

# Multi-objective location and routing problem for optimized post-disaster relief distribution

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## Abstract

The effective distribution of relief plays a critical role for rescue operations in post disaster. This paper constructs a multi-objective location-routing model for relief distribution problem. Non-dominated sorting differential evolution algorithm is introduced to solve the model. Case studies expound the application of the model and algorithm in practice.

**Keywords:** Multi-objective, Relief Distribution, Location Routing problem

## Introduction

The “5.12” Wenchuan earthquake killed almost 70 thousand people in China on May 12, 2008. Recent deadly earthquake took place in Haiti in January 2010, Chile in March 2011, and Japan in April in 2011. Once earthquake occurs, successful disaster rescue efforts can reduce the damage severity. Emergency managers need to make an optimal schedule for relief distributions with limited available time, funds and resources. Relief distribution in emergency logistics is more complicated compared to the traditional logistics. First, the secondary disasters often occur following the fatal earthquake. Hence, time minimizing measures are important for supplies delivery. Second, the security of rescue workers is another important consideration. Security is positive correlated with the reliability of vehicle routes. With limited funds and resources for salvage, the rescue cost also needs to be considered.

In this paper, relief distribution involves the location of distribution centers (DCs) and vehicle routing and scheduling. It can be classified as an open location-routing problem (OLRP) with multi-objective. Split delivery is applied, which makes the problem closer to the real situation.

In general location of DCs and vehicle routing are addressed individually for emergency logistics (Barbaroso and Arda 2004, Haghani and Oh 1996, Özdamar et al. 2004). However they are highly correlated (Ronald 1993). There is a lack of studies on the design of mathematical models and algorithms for the integration of location and route problem in a post-disaster situation. In (Fiedrich 2000) DCs are definite and fixed in the distribution network, and resources are directly sent from DCs to demand points. An integrated location-distribution model is described in by Yi and Özdamar (2007) for coordinating resource supply and wounded people evacuation operations in response activities. Vehicle routing and scheduling is not considered in

the above. A hybrid fuzzy clustering-optimization approach for multi-objective dynamic programming model is presented by Sheu (2007). However, the reliability of the infrastructure is neglected. An original multi-criteria optimization model for humanitarian aid distribution is presented in reference (Vitoriano et al. 2011), which providing helps in the selection of vehicles and the design of routes. The location of the DCs is not considered.

In comparison with typical LRP, vehicles in OLRP may wait at their last node without returning to DCs until the next order is specified. The open vehicle routing problem has been extensively discussed in traditional logistics (Sariklis and Powell 2000, Schrange 1981). However, few works has been focused on OLRP in emergency logistics. As stated above, the OLRP is a generalization of LRP, and is thus NP-hard problem (Balakrishnan 1987). Hence, the use of heuristics should be justified.

Over the past few years, some researchers have been engaged in the development of multi-objective evolutionary algorithms. Strength Pareto evolution algorithm (Georgiadou 2010), non-dominating sorting genetic algorithm-II (Liao et al. 2011, Nolz et al. 2010), etc., constitute the pioneering multi-objective methods that have been used to solve the LRP problem. Recently, differential evolution (DE) originally proposed by Storn and Price (1997) has been developed as a novel evolutionary method. It is simple, yet fast and robust compared to other evolution algorithms, which makes it very attractive in numerical studies. Non-dominated sorting differential algorithm (NSDE) is proposed in (Rakesh and Babu 2005) as an extension of DE to solve multi-objective problems. By integrating non-dominated sorting and ranking mechanism with DE operations, NSDE is able to achieve Pareto-optimal solutions with high superiority. But to our knowledge, works that using NSDE for multi-objective OLRP in emergency logistics are very scare. An improved NSDE is designed to solve multi-objective OLRP model in this paper.

Aiming at the scope of the study defined above, the proposed relief distribution model and approach in emergency logistics is unique with the following distinctive features:

- DCs location and open routes scheduling of heterogeneous vehicles strategies are considered simultaneously to coordinate supplies and demands of relief.
- Reliability as well as response time and total cost is considered as an objective, which ensures security for distribution operation.
- Mixed split deliveries are allowed when the loading of the service vehicle is not enough for the total demands of some disaster area.
- An improved NSDE algorithm is prosed to solve the multi-objective OLRP model.

