Absenteeism and Nurse Staffing

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We use data from multiple nursing units of two hospitals to study which factors, including unit culture, short-term workload, and shift type explain nurse absenteeism. This analysis forms the basis for a staffing model with heterogeneous nurses. The proposed staffing strategy is cost-effective and useful for adjusting staffing levels and allocating nurses among multiple units.

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1. Introduction

Inpatient units are often organized by nursing skills required to provide care – e.g. a typical classification of nursing units includes the following tiers: intensive care (ICU), step-down, and medical/surgical. Several units may exist within a tier, each with a different specialization. For example, different step-down units may focus on cardiac, neurological, and general patient populations. Each nurse is matched to a home unit in which skill requirements are consistent with his or her training and experience. Nurses prefer to work in their home units and their work schedules are fixed several weeks in advance. Once finalized, schedules may not be changed unless nurses agree to such changes. A finalized staffing schedule is also subject to random changes due to unplanned nurse absences. These facts complicate a nurse manager's job of scheduling nurses to match varying demand and supply. To illustrate these points, we provide in Figure 1 a time-series plot of percent of occupied beds (i.e. average census divided by bed capacity), and percent of absent nurse shifts (unplanned) from three step-down units of an urban 466-bed community hospital between Jan 3rd, 2009 and Dec 4th, 2009. Note that patient census and nurse absentee rate vary significantly from one day to the next, which makes staffing decisions challenging.

Figure 1: Percent occupied beds (upper panel) and absent nurse shifts (lower panel) by shift.
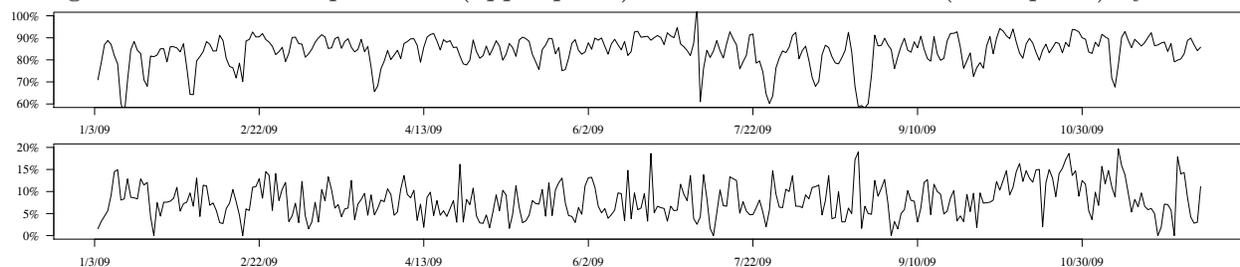
Nurses' absence from work may be either planned or unplanned. Planned absences, such as scheduled vacations, and continuing education classes and training, are easier to cope with because a nurse manager has advance warning of potential staff shortages created by such absences. In contrast, unplanned absences often require the use of expensive contingent workforce and may compromise patient safety or quality of care because replacements can be difficult to find at a short notice. For these reasons, our focus in this paper is on unplanned absences. There are a whole host

of reasons why nurses may take unplanned time off; see Davey et al. (2009) for a systematic review. This literature suggests that causes of absenteeism vary among different groups of nurses in the same hospital, and fluctuate overtime(Johnson et al. 2003). It also concludes that nurse absences are associated with organization norms, nurses' personal characteristics, chronic work overload and burnt out. Green et al. (2011) uses an econometric model to show that nurses' anticipated workload (measured by the staffing level relative to long-term average census) is positively correlated with their absentee rate in data from one emergency department (ED) of a single hospital. This motivates us to investigate using a larger data set from multiple hospitals whether anticipated workload explains variation in nurse absenteeism and how nurse managers may use this information to improve staffing decisions.

Absences may be either voluntary or involuntary. Sickness, caregiver burden, major weather events, and traffic disruptions are examples of involuntary reasons for absence and their root causes cannot be affected by nurse managers. In contrast, work stress and undesirable shift times are examples of reasons whose root causes may be addressed by hospital management. Although it is useful for nurse managers to understand the difference between voluntary and involuntary reasons for being absent, it is often not possible to distinguish between these causes of absence from historical data. We can also clarify absence into two groups based on whether absence is due to unit-level or individual-level (nurse-specific) reasons. For example, unit culture (the extent to which nurses feel responsible for showing up as scheduled), unit manager's effectiveness, and workload are unit-level factors. In contrast, how each nurse copes with caregiver burden and work stress are examples of nurse-specific factors.

When unplanned absences are high, a hospital can benefit from having a predictive model that explains absences as a function of observable explanatory variables. This model may be used to improve staffing decisions as well as to develop coping and pro-active strategies for reducing unplanned absences. In this paper, we utilize the second classification scheme mentioned earlier and obtain models that predict nurses' absentee rate as a function of unit-level and individual-level factors. Unit-level factors include long-term average workload, short-term anticipated or realized workload, shift time and day of week. The individual-level factor considered in this paper is the history of absence of each nurse. We also propose models that can achieve more robust staffing based on individual absence patterns. A summary of our models and results is presented next.

We used data from multiple inpatient units of two hospitals to evaluate predictors of absenteeism via two statistical models. The first model assumes nurses are homogeneous decision makers and finds the extent to which the variability in nurse absences is explained by unit-level factors such as unit index (which captures unit culture, manager effectiveness and long-term workload), shift time, short-term anticipated workload, and interactions among these factors. The second model assumes that absentee rates are not homogeneous and tests the hypothesis that nurses' past absence records can be used to predict their absences in the near future.

For the first statistical model, we propose three possible metrics for quantifying short-term anticipated workload depending on the stability of census and planned staffing levels across shifts.

1. Ratio of planned staffing level and long-run average census – suitable for stable census.

2. Ratio of previous m-shift average census and planned staffing level – accommodates both census and staffing level variations.

3. Previous m-shift average census – suitable for stable planned staffing level.

Parameter m was varied from 1 to 12 and a factorial design with all two-way interactions was employed when carrying out the analysis. We found that unit index had a significant effect on how nurses as a group responded to the anticipated workload, but that there did not exist a consistent relationship between workload and nurses' absenteeism after controlling for other factors.

In the second model, which utilized individual-level data, we found that nurses had heterogeneous absentee rates and each nurse's absentee rate was relatively stable over the period of time for which data was obtained. Consistent with the literature (see, e.g. Davey et al. 2009), we also found that a nurse's history of absence from an earlier period was a good predictor of his or her absentee rate in a future period. Therefore, we conclude that a nurse manager must account for heterogeneous attendance history when making staffing plans with a cohort of staff nurses. This forms the basis of the model-based investigations presented in this paper. In particular, we present a model to determine near-optimal nurse assignment to interchangeable units and shifts. This model shows that hospitals can achieve a robust operational performance by taking into account the differences in nurses' absentee rates.

The results from our analysis of data are significantly different from those reported in Green et al. (2011) who found that greater short-term workload was correlated with greater absenteeism.

