

(4) If $C_s < a_2R_s < T_s - (1-a_1)R_s + C_s$, $C_m < b_2R_m < T_m - (1-b_1)R_m + C_m$, a_2R_s and b_2R_m are larger than C_s and C_m. Because T_s and T_m are larger than R_s^b and R_m^b, population *S* and population *M* are more inclined to implement free-riding behavior. But if they all don't invest, the free-riding behavior can't be implemented. Therefore system 1 will eventually evolve to two ESS. See Fig. 1(d), (0, 0) and (1, 1) are unstable points, (p_0, q_0) is a saddle point, (0, 1) and (1, 0) are ESS of system 1. In other words, population *S* does not invest and population *M* invest or population *M* does not invest and population *S* invest.

(5) If $a_2R_s > T_s - (1-a_1)R_s + C_s$, $b_2R_m > T_m - (1-b_1)R_m + C_m$, R_s^b and R_m^b are larger than T_s and T_m, a_2R_s and b_2R_m are larger than C_s and C_m. See Fig.1 (e), (0,0) is unstable, (0,1) and (1,0) are saddle points, (1,1) is the ESS. In other words, populations all invest.

Evolutionary analysis under government control

In order to improve the resilience of the whole supply chain and maximize the revenue, we should let (1, 1) be the unique ESS, in other words, populations all invest for resilience. Then we consider that the government should get involved in the management of supply chain, set up mechanisms to regulate the behaviors of populations.

Evolutionary analysis under punitive mechanism

If $C_s < a_2 R_s < T_s - (1-a_1) R_s + C_s, C_m < b_2 R_m < T_m - (1-b_1) R_m + C_m$, the ESS of system 1 are (0,1) and (1,0), the resilience of the whole supply chain cannot be fully improved, so the supply chain cannot achieve optimal operational efficiency. Therefore government should set up punitive mechanism to punish free-riders to prompt them to invest. For case 4 of system 1, assume punishment for free-rider is P^*, the payoff matrix is shown as Table 4.

Table 4-Payoff matrix of individual S and individual M under punitive mechanism

S	*M*	
	I	*N*
I	$(1-a_1+a_2)R_s - C_s$, $(1-b_1+b_2)R_m - C_m$	$(1-a_1+a_2)R_s - C_s$, $T_m - P^*$
N	$T_m - P^*$, $(1-b_1+b_2)R_m - C_m$	$(1-a_1)R_s$, $(1-b_1)R_m$

The replicator dynamic system under punitive mechanism (denoted by system 2) is

$$\begin{cases} \dfrac{dp}{dt} = p(1-p)[q(1-a_1)R_s + a_2 R_s - C_s - q(T_s - P^*)] \\ \dfrac{dq}{dt} = q(1-q)[p(1-b_1)R_m + b_2 R_m - C_m - p(T_m - P^*)] \end{cases} \tag{11}$$

Proposition 3. For the system 2 given by Eq. (11), we derive the following:

(1) (0, 0), (0, 1), (1, 0) and (1, 1) are its equilibriums.

(2) If inequation (12) satisfies, (p_0^1, q_0^1) is also an equilibrium of system 2,

$$p_0^1 = \frac{b_2 R_m - C_m}{T_m - (1-b_1) R_m - P^*}, \quad q_0^1 = \frac{a_2 R_s - C_s}{T_s - (1-a_1) R_s - P^*}.$$

$$\begin{aligned} &max[T_s - (1-a_1+a_2)R_s + C_s, \; T_m - (1-b_1+b_2)R_m + C_m] < P^* \\ &< min[T_s - (1-a_1)R_s, \; T_m - (1-b_1)R_m] \end{aligned} \tag{12}$$

Proof. For system 2, we let $dp/dt = 0$, $dq/dt = 0$, obviously, we get that (0, 0),(0, 1),(1, 0) and (1, 1) are its equilibriums. If inequation (12) satisfies, we can easily know that $0 < p_0^1 < 1$, $0 < q_0^1 < 1$, then (p_0^1, q_0^1) is also an equilibrium.

Proposition 4. The necessary and sufficient condition under which (1, 1) is the unique ESS of system 2 is:

$$P^* > \max[T_s - (1-a_1+a_2)R_s + C_s, T_m - (1-b_1+b_2)R_m + C_m] \tag{13}$$

Obviously, inequation (13) is equivalent to $P^* > \max[T_s - R_{s2}, \; T_m - R_{m2}]$.

Proof. First we know that the necessary and sufficient condition is *trJ<0,detJ>0*. Easily, we get $-[(1-a_1+a_2)R_s - C_s - (T_s - P^*)] - [(1-b_1+b_2)R_m - C_m - (T_m - P^*)] < 0$, and $[(1-a_1+a_2)R_s - C_s - (T_s - P^*)][(1-b_1+b_2)R_m - C_m - (T_m - P^*)] > 0$, then we can get $(1-a_1+a_2)R_s - C_s - (T_s - P^*) > 0$ and $(1-b_1+b_2)R_m - C_m - (T_m - P^*) > 0$. Therefore we see that $P^* > \max[T_s-(1-a_1+a_2)R_s+C_s,\ \ T_m-(1-b_1+b_2)R_m+C_m]$. Secondly when inequation (13) satisfies, based on Table 5 we can obtain (0, 0) is an unstable point of system2, (0, 1) and (1, 0) are saddle points, (1, 1) is the unique ESS of system 2.