### The Problem Description

The emergency logistics can be described as an undirected  $G = (V, E)$ , where  $V$  is the vertex set and  $E$  is the set of all available arcs in the post-disaster transportation network. Vertex set  $V$  contains two subsets:  $M$  and  $N$ , the former represents the locations of candidate DCs; while the latter represents the disaster areas requiring critical supplies. The distance matrix  $D = (d_{ii'})$  is assumed to satisfy the triangle inequality, i.e.,  $d_{ii'} < d_{ik} + d_{ki'}$  for all  $i, i', k \in V$ . To ensure serving vehicles to move on proper links, the allowable velocity matrix for crossing links  $MV = (v_{ii'})$  and reliability matrix  $R = (r_{ii'})$  of the probability for crossing links safely are also defined on  $A$ . In order to select appropriate type of vehicles, some nonnegative weights related to vehicles are defined:  $v_k$  means the normal velocity of  $k$  type vehicles,  $c_k$  denotes the cost per unit of length, and  $L_k$  describes the loading capacity. Further more, sleeping bags and water are critical supplies

considered in this study, which are urgently required in case of an earthquake. The reliefs are considered for one day needed by helpless people, and mixed split deliveries are required.

In this paper, the following three objectives are considered for the OLRP in emergency logistics: (1) minimization of the maximum vehicle route travelling time; (2) minimization of the total cost, including the fixed establishing costs of DCs and the vehicle travelling cost; (3) maximization of the minimum route reliability for all the serving vehicles.

## Model Formulation

### Assumptions

(1) The number of disaster areas and candidate DCs is known, and the corresponding geographic relationships can be obtained by advanced disaster detection technology in real time. (2) Each vehicle is allowed to stow multiple types of relief in any given transportation assignment. (3) Only the disaster areas that are still reachable through the current traffic network will be considered.

### Notations and definitions

(1) Transportation Network.  $i, i'$ : Indices to nodes,  $i, i' \in V$ ;  $m$ : Number of candidate DCs;  $n$ : Number of disaster areas;  $f_j$ : Fixed cost of establishing DC at candidate DC  $j$ ,  $j \in M$ ; (2) Relief.  $L$ : Set of reliefs;  $l$ : Indices to relief,  $l \in L$ ;  $D_i^l$ : quantity of relief  $l$  demanded by disaster area  $i$ ,  $l \in L, i \in N$ ;  $V_l$ : the unit volume of relief  $l$ ,  $l \in L$ ;  $Q_l$ : the amount of relief  $l$  available in traffic network,  $l \in L$ ; (3) Load flow.  $Q_{ii'}$ : Amount of relief type  $l$  carried from  $i$  to  $i'$ ,  $(i, i') \in E$ ;  $QA_{ii'}$ : Amount of relief type  $l$  supplied at node  $i$ ,  $i \in N$ ;  $QF_{ii'}$ : Amount of relief type  $l$  staying at node  $i \in N$  at the end of the operation; (4) Lorry flow.  $VA_{ik}$ : 1, if vehicle  $k$  is available at node  $i$ ; 0, else,  $i \in N$ ;  $VF_{ik}$ : 1, if the last demand point served by vehicle  $k$  is node  $i \in N$ ; 0, else; (5) Decision variables.  $x_j$ : 1, if candidate DC  $j$  are opened, 0, else,  $j \in M$ ;  $y_{ii'k}$ : 1, if vehicle  $k$  serve link  $(i, i')$  from  $i$  to  $i'$ , 0, else,  $k \in K, (i, i') \in E$ ;  $d_{ik}^l$ : The quantity of relief  $l$  distributed by  $k$  to demand point  $i$ ,  $i \in N$ .