There are a variety of explanations for these differences. First, inpatient units and EDs face different demand patterns and patients' length-of-stay. Patients stay significantly longer in inpatient units[1]. Second, it may be argued that EDs present a particularly stressful work environment for nurses and that ED nurses may react differently to workload than inpatient-unit nurses. Third, unlike Green et al. (2011), we use data from multiple units and two hospitals, which allows us to quantify the effects due to unit index and the interaction between unit index and shift index.

This paper contributes to literature by investigating whether there exist a consistent relationship between short-term workload and nurse absences, and by identifying an observable nurse characteristic that can be used to improve staffing decisions. With the exception of Green et al. (2011), previous papers have not focused on incorporating nurse absence in staffing decisions. Much of the health service research literature concerns the impact of inadequate staffing levels on quality of care, patients' safety and length of stay, nurses' job satisfaction, and hospital's financial performance (e.g. Unruh (2008), Aiken et al. (2002), Needleman et al. (2002), Cho et al. (2003), Lang et al. (2004), and Kane et al. (2007)). In contrasts, operations research/management literature has mainly focused on developing nurse schedules to minimize costs and maximizes nurses' work preferences to meet target staffing levels; see Lim et al. (2011) for a recent review of nurse scheduling models. Green et al. (2011) is the first paper that investigates endogenous nurse absentee rate and models it as a function of staffing level based on data from an ED. However, the generalizability of this model to other settings has not been established. We propose a model that is suitable for inpatient units.

Nurse managers are frequently charged with deciding whether to add or subtract nurse shifts from the planned staffing level based on the projected demand one or two shifts in advance. It is generally cheaper and more efficient to recruit nurses (part-time or full-time) for extra assignments (as either extra-time or overtime) on a volunteer basis than to hire agency nurses. The former requires nurse managers to plan ahead. By utilizing the method proposed in this paper, nurse managers can calculate the minimum number of extra shifts needed by taking into account absentee rates of previously scheduled nurses and those recruited for extra assignments.

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[1] According to two surveys done in 2006 and 2010, average length of stay in emergency rooms (delay between entering emergency and being admitted or discharged) was 3.7 hours and 4.1 hours respectively (Ken 2006, Anonymous 2010). In contrast, the average length of inpatient stay in short-stay hospitals was 4.8 days according to 2007 data (Table 99, part 3 in National Center for Health Statistics 2011).

The remainder of this paper is organized as follows. In Section 2, we present institutional background and data summary. Statistical models and analyses are presented in Section 3. We find that nurses have heterogeneous absentee rates and that nurses' absentee rates remain unchanged in our data set. These observations lead to a cost-evaluation model proposed in Section 4. We evaluate the impact of ignoring absentee rate and compare the performance of different staffing strategies in Section 5. Section 6 concludes the study.

2. Institutional Background

We studied de-identified census and absentee records from two hospitals located in Minneapolis–Saint Paul metropolitan area. Basic information about these hospitals from fiscal year 2009 is summarized in Table 1. The difference between maximum and minimum patient census was 52.7% and 49.5% of the average census for Hospitals 1 and 2, respectively, indicating that the overall variability in nursing demand was high. Patients' average lengths of stay were 3.99 and 4.88 days and register nurse (RN) salary accounted for 17.9% and 15.5% of total operating expenses of the two hospitals. This indicates that even a small percent improvement in nurse staffing cost could result in a significant cost savings.

Table 1: Fiscal year 2009 statistics for the two hospitals.

	Hospital 1	Hospital 2
Available beds	466	284
Maximum Daily Census	359	242
Minimum Daily Census	205	149
Average Daily Census	292	188
Admissions	30,748	14,851
Patient Days	114,591	68,924
Admissions through ER	14,205	6,019
Acute Care Admissions	26,751	13,694
Acute Patient Days	106,625	66,842
Average Acute Care Length of Stay (days)	3.99	4.88
Number of RN FTEs	855.93	415.42
RN salary expenses (in 000s)	$73,190	$36,602
Total Operating Revenue (in 000s)	$431,218	$231,380
Total Operating Expenses (in 000s)	$408,472	$235,494
Total Operating Income (in 000s)	$22,745	$-4,117

Available beds = number of beds immediately available for use (ties to staffing level).
RN = registered nurse. FTE = full-time equivalent.

Census and absentee data from Hospital 1 were for the period January 3, 2009 through December 4, 2009, whereas Hospital 2's data were for the period September 1, 2008 through August 31, 2009. Hospital 1 had five shift types. There were three 8-hour shifts designated Day, Evening, and Night shifts, which operated from 7 AM to 3 PM, from 3 PM to 11 PM, and from 11 PM to 7 AM, respectively. There were also two 12-hour shifts, which were designated Day-12 and Night-12 shifts. These operated from 7 AM to 7 PM, and 7 PM to 7 AM, respectively. Hospital 2 had only three shift types, namely the 8-hour Day, Evening, and Night shifts. Hospital 1's data pertained to three step-down (telemetry) units labeled T1, T2, and T3 with 22, 22, and 24 beds, and Hospital 2's data pertained to two medical/surgical units labeled M1 and M2 with 32 and 31 beds. The common data elements were hourly census, hourly admissions discharges and transfers (ADT), planned/realized staffing levels, and the count of absentees for each shift. Hospital 1's data also contained individual nurses' attendance history. The two health systems' data were analyzed independently because (1) the data pertained to different time periods, (2) the target nurse-to-patient ratios were different for the two types of nursing units, and (3) the two hospitals used different staffing strategies.

Hospital 1's target nurse-to-patient ratios for telemetry units were 1:3 for Day and Evening shifts during week days and 1:4 for Night and weekend shifts. Hospital 2's target nurse-to-patient ratios for medical/surgical units were 1:4 for Day and Evening shifts and 1:5 for Night shifts. Hospital 1's planned staffing levels were based on the mode of the midnight census in the previous planning period. Nurse managers would further tweak the staffing levels up or down to account for holidays and to meet nurses' planned-time-off requests and shift preferences. Hospital 2's medical/surgical units had fixed staffing levels based on the long-run average patient census by day of week and shift. In both cases, staff planning was done in 4-week increments and planned staffing levels were posted 2-weeks in advance of the first day of each 4-week plan. Consistent with the fact that average lengths of stay in these hospitals were between 4 and 5 days, staffing levels were not based on a projection of short-term demand forecast. When the number of patients exceeded the target nurse-to-patient ratios, nurse managers attempted to increase staffing by utilizing extra-time or overtime shifts, or calling in agency nurses. Similarly, when census was less than anticipated, nurses were assigned to indirect patient care tasks or education activities, or else asked to take voluntary time off. These efforts were not always successful and realized nurse-to-patient ratios often differed from the target ratios. For example, Hospital 1's unit T3 on average staffed lower than the target ratios,

whereas Hospital 2's unit M2 on average staffed higher than the target ratios during weekends; see Table 2.

Table 2: Number of patients per nurse by shift type.