Table 5-Stability analysis of equilibriums for system 2

Equilibrium	*trJ*	*detJ*
(0, 0)	$(a_2R_s-C_s)+(b_2R_m-C_m)$	$(a_2R_s-C_s)(b_2R_m-C_m)$
(0, 1)	$[(1-a_1+a_2)R_s-C_s-(T_s-P^*)]-(b_2R_m-C_m)$	$-(b_2R_m-C_m)[(1-a_1+a_2)R_s-C_s-(T_s-P^*)]$
(1, 0)	$[(1-b_1+b_2)R_m-C_m-(T_s-P^*)]-(a_2R_s-C_s)$	$-(a_2R_s-C_s)[(1-b_1+b_2)R_m-C_m-(T_s-P^*)]$
(1, 1)	$-[(1-a_1+a_2)R_s-C_s-(T_s-P^*)]-$ $[(1-b_1+b_2)R_m-C_m-(T_s-P^*)]$	$[(1-a_1+a_2)R_s-C_s-(T_s-P^*)]$ $[(1-b_1+b_2)R_m-C_m-(T_s-P^*)]$

This illustrates that when $P^* > \max[T_s-R_{s2},\ \ T_m-R_{m2}]$, the punishment P^* is larger than the difference between free-rider's revenue and the revenue when firms invest, the revenue when firms invest is larger than the revenue of free-riding, then supply chain members all tend to invest for resilience, the resilience of the whole supply chain can be improved.

Evolutionary analysis under subsidy mechanism

If $0<a_2R_s<C_s$, $0<b_2R_m<C_m$, the ESS of system 1 is (0,0), the resilience cannot be improved, and this will form a vicious circle. So the government should set up subsidy mechanism to complement the firms to prompt them to invest. For case 1of system 1, assume subsidy for firms is S^*, then we get the payoff matrix shown as Table 6.

Table-6 Payoff matrix of individual S and individual M under subsidy mechanism

S	*M*	
	I	*N*
I	$(1-a_1+a_2)R_s-C_s+S^*,\ \ (1-b_1+b_2)R_m-C_m+S^*$	$(1-a_1+a_2)R_s-C_s+S^*,\ \ T_m$
N	$T_s,\ \ (1-b_1+b_2)R_m-C_m+S^*$	$(1-a_1)R_s,\ \ (1-b_1)R_m$

The replicator dynamic system under subsidy mechanism (denoted by system 3) is

$$\begin{cases} \dfrac{dp}{dt}=p(1-p)[q(1-a_1)R_s+a_2R_s-C_s+S^*-qT_s] \\ \dfrac{dq}{dt}=q(1-q)[p\left(1-b_1\right)R_m+b_2R_m-C_m+S^*-pT_m] \end{cases} \tag{14}$$

Proposition 5. For the system 3 given by Eq. (14), we derive the following:

(1) (0, 0), (0, 1), (1, 0) and (1, 1) are its equilibriums.

(2) If inequation (15) satisfies, (p_0^2,q_0^2) is also an equilibrium of system 3.

$p_0^2=\dfrac{b_2R_m-C_m+S^*}{T_m-\left(1-b_1\right)R_m}$, $q_0^2=\dfrac{a_2R_s-C_s+S^*}{T_s-\left(1-a_1\right)R_s}$.

$$\max[(C_s - a_2R_s),(C_m - b_2R_m)] < S^* \\ < min[T_s - (1+a_2 - a_1)R_s + C_s,\ \ T_m - (1+b_2 - b_1)R_m + C_m] \tag{15}$$

Proof. Similar to Proposition 3.

Proposition 6. The necessary and sufficient condition under which (1, 1) is the unique ESS of system 3 is:

$$S^* > \max[T_s - R_{s2},\ \ T_m - R_{m2}] \tag{16}$$

Proof. Similar to Proposition 4.

This illustrates that when $S^* > \max[T_s - R_{s2},\ T_m - R_{m2}]$, the subsidy S^* is larger than the difference between free-rider's revenue and the revenue when firms investing. Suppliers or manufacturers selecting the invest strategy will get more revenue than free-riding, then they all tend to invest. So when a_2R_s and b_2R_m are relatively small, government should use subsidy method to control the behaviors of firms to let them invest for resilience.

Conclusion

This paper first studies the factors and improvement method of supply chain resilience, then analyzes the investment decision of suppliers and manufacturers based on Evolutionary Game theory. We find that with the increase of a_2R_s and b_2R_m, there appears different ESS such as (0, 0), (0, 1), (1, 0), (1, 1). In order to improve the resilience of the whole supply chain and maximize the revenues of supply chain members, we consider that government should use punitive and subsidy methods to prompt firms to invest. Thus the resilience can be improved and the disruption loss can be reduced.

There are several areas to extend this research. First we can study resilience improvement methods of multi-echelon supply chain. Secondly, we can consider coordination contracts of supply chain in the investment decision based on Evolutionary Game theory.

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