### Multi-objective Relief Distribution

Objective 1: Minimization of the maximum vehicle route travelling time. Here, the time  $t_{i'ik}$  for vehicle  $k$  to traverse arc  $(i', i)$  can be calculated as:  $t_{i'ik} = d_{i'i} / \min(v_{i'i}, v_k)$ . Without serving time considered, the leaving time at node  $i$  of vehicle  $k$  is  $t_{ik} = t_{i'k} + t_{i'ik}$ , in which  $t_{i'k} = 0$  if  $x_{i'} = 1$ . The leaving time for vehicle from DC is assumed as 0. The travelling time  $t_k$  of route served by vehicle  $k$  ( $k \in K$ ) is equal to the serving finished time at the last node:  $t_k = t_{ik}$  for  $i \in N, VF_{ik} = 1$ . The formulation of objective 1 is as follows:

$$f_{time} = \text{Minimax} \{ t_k, k \in K \} \quad (1)$$

Objective 2: Minimization of relief distribution cost. Here distribution cost contains two components: the fixed cost  $f_j$  for establishing DC ( $j \in M$ ), and the vehicle flow cost

$\sum_{(i,i') \in A} c_k d_{ii'} y_{ii'k}$  ( $k \in K$ ). The objective function is as follows:

$$f_{cost} = \text{Min} \sum_{j \in M} f_j x_j + \sum_{k \in K} \sum_{(i,i') \in A} c_k d_{ii'} y_{ii'k} \quad (2)$$

Objective 3: Maximization of the minimum route reliability. Let's  $P_k$  denote the reliability for vehicle  $k$  to accomplish the corresponding distribution mission successfully. Assuming the links on route are independent of each other,  $P_k$  is the possibility of completing each link of the route  $k$ ,  $P_k = \prod_{(i,i') \in A, y_{ii'k}=1} r_{ii'}$ . The formulation for objective 3 is as follows:

$$f_{reliability} = \text{Maximin} \{ P_k, k \in K \} \quad (3)$$

Subject to

$$\sum_{i'/(i',i) \in A} \sum_{l \in L} Q_{i'il} + \sum_{l \in L} Q A_{il} = \sum_{i'/(i,i') \in A} \sum_{l \in L} Q_{ii'l} + \sum_{l \in L} Q F_{il}, \forall i, l \quad (4)$$

$$\sum_{i \in V} \sum_{l \in L} Q A_{il} = \sum_{i \in V} \sum_{l \in L} Q F_{il} \quad (5)$$

$$\sum_{i'/(i',i) \in A} \sum_{k \in K} y_{i'ik} + \sum_{k \in K} V A_{ik} = \sum_{i'/(i,i') \in A} \sum_{k \in K} y_{ii'k} + \sum_{k \in K} V F_{ik}, \forall i, k \quad (6)$$

$$\sum_{i \in V} \sum_{k \in K} V A_{ik} = \sum_{i \in V} \sum_{k \in K} V F_{ik} \quad (7)$$

$$V F_{ik} \leq y_{i'ik}, \forall i, k, (i', i) \in A \quad (8)$$

$$\sum_{i \in V} V F_{ik} = 1, \forall k \quad (9)$$

$$\sum_{i \in V} y_{ii'k} - \sum_{i \in V} y_{i'ik} = 0, \forall i' \in V, k \in K \quad (10)$$

$$D_i^l \geq \sum_{k \in K} d_{ik}^l > 0, \forall i, k, l \quad (11)$$

$$\sum_{k \in K} \sum_{i \in V} d_{ik}^l \leq Q_l, l \in L, i \in V \quad (12)$$

$$\sum_{l \in L} \sum_{i \in V} d_{ik}^l V_l \leq L_k, k \in K \quad (13)$$

$$d_{ik}^l \geq 0 \quad (14)$$

Constraints (4) are equations of load flow balance at disaster areas and DCs. Constraint (5) ensures that at the end of the distribution work the total relief remaining at the disaster areas and DCs is equal to the available relief. Constraints (6) are the equations of flow balance of the dispatched vehicles. Constraints (7) ensure that total number of vehicles staying at the disaster areas and DCs at the end of the distribution mission are the same as the available vehicles. Constraints (8) characterize that the disaster areas or DC at the end of the route  $k$  should be served by vehicle  $k$ . Constraints (9) mean that each vehicle should stay at only one disaster area or DC finally. Constraints (10) ensure that the vehicle arrives at and departs from the same point it serves. Constraints (11) guarantee that the total quantity of relief  $l$  distributed to a node  $i$  do not exceed the amount of demands in node  $i$ . Constraints (12) mean that the amount of relief served for relief demand point does not exceed the corresponding amount available. Constraints (13) represent the volume of all the relief loaded by vehicle  $k$  without exceeding its capacity. Constraints (14) mean that the relief  $l$  served by vehicle  $k$  for point  $i$  is not less than 0.