Shift Type	TR	Unit	Mean	SD	95% CI
Weekday	3	T1	2.98	0.60	(2.92, 3.03)
Day or	3	T2	3.25	0.63	(3.20, 3,31)
Evening shifts	3	T3	2.81	0.81	(2.73, 2.88)
Weekends	4	T1	3.85	1.04	(3.76, 3.94)
and	4	T2	3.77	1.29	(3.66, 3.88)
Night shifts	4	T3	3.04	0.67	(2.98, 3.09)
Weekdays	4	M1	2.99	0.56	(2.95, 3.03)
	4	M2	3.14	0.39	(3.12, 3.17)
Weekends	5	M1	4.97	1.07	(4.90, 5.05)
	5	M2	5.21	0.80	(5.15, 5.27)

TR = target number of patients per nurse

SD = standard deviation. CI = confidence interval.

Absentee rate statistics are summarized in Table 3. This table shows that the absentee rate for Hospital 1's three nursing units varied from 3.4% (T1, Sunday, Day Shift) to 18.3% (T1, Saturday, Night Shift) depending on unit, shift time, and day of week. Among the 154 nurses who worked in the three units of Hospital 1, the average absentee rate within our data set was 10.78% with a standard deviation of 9%. The first and third quartile were 4.8% and 14.9% respectively. Similarly, the absentee rate for Hospital 2's two nursing units varied from 2.99% (M1, Wednesday, Evening Shift) to 12.98% (M2, Tuesday, Evening Shift) depending on unit, shift time, and day of week. At first glance, these statistics suggest that absentee rates were significantly different by nursing unit and shift, but not by the day of week (95% confidence intervals do not overlap across unit and shift type). We also compared absentee rates for regular days and holidays, and fair-weather days and storm days. At 5% significance level, holidays had a lower average absentee rate than non holidays for Hospital 2 and bad weather days had a higher average absentee rate for Hospital 1.

3. Statistical Models & Results

We next present two models to evaluate different predictors of nurse absenteeism. The choice of potential predictors was limited to those factors that were included in the de-identified data obtained from historical records as described in Section 2.

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Table 3: Absentee Rate by Unit, Day of Week (DoW), Shift, Holidays, and Storm Days. SD = Standard Deviation. CI = Confidence Interval.

Hospital 1				Hospital 2			
Unit	Mean	SD	95% CI	Unit	Mean	SD	95% CI
T1	0.101	0.134	(0.093, 0.109)	M1	0.062	0.104	(0.057, 0.068)
T2	0.082	0.111	(0.075, 0.089)	M2	0.091	0.118	(0.085, 0.098)
T3	0.067	0.091	(0.061, 0.073)	–	–	–	–
DoW	Mean	SD	95% CI	DoW	Mean	SD	95% CI
Sun	0.077	0.118	(0.066, 0.089)	Sun	0.076	0.112	(0.065, 0.087)
Mon	0.093	0.124	(0.081, 0.104)	Mon	0.082	0.120	(0.071, 0.094)
Tue	0.077	0.103	(0.068, 0.087)	Tue	0.084	0.120	(0.072, 0.095)
Wed	0.074	0.100	(0.065, 0.083)	Wed	0.072	0.108	(0.062, 0.083)
Thu	0.086	0.111	(0.075, 0.096)	Thu	0.077	0.112	(0.066, 0.087)
Fri	0.088	0.116	(0.077, 0.099)	Fri	0.073	0.102	(0.063, 0.083)
Sat	0.086	0.123	(0.074, 0.098)	Sat	0.075	0.107	(0.064, 0.085)
Shift	Mean	SD	95% CI	Shift	Mean	SD	95% CI
Day	0.058	0.090	(0.053, 0.064)	Day	0.057	0.083	(0.051, 0.063)
Evening	0.085	0.096	(0.079, 0.091)	Evening	0.075	0.098	(0.067, 0.082)
Night	0.107	0.143	(0.098, 0.116)	Night	0.088	0.129	(0.081, 0.094)
Day 12-hour	0.134	0.341	(0.112, 0.156)	–	–	–	–
Night 12-hour	0.228	0.420	(0.201, 0.256)	–	–	–	–
Holiday[a]	Mean	SD	95% CI	Holiday[a]	Mean	SD	95% CI
Non holiday	0.083	0.113	(0.079, 0.087)	Non holiday	0.078	0.112	(0.074, 0.082)
Holiday	0.084	0.138	(0.054, 0.114)	Holiday	0.051	0.101	(0.032, 0.070)
Storm[b]	Mean	SD	95% CI	Storm[b]	Mean	SD	95% CI
No	0.082	0.114	(0.078, 0.087)	No	0.077	0.112	(0.073, 0.081)
Yes	0.129	0.149	(0.088, 0.171)	Yes	0.10	0.126	(0.052, 0.143)

[a]Holidays include US federal holidays and the day before Thanksgiving and Christmas.
[b]Storm days were 2/26/09, 5/5/09, 8/2/09, 8/8/09, 10/12/09, 12/8/09, and 12/23/09 according to National Climatic Data Center (2012).

3.1 Unit-Effects Model

In the first model, nurses' absentee rate for a particular shift is assumed to depend both on factors that are relatively stable and factors that vary. Factors in the former category include long-term average demand and staffing levels, unit culture, and desirability of certain shift start times. These factors are represented by fixed effects for unit, day of week, and shift. The factors that vary within our data are census levels and nurse availability, which is represented by the short-term anticipated workload w_t. Finally, all the unexplained variation in absentee rate is considered as random effects.

Given unit index $i \in \{1, \cdots, u\}$, shift type $j \in \{1, \cdots, v\}$, and day-of-week $j \in \{1, \cdots, 7\}$, a

logistic regression model was used to estimate π_t, the probability that a nurse will be absent in shift t if the anticipated workload for that shift is w_t. Note that the index t is the notation we use to represent an arbitrary shift in the data. Each shift t can be mapped to exactly one (i, j, k) triplet, each (i, j, k) may map to several shifts with different shift indices in our data. A full factorial model for estimating π_t is

$$
\begin{aligned}
log(\frac{\pi_t}{1 - \pi_t}) &= \mu + \sum_{i=2}^{u} \beta_i U_i + \sum_{j=2}^{v} \alpha_j S_j + \sum_{k=2}^{7} \xi_k D_k + \rho H_t + \lambda Y_t + \gamma w_t \\
&+ \sum_{i=2}^{u} \sum_{j=2}^{v} \eta_{i,j}(U_i * S_j) + \sum_{i=2}^{u} \sum_{k=2}^{7} \vartheta_{i,k}(U_i * D_k) + \sum_{i=2}^{u} \iota_i(U_i * w_t) \\
&+ \sum_{j=2}^{v} \sum_{k=2}^{7} \varsigma_{j,k}(S_j * D_k) + \sum_{j=2}^{v} \phi_j(S_j * w_t) + \sum_{k=2}^{7} \nu_j(D_k * w_t) \\
&+ \text{(the remaining higher-order interaction terms)},
\end{aligned}
\tag{1}
$$

where U_i, S_j, and D_k are indicator variables. In particular, $U_i = 1$ if the nurse under evaluation worked in unit i and $U_i = 0$ otherwise. Similarly, $S_j = 1$ (respectively $D_k = 1$) if the nurse was scheduled to work on a type-j shift (respectively day k of the week). H_t and Y_t are also indicator variables that are set equal to 1 if shift t occurred on a holiday or bad-weather day, respectively. The unit, shift, and day-of-week with the smallest indices are used as the benchmark group in the above model. We use $(a * b)$ to denote the interaction term of a and b. In the ensuing analysis, all two-way interactions are included in the initial model whereas higher-order interaction terms are omitted. This is done because higher order interaction terms do not have a practical interpretation (see Faraway 2006 for details). Notation and assumptions are also summarized in Table 4.

Nurses' perception of short-term workload is measured by w_t. We used three different versions of w_t in our analysis: (1) $w_t^{(1)} = n_t/E[C_t]$ and (2) $w_t^{(2)} = \sum_{i=1}^{m}(c_{t-m}/m)(1/n_t)$, and (3) $w_t^{(3)} = \sum_{i=1}^{m}(c_{t-m}/m)$, where n_t is the planned staffing level, c_t is the start-of-shift census for shift t, and $E[C_t]$ is the long-run expected census. Put differently, $w_t^{(1)}$ equals the anticipated nurse-to-patient ratio; $w_t^{(2)}$ equals the m-period moving average of estimated number of patients per nurse; and $w_t^{(3)}$ equals the m-period moving average census. The choice of $w_t^{(1)}$ is appropriate for units with stable nursing demand, $w_t^{(2)}$ for units in which a demand surge can cause high census to last several shifts

Table 4: Unit-Effects Model Notation and Assumptions

Covariate	Description	Coefficient
π_t	absentee rate for a shift t	none
U_i	indicator variable for unit i.	β_i
S_j	indicator variable for shift type j	α_j
D_k	indicator variable for day k of the week	ξ_k
H_t	indicator variable for holiday shifts	ρ
Y_t	indicator variable for storm-day shifts	λ
w_t	short-term anticipated workload for shift t	γ
$(U_i * S_j)$	unit and shift interaction	$\eta_{i,j}$
$(U_i * D_k)$	unit and day of week interaction	$\vartheta_{i,k}$
$(U_i * w_t)$	unit and workload interaction	ι_i
$(S_j * D_k)$	shift and day of week interaction	$\varsigma_{j,k}$
$(S_j * w_t)$	shift and workload interaction	ϕ_j
$(D_k * w_t)$	day of week and workload interaction	ν_j

Assumptions:

1. Independent and homogeneous nurses.

2. A nurse's attendance decision for a particular shift is independent of his/her decisions for other shifts.

due to patients' lengths of stay, and $w_t^{(3)}$ for units that have constant staffing levels (such as in Hospital 2). We tested $w_t^{(2)}$ and $w_t^{(3)}$ with $m = 1, 2, \cdots, 12$. The results were consistent in the sign and significance of the parameters estimated and there was little variation in the estimated values of coefficients.

The model in (1) has the following interpretation: μ represents the long-term effect of being scheduled to work in unit 1, shift 1, and day 1 of the week (benchmark group). The effect of being scheduled to work in a different unit i for shift j on day k of the week can be obtained relative to the benchmark group. For example, the odds ratio of being absent for a test-case (i, j, k) schedule as compared to the $(1, 1, 1)$ benchmark schedule is captured by $exp(\beta_i + \alpha_j + \xi_k + \eta_{i,j} + \theta_{i,k} + \varsigma_{i,j})$ when both test-case and benchmark cases fall on non holidays with normal weather and short-term anticipated workload does not affect absentee rate. The explanatory variables in (1) capture the systematic variation in nurses' absentee rates due to unit, shift time, day of week and their interactions. Because long-term workload is included in these factors, we do not include that as a separate predictor. We also do not include week- or month-of-year effect because of data limitations[2].

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[2]With approximately 1 year of data, observations of higher/lower absentee rate in certain weeks are not informative about future absentee rates in those weeks. Also, when week of year was included as a explanatory variable, this

We used stepwise variable selection processes to identify significant explanatory factors. A summary of our results with $m = 6$ is reported in Table 6. It shows that Unit and Shift effect were significant for both datasets, but bad weather effect was not. However, holiday effect was significant for Hospital 2's data. Two of three workload measures produced consistent results for both hospitals' data – neither $w_t^{(1)}$ nor $w_t^{(2)}$ were statistically significant. Anticipated workload was significant only for Hospital 2's data when $w_t = w_t^{(3)}$, and the coefficient was positive for one unit and negative for another. In particular, higher workload would decrease the odds ratio of probability of being absent in M1 and increase this quantity in M2. We also tried a variant of our model in which w_t was replaced by the m-shift realized nurse-to-patient ratios[3] for both datasets and the conclusion that there was not a consistent relationship between short-term anticipated workload and nurses' absenteeism remained intact. The presence of inconsistent relationship makes it difficult to incorporate short-term workload related absenteeism in staffing decisions.

Table 5: Hospital 1 Summary.

Coefficients	Estimate	SE	Wald Test p-value
(Intercept)	-2.76	0.085	< 0.001
T2	0.159	0.119	0.182
T3	-0.317	0.122	0.010
Evening	0.499	0.110	< 0.001
Night	1.044	0.112	< 0.001
T2*Evening	-0.150	0.155	0.333
T3*Evening	0.184	0.155	0.236
T2*Night	-0.759	0.164	< 0.001
T3*Night	-0.430	0.164	0.009

Benchmark unit = T1.
Null deviance: 3345.5 on 3023 degrees of freedom.
Residual deviance: 3163.3 on 3015 degrees of freedom.
Goodness of fit test: p-value = 0.029.

Table 6: Hospital 2 Summary.

Coefficients	Estimate	SE	Wald Test p-value
(Intercept)	-2.471	0.277	< 0.001
M2	-1.004	0.442	0.023
Evening	0.150	0.123	0.223
Night	0.644	0.106	< 0.001
Holiday	-0.413	0.176	0.019
M2*Evening	0.208	0.154	0.176
M2*Night	-0.306	0.138	0.027
M1*w_t	-0.025	0.011	0.017
M2*w_t	0.035	0.013	0.006

Benchmark unit = M1; $w_t = w_t^{(3)}$.
Null deviance: 3232.8 on 2907 degrees of freedom.
Residual deviance: 3076.1 on 2899 degrees of freedom.
Goodness of fit test: p-value = 0.011.

Upon further examination, we found that the logistic regression model did not fit the data very well – the goodness of fit test rejected the null hypothesis that the model was a good fit. For example, the two models in Tables 2 and 3 respectively resulted in a p-value of 0.029 and 0.011. This happened because of the large residual deviances of the unit-level model. The lack of fit may

resulted in some covariate classes with too few observations. For example, there were only 3 nurses who were scheduled to work during week 2 (the week of 1/4/09 – 1/10/09) Monday Night shift in Unit 1 of Hospital 1.