## NSDE Algorithm

### NSDE Operations

The conceptual schema of NSDE is illustrated in Figure 1. Series of sets solutions called generations are computed by NSDE. Each generation consists of  $NP$  chromosomes.

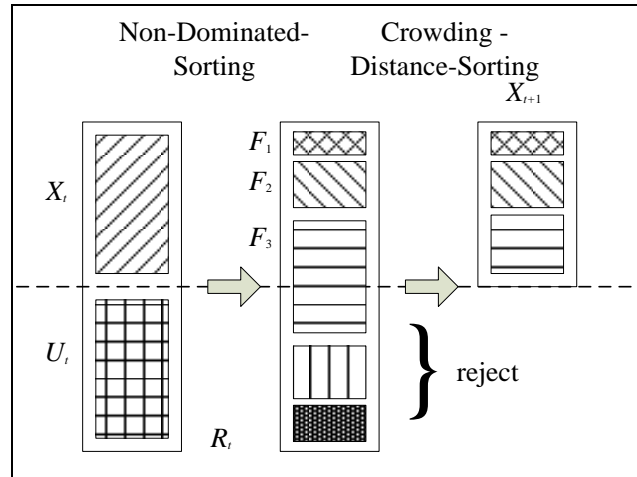


Figure 1: NSDE conceptual schemas, based on [22]

The initial operation is used to produce the initial population. Mutation and crossover operations are applied to get trial population  $U_t$  from  $X_t$  in  $t$  generation. The operations of NSDE are improved from original DE to solve the particular multi-objective OLRP in this paper. Selection operation is employed to choose new individuals for the  $t + 1$  generation. First combine the current population  $X_t$  and corresponding trial population  $U_t$  to form  $R_t$  with  $2NP$  size. Fast-Non-Dominated-Sort approach is used to sort them to get Pareto-optimal fronts in the present work (Deb 2002). Then sort individuals in front  $F_i$  using Harmonic-Average-Distance-Assignment operation (Huang 2005).  $NP$  individuals of generation  $P_{t+1}$  are selected according to their ranks and dominance distance.

### NSDE Procedure

The procedures of NSDE in present study are summarized as follow:

Step 1: Generate parent population  $X_t$  of size  $NP$ . Initial population  $X_0$  ( $t = 0$ ) is generated

randomly.

Step 2: Perform *DE* operations (mutation, crossover) over each individual in the population  $X_t$ . In this way trial vectors  $U_t$  of size  $NP$  are generated.

Step 3: Combine the parent population of  $X_t$  and trial population  $U_t$  together to form population  $R_t$ . Compute the objective values for each chromosome in  $R_t$ . Since all parent and current population are included in  $R_t$ , elitism is guaranteed.

Step 4: classify all the individuals in  $R_t$  into several ranks based on non-domination obtained by applying fast-non-dominated-sorting operation.

Step 5: calculate the crowding distance for each individual in any front  $F$  of  $R_t$ . Then sort all the individuals in front  $F$  in ascending order of magnitude according to crowding distance.

Step 6: select the best  $NP$  individuals based on their ranking and crowding distances. In the next generation, these  $NP$  individuals will act as the parent population.

Step 7: the procedure stops if the generation  $t$  is bigger than maximum of iteration times, else turn to Step 2.

## Case Study

### Test Instances

The 5.12 Wenchuan earthquake is studied in this part. The study is aimed at the 11 most severely and reachable disaster areas with 3 candidate DCs.

(1) Candidate DCs information. The candidate DCs in this case are Chengdu, Mianyang, Guangyuan which are shown in Table 1.

Table1–Candidate DCs parameters

	Chengdu ( $J_1$ )	Mianyang ( $J_2$ )	Guangyuan ( $J_3$ )
Capacity	30000	25000	20000
Fixed cost(yuan)	10000	15000	20000

(2) Relief. The size of a tent is used as the criterion for measuring volume equivalents. The parameters of relief can be found in Table 2.

Table2– Parameters of relief commodity

Item	Available quantity	Calculation unit	Unit volume( $\text{cm}^3$ )	Volume equivalent	Per need
Tent $L_1$	2000	Each	$45 \times 25 \times 11$	1.00(2 person)	0.50
Mineral water $L_2$	6000	Box	$36 \times 26 \times 30$	2.27(6 person)	0.38

(3) Vehicle. The vehicles used for transportation are of 3 types (10 big Military vehicles, 6 medium Military vehicles and 9 small Civilian trucks). For more detail see Table 3.

Table3–Parameters of the vehicles

Vehicle name	Loading capacity ( $\text{cm}^3$ )	Volume equivalent	Normal velocity (kilometer / hour)	Cost per unit of length(Yuan/kilometer)
Military vehicle $K_1$	$600 \times 250 \times 175$	2121.21	50	10.0
Military vehicle $K_2$	$280 \times 200 \times 145$	656.16	30	3.1
Civilian truck $K_3$	$231 \times 150 \times 130$	364.00	20	1.7

(4) Demand information. Here we study on the first day of the earthquake. The relief demands for each disaster area are as follow Table 4.

Table 4–Parameters of the disaster areas

Demand points	Demands (nylon sleeping bags, mineral water)	Demand points	Demands (nylon sleeping bags, mineral water)
Wenchuan $V_1$	(3458,1153)	Anxian $V_7$	(1348,449)
Jinzhu $V_2$	(3647,1216)	Pingwu $V_8$	(3215,1072)
Beichuan $V_3$	(969,323)	Pengzhou $V_9$	(577,192)
Qingchuan $V_4$	(1545,515)	Jiangyou $V_{10}$	(1002,334)
Maoxian $V_5$	(818,273)	Deyang $V_{11}$	(3199,1066)
Dujiangyan $V_6$	(439,146)		

(5) Traffic network status. The state and provincial highway, county roads are taken into consideration in this test instance study. The topology of the transportation network is illustrated in Figure2. The Demanding nodes labeled from 1 to 11 and 3 candidate DCs labeled from 12 to 14 are shown in the network. The node labeled 15 is working as transship node.

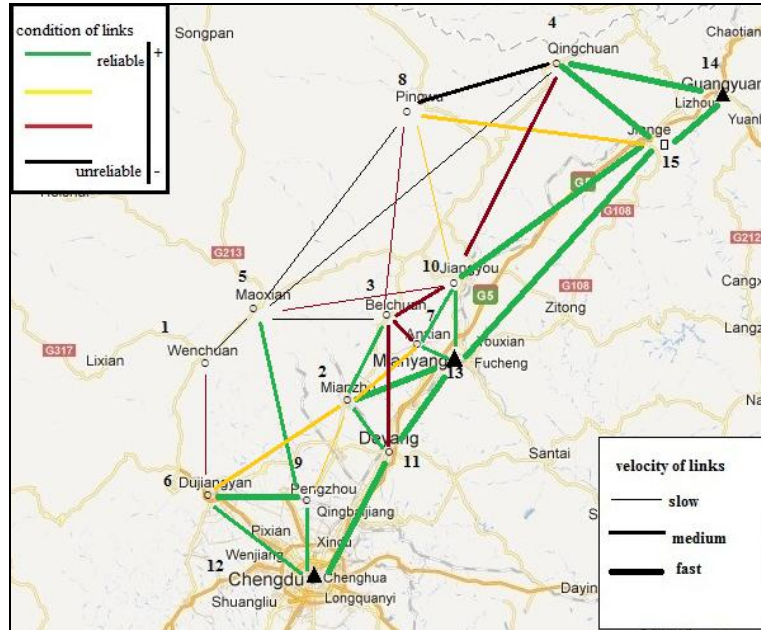


Figure 2–Transport network at Sichuan

### Computational Results

The algorithm described in Section4 is programmed at Matlab®6.5. With considerable research data, the best values of control variables are:  $NP=66$ , for NSDE  $F=0.5$ ,  $CR=1-t/\maxgen$ ; for NSGA-II  $p_m=0.7$ ,  $p_c=0.7$ .