[3]Note, this assumes nurses have advance knowledge of how many of nurse shifts will be short relative to n_t net of absences and management action to restore staffing levels in shift t when making their attendance decisions.

be caused by a variety of reasons. For example, it is possible that the unit, shift, and day of week pattern in absentee rate are confounded with individual nurses' work patterns – some high absentee rate nurses may have a fixed work pattern that contributed to the high absentee rates for some shifts. Therefore, we also evaluated whether unit, shift, and day of week effects still exist after we account for individual nurses' work patterns. We used a generalized estimating equation (GEE) to fit a repeated measure logistic regression model with Hospital 1's data. GEE, introduced by Zeger and Liang (1986), can be used to analyze correlated data in which subjects are measured at different points in time. The difference between a standard generalized linear model (GLM) and the GEE model is that the GEE model takes into account correlated observations (same nurses are scheduled multiple times) whereas a standard generalized linear model assumes independent observations. Because we had nurse-level data only from Hospital 1, we could apply GEE model to Hospital 1' data only.

Table 7: Hospital 1 GEE Model Summary.

Coefficients	Estimate	SE	Wald Test p-value
(Intercept)	2.455	0.185	< 0.0005
T2	-0.015	0.205	0.942
T3	0.243	0.215	0.259
Evening	-0.543	0.136	< 0.0005
Night	-0.736	0.193	< 0.0005
Mon	-0.095	0.124	0.443
Tue	0.158	0.1232	0.199
Wed	0.254	0.1202	0.035
Thu	0.071	0.1055	0.501
Fri	0.127	0.1197	0.29
Sat	-0.126	0.0956	0.188

Benchmark unit = T1; Benchmark shift = Day;
Benchmark day of week = Sunday.

The result in Table 7 shows that shift effect and day of week effects were significant while accounting for individual nurses' effect. However, unit effect was no longer significant. This observation was different from the model in which we assumed independent and homogeneous nurses. The differences in results from the GLM and GEE models suggest that it is not reasonable to ignore differences among nurses. Therefore, we next investigate a nurse-effects model and its implications

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for staffing decisions.

3.2 Nurse-Effects Model

We divided Hospital 1's staffing data into two periods – before and after June 30, 2009. There were 146 nurses who worked for more than 10 shifts in both periods. Among these nurses, we calculated the absentee rate prior to June 30, 2009 for each nurse. The mean and median absentee rates among those nurses were 11.0% and 7.6%, respectively, and the standard deviation was 12.0%. We identified nurses whose absentee rates were higher than 7.6% before June 30 and categorized these nurses as type-1 nurses. The remaining nurses from the cohort of 146 nurses were categorized as type-2.

For each shift, we model the impact of nurse-effects via the percent of type-1 nurses scheduled for that shift. We used the data between July 1st and December 4th, 2009 to evaluate the impact of having different proportions of type-1 nurses scheduled for a shift. We fitted the following model:

$$
log(\frac{\pi_t}{1 - \pi_t}) \quad = \quad \mu + \sum_{i=2}^{u} \beta_i U_i + \sum_{j=2}^{v} \alpha_j S_j + \sum_{k=2}^{7} \xi_k D_k + \gamma w_t + \nu z_t + \text{(two-way interaction terms)},
$$

$$(2)$$

where π_t is the absentee rate for a given unit, shift, and day of week in which w_t (measured by $w_t^{(1)}$, $w_t^{(2)}$, or $w_t^{(3)}$) is workload, and $(100 \times z_t)\%$ of the scheduled nurses are type 1. Coefficients γ and ν in Equation (2) capture the workload and nurse-effects in each shift, whereas β_i, α_j, and ξ_i captures the effect of unit i, shift j, and day of week k relative to the benchmark groups as before.

We found that w_t did not have a consistent effect on absentee rates. Specifically, $w_t^{(1)}$ was positively correlated with absentee rate, $w_t^{(2)}$ was negatively correlated with absentee rate, and $w_t^{(3)}$ was positively correlated with Thursday's absentee rate and negatively correlated with Sunday's absentee rate. In contrast, the percent of type 1 nurses scheduled (z_t) was positively correlated with absentee rates after controlling for unit, shift, and day of week effects.

In the spirit of using a parsimonious model to explain absentee rates, we next fitted an absentee rate model with unit effect, shift effect, unit and shift interaction, and z_t. That is, we dropped w_t in (2). The model was a reasonably good fit as the residual deviance is 1337.2 with 1314 degrees of

freedom and p-value $= 0.32$, which failed to reject the null hypothesis that the model was a good fit. The results, shown in Table 8, suggest that individual nurses' attendance history could be used to explain future shifts' absentee rates, and that nurse managers may benefit from accounting for individual nurses' likelihood of being absent in staffing decisions.

Table 8: Nurse-Effects Model Summary

	Estimate	Std. Error	Wald z value	Wald Test p-value
(Intercept)	-2.85027	0.13259	-21.497	< 2e-16
T2	0.06239	0.18507	0.337	0.73602
T3	-0.23109	0.18344	-1.260	0.20774
Evening	0.33266	0.17061	1.950	0.05120
Night	1.07602	0.16914	6.362	1.99e-10
z_t	0.27449	0.08684	3.161	0.00157
T2*Evening	-0.04423	0.23367	-0.189	0.84987
T3*Evening	0.20904	0.24139	0.866	0.38649
T2*Night	-0.70931	0.24933	-2.845	0.00444
T3*Night	-0.46662	0.23987	-1.945	0.05173

Null deviance: 1445.1 on 1323 degrees of freedom.
Residual deviance: 1337.2 on 1314 degrees of freedom.
Goodness of fit test: p-value $= 0.32$.

The take away from Section 3 is that (1) if nurses are assumed to be homogenous and their decisions independent across shifts, then short-term workload either does not explain shift absentee rate or its effect is both positive and negative (which makes it non-actionable for nurse managers), and (2) if nurses are assumed to be heterogenous but consistent decision makers, then nurse-effects explain shift absentee rates reasonably well. Consistent with these findings, we next use two examples to illustrate the need for a staffing model that accounts for nurse-effects.

In the first example, we consider u independent nursing units each with its own demand for beds but common nursing skill requirements. The nurse manager needs to allocate available nurses to work in these nursing units. Because nurses may be on planned leave, attending training/education events, and may prefer to work in certain shifts, the total number of available nurses for a shift varies over different planning periods. Assume the nursing demand for these units are independent and identically distributed random variables, denoted by $X_i \ \forall \ i = 1, \cdots, u$. Let $g(q_i) = E[c_o(X_i - q_i)^+]$ be the expected understaffing cost for staffing at a nursing level q_i, where $c_o(\cdot)$ is an increasing convex function. With the assumption of zero nurse absence, for a given a total number of available shifts

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$\bar{q}$, equalizing the staffing levels for these nursing units is better than having uneven staffing levels across nursing units. This result is based on a property of Schur-convex functions (Marshall et al. 2011), which we explain next. A vector $\vec{q} = (q_1, \cdots, q_n)$ is majorized by a vector $\vec{q'} = (q'_1, \cdots, q'_n)$ (denoted by $\vec{q} \leq_M \vec{q'}$) if $\sum_{i=1}^{k} q_{[i]} \leq \sum_{i=1}^{k} q'_{[i]}$ for $k = 1, \cdots, n-1$ and $\sum_{i=1}^{n} q_{[i]} = \sum_{i=1}^{n} q'_{[i]}$, where $q_{[i]}$ and $q'_{[i]}$ respectively denote the i-the largest value in vector $\vec{q}$ and $\vec{q'}$. If $\vec{q}$ is majorized by $\vec{q'}$ where $\sum_{i=1}^{n} q_i = \sum_{i=1}^{n} q'_i = \bar{\bar{q}}$, and $g(\cdot)$ is a convex function, then $\sum_{i=1}^{k} g(q_i) \leq \sum_{i=1}^{k} g(q'_i) \ \forall \ k = 1, \cdots, n$. For example, if $q_i = \bar{\bar{q}}/n = \bar{q}$ for all $i = 1, \cdots, n$, then $\sum_{i=1}^{n} g(q_i) \leq \sum_{i=1}^{n} g(q'_i)$. Therefore, by having a staffing plan $\vec{q} = (\bar{q}, \cdots, \bar{q})$ that equalizes staffing levels across units will lead to a lower expected total cost than a plan $\vec{q'} = (q'_1, \cdots, q'_n)$ that has unequal staffing levels. When nurses may be absent, it is natural to ask whether this is still the best strategy to allocate available nurses such that the expected number of nurses who show up would be the same across nursing units with independent and identical demand?

We use a two-unit example to illustrate the need for a more sophisticated staffing strategy. Assume that each unit's nursing demand follows an independent Poisson distribution with the mean of 4 nurses per shift. Ten nurses with (0.6, 0.6, 0.2, 0.2, 0.2, 0.2, 0, 0, 0, 0) absentee rates are to be scheduled to work in these two units. Consider two staffing plans that both utilize all 10 nurses: (1) 6 nurses with absentee rates (0.6, 0.6, 0.2, 0.2, 0.2, 0.2) in Unit 1 and 4 nurses with zero absentee rates in Unit 2; and (2) 5 nurses with absentee rates (0.6, 0.2, 0.2, 0, 0) in unit 1 and 5 nurses with absentee rates (0.6, 0.2, 0.2, 0, 0) in unit 2. The expected number of nurses show up in a shift in each unit is 4 in these two arrangements. However, the average number of shift short under Plan (1) is 57% higher than Plan (2).

In the second example, a nursing unit's nurse manager is able to obtain sufficient information (such as inpatients' conditions, scheduled admissions, and anticipated discharges) to accurately project his/her unit's demand one or two shifts in advance. If this projection is higher than the planned staffing level, then the manager may wish to recruit some part-time nurses to work extra shifts at pay rate r per shift to satisfy the excess demand. This is cheaper and less stressful than finding overtime or agency nurses at pay rate $r' > r$ on a short notice. The nurse manager often needs to announce the opportunity to pick up extra shifts to all nurses who are qualified for these shifts, and union rules may have a rank order in which extra shift requests must be granted; e.g. Hospital 1 prioritizes nurses by seniority. Consequently, the nurse manager has little

control over who may be selected to work extra shifts. Nurses' absentee rates could differ and the cost per shift may also differ by nurse due to skill level and seniority. All these factors make the determination of the number of extra shifts (or target staffing level) difficult.

Assume that the projected excess demand equals 5 RN shifts (deterministic), and some of the available nurses have a 5% absentee rate whereas the others have a 15% absentee rate. If the average absentee rate for the available nurses is 10%, the nurse manager assumes independent homogeneous absentee rates, and $r' = 1.5r$, then he or she will recruit 6 extra-shift nurses based on the expected under- and over-staffing cost: $1.5r \sum_{q=1}^{n}(5-q)^+ P(Q=q) + r \sum_{q=1}^{n}(q-5)^+ P(Q=q)$, where Q is a binomial random variable representing the number of nurses who show up for work among the n scheduled nurses. However, if the 6 nurses selected for the extra shifts happen to all have absentee rate of 5%, then the optimal number of nurse shift is 5 and the expected cost with 6 nurses will be twice that of 5 nurses.

The two examples discussed in this section illustrate problems that nurse managers need to solve on a regular basis and in real time. Both problem are not easy to solve to optimality in short amount of time. Therefore, we model the nurse staffing problem with heterogeneous absentee rates, and develop a heuristic that performs an online cost-benefit evaluation to assign one nurse at a time. This method can be applied to both problem scenarios described in the examples above.

4.   Formulation and Proposed Model

In this section, we develop staffing models for nurses with heterogeneous absentee rates. For this purpose, we need the following additional notation. Nurse ℓ has attendance probability p_ℓ. Let $Q(\bar{s}_i, \bar{p})$ be the number of nurses who will show up for work in unit i, where $\bar{s}_i$ is a n-vector with its ℓ-th element equal to 1 if nurse ℓ is scheduled to work in unit i and 0 otherwise, and $\bar{p} = (p_1, \cdots, p_n)$. We assume that nurse managers can obtain $\bar{p}$ from historical data. Recall that n denotes the number of available nurses or nurse shifts. Let $c_o(\cdot)$ be an increasing convex underage cost function, X_i be the demand for unit i, and $s = (\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_u)$ be a $n \times u$ staffing plan matrix. Table 9 summarize the notation.

The best staffing plan s can be obtained by solving the following non-linear discrete optimization problem.

Decision Variables

$\bar{s}_i$	A n-vector; $s = (\bar{s}_1, \cdots, \bar{s}_u)$.
s	A $n \times u$ matrix where (ℓ, i)-th element $= 1$ if nurse ℓ is scheduled to work in unit i.

Parameters

$\bar{p}$	A n-vector where the ℓ-th element is the show probability for the ℓ-th nurse.
$c_o(\cdot)$	An increasing convex underage cost function.