Figure 3 illustrates the Pareto-optimal solution based on Objective 1. It can be seen that the DCs labeled 12, 13 are open. The longest route travelling time is 7.6 hours. Only the Military vehicle  $K_1$  and  $K_2$  with high velocity and large capacity are used for nodes far from DCs, such as



disaster areas 1, 4, 5, 8. Figure 4 describes the solution of Objective 2. Compared to Figure 3, more links with high travel speed are chosen. Only the candidate DC labeled 12 is chosen for reducing high fixed cost. However, Unreliable link 4-8 and 1-5 are used in Figure 3 and 4 respectively, the solution illustrated in Figure 5 for Objective 3 does not include unreliable links.

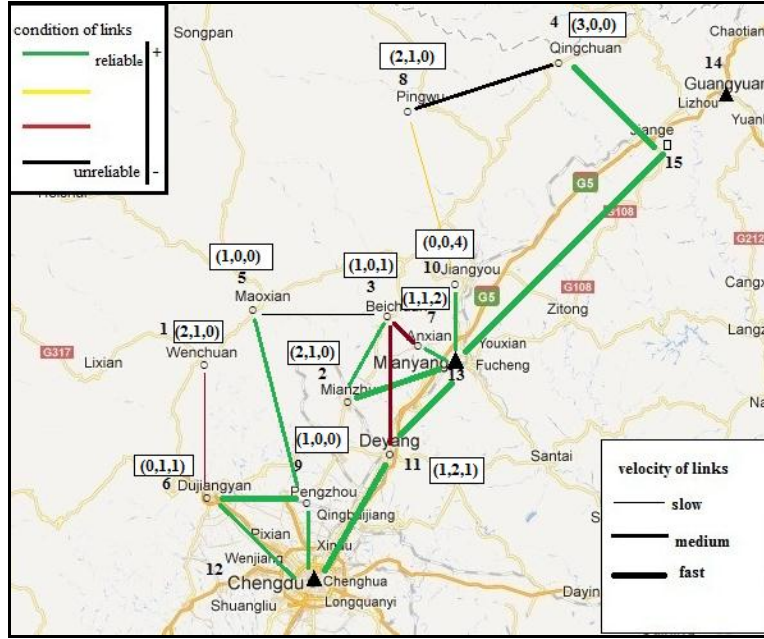


Figure 3–Solution minimizing the maximum route time

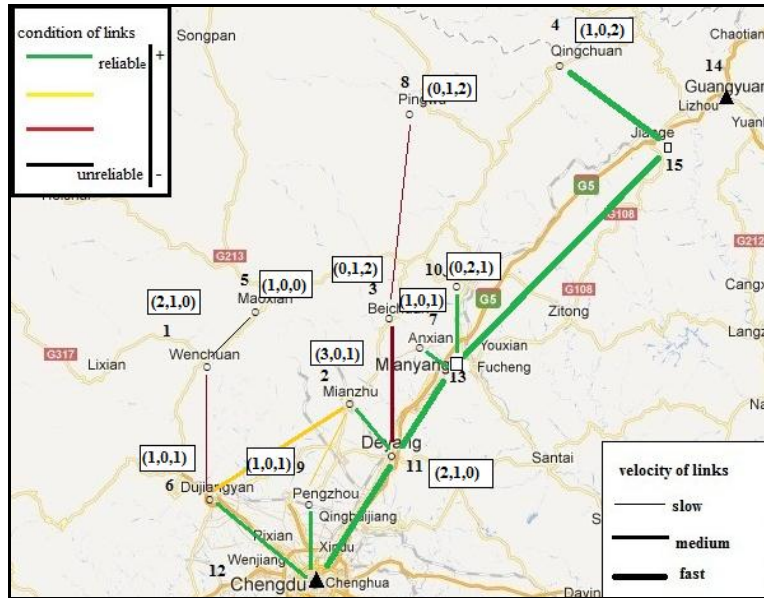


Figure 4–Solution minimizing total cost

### Comparison with NSGA-II

Pareto optimal solutions obtained on test instance are shown in Figure 6. Approximately 31 non-dominated solutions are got by NSDE algorithm while only 10 obtained by NSGA-II. And solutions got by NSDE dominate most of the NSGA-II solutions.