Random Variables

X_i	Nursing requirements for unit i.
$Q(\bar{s}_i, \bar{p})$	The number of nurses who show up in unit i.

$$\min_s \Pi(s, \bar{p}) = \sum_{i=1}^{u} E\big[c_o(X_i - Q(\bar{s}_i, \bar{p}))^+\big] \qquad (3)$$

subject to:

$$\sum_{i=1}^{u} s_{\ell i} = 1 \qquad \forall\, \ell = 1, \cdots, n \qquad (4)$$

$$s_{\ell i} \in \{0, 1\} \qquad \forall\, \ell = 1, \cdots, n;\ i = 1, \cdots, u, \qquad (5)$$

where the objective function is the sum of the expected underage cost for the u units, and n is the total number of nurse shifts available. Note that the probability distribution for $Q(\bar{s}_i, \bar{p})$ is not a binomial distribution. The objective function in (3) becomes analytically intractable as the number of heterogeneous nurses increases, making it difficult to identify an optimal staffing strategy. The objective function has some properties. First, the expected benefit of assigning one additional nurse to unit i to an existing assignment (s) is non-negative (i.e. $\Pi(s, \bar{p}) - \Pi(s', \bar{p}') = (p_{n+1})E[c_o(X_i - Q(\bar{s}_i, \bar{p}))^+ - c_o(X_i - 1 - Q(\bar{s}_i, \bar{p}))^+] \geq 0$ for all i, where s' is the staffing matrix with the additional $(n+1)$-th nurse assigned to unit i as compared to s). Second, if all nurses have the same show probability, then the expected marginal benefit of adding a nurse to unit i diminishes as the number of nurses assigned to unit i increases (explained later in this section). However, this diminishing expected benefit property does not hold with heterogeneous nurses. For example, if $p_{n+1} = 0$ and $p_{n+2} > 0$, then the expected benefit of adding the $(n+1)$-th nurse to unit i is zero whereas the expected benefit from adding the (n+2)-th nurse is non-negative. Because the nurse

manager cannot ignore the difference in nurses absentee rate, due to the large number of possible assignments, there is not a simple structure about how different nurse assignments affect the the objective function.

The complexity of the problem and the need for solving the problem efficiently led us to search for an implementable staffing strategy that utilize nurses' absentee rates. We describe our approach next.

Table 10: Simplified Notation with Two Classes

Parameters	
n_ℓ	Total number of type-ℓ nurse.
$n_\ell^{(i)}$	Number of type-ℓ nurse scheduled for unit i.
p_ℓ	Show probability of type-ℓ nurse.
Random Variable	
$Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2)$	The number of nurses show up for unit i.

For sake of clarity and simplicity of notation, we hereafter assume that there are two classes of nurses based on their historical show probability. The simplified notation is summarized in Table 10. Let n_1 type-1 nurses have show probability p_1 and n_2 type-2 nurses have show probability p_2. In what follows, we characterize the marginal benefit from adding a type-1/type-2 nurse to each unit for a given assignment (n_1^i, n_2^i). The benefit of adding one more type-1 nurse to unit i is

$$
\begin{aligned}
\delta_1^i(n_1^{(i)}, n_2^{(i)}) &= E\big[c_o(X_i - Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2))^+\big] - E\big[c_0(X_i - Q(n_1^{(i)}+1, n_2^{(i)}, p_1, p_2))^+\big] \\
&= p_1 E\big[c_o(X_i - Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2))^+ - c_o(X_i - 1 - Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2))^+\big] \\
&\geq 0.
\end{aligned}
\tag{6}
$$

Similarly, the benefit of adding one more type-2 nurse to unit i is

$$
\begin{aligned}
\delta_2^{(i)}(n_1^{(i)}, n_2^{(i)}) &= p_2 E\big[c_o(X_i - Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2))^+ - c_o(X_i - 1 - Q(n_1^{(i)}, n_2^{(i)}, p_1, p_2))^+\big] \\
&\geq 0.
\end{aligned}
\tag{7}
$$

By comparing $\delta_1^{(i)}$ and $\delta_2^{(i)}$, we obtain the following results.

Lemma 1.

1. If $p_1 \geq p_2$, then $\delta_1^{(i)}(n_1^i, n_2^{(i)}) \geq \delta_2^{(i)}(n_1^{(i)}, n_2^{(i)})$.

2. $\delta_1^{(i)}(n_1^{(i)}, n_2^{(i)}) - \delta_1^i(n_1^{(i)} + 1, n_2^{(i)}) \geq 0$ and $\delta_2^{(i)}(n_1^{(i)}, n_2^{(i)}) - \delta_2^i(n_1^{(i)}, n_2^{(i)} + 1) \geq 0$.

3. $\delta_1^{(i)}(n_1^{(i)}, n_2^{(i)}) - \delta_1^i(n_1^{(i)}, n_2^{(i)} + 1) \geq 0$ and $\delta_2^i(n_1^{(i)}, n_2^{(i)}) - \delta_2^i(n_1^{(i)} + 1, n_2^{(i)}) \geq 0$.

The first bullet in Lemma 1 confirms that it is always better to add a nurse with a higher show probability. The second bullet says that the benefit of adding one more type-j nurse to a particular unit diminishes in the number of type j nurses in the unit when the staffing level of the other group is held constant. The last bullet says that the benefit of adding one more type-j nurse to a particular unit diminishes as the staffing level of the other group increases. The second and third statements can also be described as submodularity of the objective function in (3) (Topkis 1998). These arguments also apply when there are an arbitrary number of nurse classes.

The optimal solution needs to simultaneously consider the assignment of all nurses, which is difficult because of the large number of possible ways in which nurses may be assigned and a complicated attendance probability distribution associated with each assignment. However, the above analysis suggests that assigning nurses based on the marginal benefit of each assignment may lead to a good solution overall. It has been shown that a greedy heuristic works well when the objective function is monotone submodular (Wolsey 1982). To utilize a greedy algorithm, the nurse manager may wish to sort nurses by decreasing show probabilities, and add the nurses sequentially to maximize marginal benefit from each assignment so long as all nurses are exhausted. For the problem of adjusting staffing levels, the nurse manager can accept extra shift volunteers in the sequence dictated by union rules until the cost of adding the next volunteer is higher than the expected benefit. We denote this heuristic as H, and provide a formal description below.

H: If there is no pre-determined assignment sequence among a group of nurses, assign one nurse at a time to a nursing unit that generates the highest expected marginal benefit. If there is a pre-determined sequence and each assignment incurs a cost, assign nurses according to the sequence until the expected marginal benefit is at least as large as the cost.

The model with homogeneous absentee rates can be viewed as a special case in which the marginal benefit method yields an optimal solution. Suppose n independent and identical nurses each with attendance rate p are available for a particular shift t. Then the problem faced by the

nurse manager is to minimize the expected shortage cost $E\big[c_o(\sum_{i=1}^{u}(X_i - Q(n^{(i)}, p))^+)\big]$, subject to $\sum_{i=1}^{u} n^{(i)} = n$. The expected marginal benefit of adding one more nurse to a particular unit i is

$\delta^{(i)}(n^{(i)}) = E\big[c_o(X_i - Q(n^{(i)}, p))^+ - c_o(X_i - Q(n^{(i)}+1, p))^+\big] = pE\big[c_o(X_i - Q(n^{(i)}, p))^+ - c_o(X_i - 1 - Q(n^{(i)}, p))^+\big]$. With some algebra, it can be shown that the expected benefit of adding an additional nurse to unit i diminishes as $n^{(i)}$ increases (i.e. $\delta^i(n^{(i)}) \geq \delta^j(n^{(i)}+k)$ for all $i = 1, \cdots, u$; $k \geq 0$), and the cost function is convex in $n^{(i)}$. In addition, if $\delta^i(n^{(i)}) \geq \delta^j(n^{(j)}) \ \forall \ j \neq i$, then $\delta^i(n^{(i)}) \geq \delta^j(n^{(j)} + k)$. This suggests that if a nurse manager assigns a nurse to a unit i that has the largest expected marginal benefit, then the nurse manager will never regret assigning this nurse to unit i because no other possible assignment (now or later to any unit) can generate a higher expected benefit. Therefore, the optimal staffing level can be achieved by assigning the n nurses to the u units one at a time such that each additional nurse assignment generates the highest marginal benefit. Because each increment in the nurse assignment maximizes the expected benefit it generates, the staffing levels chosen in this manner are optimal. We evaluate the potential impact of ignoring heterogeneous absentee rates next.

5. Straw Policies and Performance Comparison

In this section, we propose and evaluate three straw policies for assigning nurses with heterogenous attendance rates to multiple units. The notation used in this section is identical to that in the previous section. Also, all comparisons are performed with the assumption that there are two units with independent and identically distributed nursing requirements.

- Arbitrary Assignment: This policy, denoted S1, randomly assigns each nurse to the two units while ensuring that the total number assigned to each unit is proportional to the expected demand for that unit. Staffing level in each unit under S1 will be either $\lceil (n_1 + n_2)/2 \rceil$ or $\lfloor (n_1 + n_2)/2 \rfloor$.

- Segregated Assignment: This policy, denoted S2, tries to maintain homogeneity among nurses assigned to each unit. For example, if there are two nurse types with high and low show probabilities, this policy will assign mostly one type of nurses to each unit. In particular, if there are n_ℓ type-ℓ nurses with show rates p_ℓ, $\ell \in \{1, 2\}$ and $n_1 p_1 \geq n_2 p_2$, then the nurse manager will assign $(n_1 - m)$ type-1 nurses to one unit and a combination of type-1 and

type-2 nurses to the other unit. This combination will have m type-1 and n_2 type-2 nurses, where m is the largest integer such that $(n_1 - m)p_1 \geq mp_1 + n_2p_2$. Similar arguments can be developed for the case when $n_1p_1 < n_2p_2$.

- Balanced Assignment: This policy, denoted S3, searches for an assignment that minimizes the difference in the expected demand-supply ratios (i.e. $E(X_i)/E(Q(\bar{s}_i, \bar{p}))$) across units. When there are multiple assignments that result in identical expected staffing levels, any one of the balanced assignments is picked at random. Given that the two units have identically distributed nurse requirements, S3 minimizes the absolute difference between $(n_1^{(1)}p_1 + n_2^{(1)}p_2)$ and $(n_1^{(2)}p_1 + n_2^{(2)}p_2)$. Recall that $n_\ell^{(i)}$ is the number of type-ℓ nurses assigned to unit i.

In computational experiments, we fixed the total number of nurses to be $n = n_1 + n_2 = 15$, and varied n_1 from 0 to 15. When $n_1 = 0$ or $n_1 = 15$, nurses have homogeneous absentee rates. We also varied p_1 from 0.8 to 1 in 0.05 increments. The show probability for type-2 nurses were set as $p_2 = \theta p_1$, where θ was varied from 0.1 to 0.9 in 0.1 increments. Nurse requirements were assumed to be Poisson distributed and independent across units with rate $\lambda = (n_1p_1 + n_2p_2)/2$, ensuring that overall mean requirements and supply were matched. This experimental design resulted in 720 scenarios. For each heuristic, we compared its expected shortage cost relative to the optimal cost upon assuming two shortage cost functions: (1) linear, i.e. $c_o(x_i - q)^+ = x_i - q$ if $x_i \geq q$ and 0 otherwise, or (2) quadratic, i.e. $c_o(x_i - q)^+ = (x_i - q)^2$ if $x_i \geq q$ and 0 otherwise. The optimal assignment and associated minimum expected shortage cost were obtained through an exhaustive search over all possible assignments. The results of the performance comparison are shown in Table 11. The left (resp. right) panel shows the performance comparison under a linear (resp. quadratic) cost function. Within each panel, the upper table reports the mean and standard deviation of the ratio of the cost associated with each heuristic and the optimal cost, expressed in percents. Each cell in the lower table summarizes the percent of scenarios in which the column strategy performed better than the row strategy.

Table 11 shows that S1 is dominated by other solution approaches. Also, both H and S3 perform better than S2 under quadratic shortage cost function. Therefore, we focus the remainder of our discussion on a comparison of H and S3. Both these approaches perform quite well. There is statistically no difference in average performance of H and S3, but S3 performs better than H in

Table 11: Performance Comparison

Linear Cost function

Cost Relative to the Optimal Cost

	H	S1	S2	S3
Avg	100.23%	106.05%	100.27%	100.03%
SD	0.31%	8.02%	0.61%	0.09%

Pairwise Comparison

	H	S1	S2	S3
H	–	0%	37.6%	43.3%
S1	87.5%	–	87.5%	87.5%
S2	25.7%	0%	–	29.3%
S3	6.3%	0%	12.6%	–

Quadratic Cost function

Cost Relative to the Optimal Cost

	H1	S1	S2	S3
Avg	100.32%	110.93%	100.56%	100.08%
SD	0.49%	14.72%	1.28%	0.24%

Pairwise Comparison

	H	S1	S2	S3
H	–	0%	32.1%	37.1%
S1	87.5%	–	87.5%	87.50%
S2	31.8%	0%	–	28.5%
S3	10.8%	0%	5.1%	–

more scenarios. However, S3 requires knowledge of all available nurses up front, whereas H can be used when we must assign one at a time in a pre-determined sequence (e.g. when adjusting staffing levels). Thus, H is more versatile in handling different problem scenarios that nurse managers face.

6. Concluding Remarks

In this paper, we show that nurse managers may use each nurse's attendance history to predict his or her likelihood of being absent in a future shift, and improve staffing decisions by utilizing relatively simple staffing policies, e.g. the heuristic labeled H or the straw policy labeled S3. We also show that there is no consistent relationship between short-term workload and group or individual absentee rate. The main contribution of this paper lies in developing detailed analyses of data from multiple nursing units and multiple hospitals to identify predictors of nurse absenteeism.

In addition to the two problem scenarios presented in Section 3, our model can be useful to nurse managers in a variety of other scenarios. For example, hospitals sometimes use a pool of nurses that float to different units, depending on realized nurse requirements. The size of the float pool is affected both by the uncertainty in nurse requirements and the availability of nurses. Our model can be used to determine an appropriate size of the float pool while taking into account heterogenous attendance record of nurses assigned to work in any particular shift among units whose requirements are covered by float-pool nurses.

This study also highlights the importance of paying attention to unit-level factors. For example, staff morale, unit culture, job satisfaction, and nurses' health may be affected by chronic staffing

shortages documented in the health service research literature. Therefore, nurse managers also need to pay closer attention to decisions such as the number of nurses, FTEs, skill levels, and work patterns, all of which affect the degree of flexibility available to cope with demand fluctuations and nurse absences. It is essential for future studies to consider how nurse absences influence staffing at each of the many levels of hierarchical decisions that determine an overall staffing strategy.

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