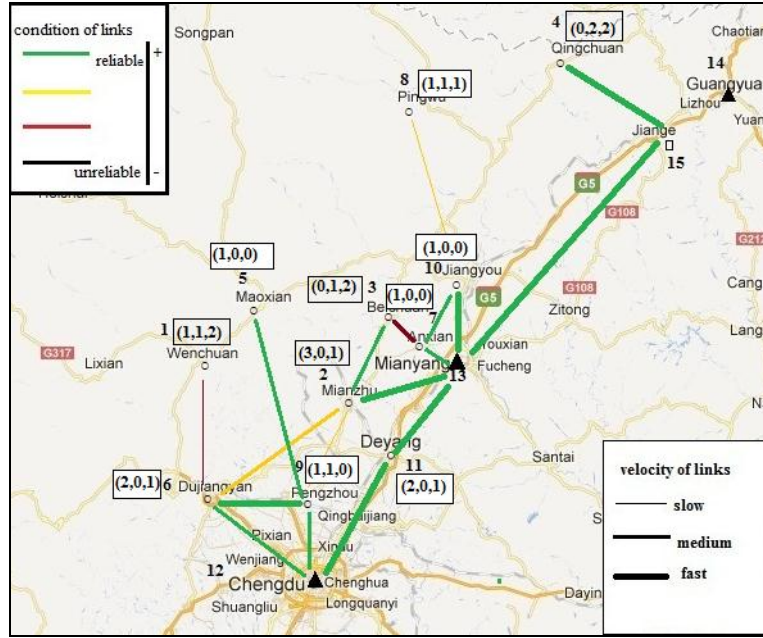


Figure 5–Solution maximizing minimum route reliability

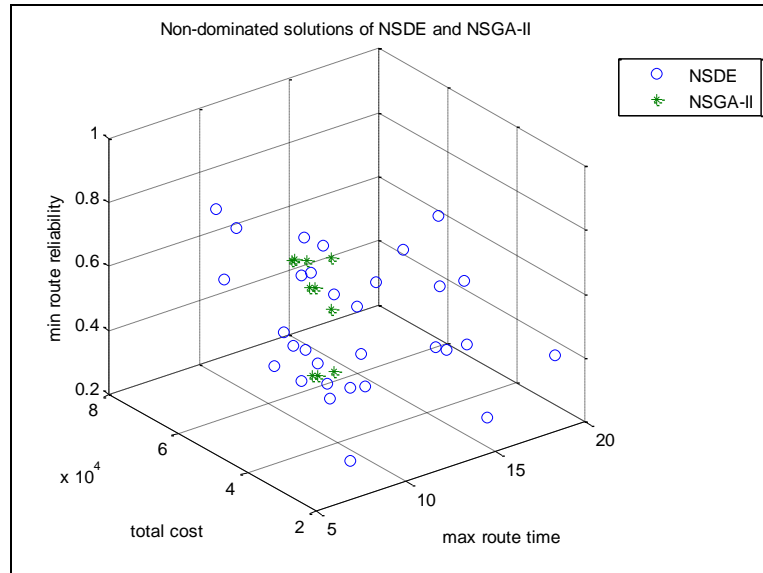


Figure 6–Non-dominated solutions of NSDE and NSGA-II

Table 5 shows performance measures for test instance with NSDE and NSGA-II. The first two metrics values of NSDE are better than those of NSGA-II. For the third one, the results are equal with each others.

Table 5–Performance measures for test instance with NSDE and NSGA-II

	Min-max route time(hour)	Min total cost(yuan)	Max-min route reliability
NSGA-II	7.8	37296	0.8
NSDE	7.6	30508	0.8
Relative improvement (%)	2.5	18.2	0

## Conclusion and Future Research

In this paper we present multi-objective OLRP model. An efficient solution approach is proposed to solve this model. With this approach, a set of Pareto-optimal solution is presented to the decision makers, which help to make decisions according to their preference. The results have shown that the solution approach works well for test instance. This provides a profound basis for extending our method to more realistic assumptions.

In the future research the multi-periods OLRP will be considered. The post disaster situation under consideration is characterized by a high grade of uncertainty. Therefore a method shall be developed considering stochastic components of the problem. There is still a potential for improving the operations performance of relief distribution in emergency logistics.

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