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Cost optimization of imperfect non-periodic inspections in a delay-time model

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Abstract

When a defect occurs in an asset, it may transition into a failure over time, known as delay time. We will develop a model to minimize the total cost of the maintenance policy including expected cost of failures, cost of inspection and expected cost of preventive maintenance.

Keywords: delay-time; inspections; condition-based maintenance

1 Introduction

Condition-based maintenance (CBM) is the practice of diagnosing the health of an expensive long-life asset by condition monitoring, and following with appropriate maintenance action. By inspecting an asset, it may be possible to obtain a better understanding of the health of the item, and thus intervene with an appropriate maintenance action just before failure.[1]

Many assets indicate some sign of defects prior to reaching a functional failure. When a defect occurs, it may transition into a failure over time. The time between the occurrence of a defect and the failure can be described as the delay time.[2]

These defects can be identified by inspecting, or taking measurements of the asset. These

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can range from low-tech visual inspections, to high-tech oil analysis and vibration monitoring. The data from inspections is processed for use in decision support systems to justify maintenance activities. In a simplified case, the machine condition can simply be reduced to the number of defects.

In order to ascertain the existence of a defect, we can make inspections of the asset. In the industry, the measurements are mostly made periodically, and many resources are spent on obtaining the observations. These inspections cost money, time, creates downtime, and occasionally causes machine failures in and of themselves [3]. Unfortunately, the number of measurements that are non-critical and indicate that no maintenance is necessary largely outnumber the critical measurements indicating a need for preventive maintenance. This implies that many resources are being used to maintain the status quo, and that there is room for improvement in the cost of inspections.

While there have been efforts to optimize for a minimum required level of reliability [4], this paper focusses on the cost of inspections, setting reliability as a secondary criterion.

The delay-time concept has been used extensively for optimizing inspection intervals. Most commonly in multi-component systems, as in [5] and [6], an optimal inspection period that minimizes the cost per unit time is sought. In [7] inspection intervals decreases with the expected cost per unit time using a recursive algorithm.

In the model addressed in this paper, inspections occur at non-periodic intervals for multi-defects in a single component machine.

2 Problem definition

In this model, suppose defects arrive according to a non-homogeneous poisson processes (NHPP) with rate λ. Each defect triggers a delay time, after which a failure occurs. The delay times have a common distribution and are independent of the time of instantiation.

The inspections are identical, and occur at scheduled times t_i. The probability of detecting a defect is β, where $\beta = 1$ implies a perfect inspection. Identified defects are repaired to good-as-new condition. When the inspection is less then perfect (i.e. $\beta < 1$), existing defects may not be identified, but there are no false positive errors, that is, non-existant defects are not identified as true defects.

The cost of one inspection is C_i, and the cost of all inspections are equal. The cost of one defect repair is C_d, and the cost of a failure repair is C_f. The time to perform repairs, preventive maintenance or inspections are negligible compared to the system life.

3 Expected number of failures

The system starts at $t_0 = 0$ with no defects. Inspections are scheduled to occur at times $t_1, ..., t_m$. As seen in [4], the expected number of failures over the inspection interval $(t_{j-1}, t_j]$ can be expressed as

$$\Lambda_\beta\left(t_{j-1}, t_j\right) = \sum_{k=1}^{j} \left\{ (1-\beta)^{j-k} \int_{t_{k-1}}^{t_k} \lambda(\tau) \left[G(t_j - \tau) - G(t_{j-1} - \tau)\right] \mathrm{d}\tau \right\}, \tag{1}$$

where $G(y)$ is the cumulative distribution function for delay time Y. By the linearity of expectation, the expected number of failures $E_\beta(t, \mathbf{T}_m)$ over the entire interval $[0, t]$ is simply the sum of the interval expectations for $j = 1 \ldots m$. The inspection policy is defined as $\mathbf{T}_m = (t_1, t_2, \ldots, t_m)$ where $t_m < t$, and the reliability is

$$R_\beta(t, \mathbf{T}_m) = \exp[-\Lambda_\beta(t_m, t)] \prod_{j=1}^{m} \exp[-\Lambda_\beta(t_{j-1}, t_j)].$$

3.1 Expected cost of inspection policy

While [4] goes on to optimize reliability, consider the three prevailing costs in this model: C_f, the cost of a functional failure, C_d, the cost of repairing a defect, and C_i, the cost of an inspection. The resulting total cost is

$$\Theta_\beta(t, \mathbf{T}_m) = C_f E_\beta(t, \mathbf{T}_m) + C_i m + \beta C_d \lambda(t),$$

or the sum of the expected total cost of failures, the total cost of inspections, and the expected total cost of repairing the detected defects.

In minimizing the cost of the maintenance policy, only the failure cost and inspection cost need to be considered. Since defects occur and are detected outside of our control, the expected cost of repairing a defect cannot be changed. It is sufficient to optimize the following truncated function

$$\theta_\beta(t, \mathbf{T}_m) = C_f E_\beta(t, \mathbf{T}_m) + C_i m, \tag{2}$$

and minimize $\theta_\beta(t, \mathbf{T}_m)$. There are two dimensions of minimization that need to occur. One is with respect to the number of inspections m, and the other is the timing of the inspections with m fixed. Let $\mathbf{T}_m^*$ denote the optimal inspection schedule $(t_1^*, \ldots, t_m^*)$ that minimizes $\theta_\beta(t, \mathbf{T}_m)$ for fixed m inspections. Similarly, $\mathbf{T}_{m^*}^*$ denotes the optimal inspection schedule for the optimal number of inspections m^*. In determining the optimal number of inspections m^*, consider that as the number of inspections in the time interval $[0, t]$ increases, we can expect the reliability to increase, thereby decreasing the expected total cost of failure.

Suppose there are m inspections scheduled. If

$$C_f \left(E_\beta(t, \mathbf{T}_m^*) - E_\beta(t, \mathbf{T}_{m+1}^*) \right) < C_i, \tag{3}$$

then the cost of the next inspection outweighs the marginal cost savings. If an m_0 can be found such that $\forall\ m \geq m_0$ equation 3 is true, then $\theta_\beta(t, \mathbf{T}_m^*)$ is increasing and m_0 is an upperbound on m^*, the optimal number of inspections.

Alternatively, we can verify the convexity of $\theta_\beta(t, \mathbf{T}_m^*)$ to identify the upperbound of m^*. Using the second derivative, the only term that needs to be considered is $C_f E_\beta(t, \mathbf{T}_m^*)$. The second term, $C_i m$ is linear with slope C_i and does not affect convexity. Determining the convexity of equation 2 is expected to be a non-trivial task as every instance of equation 1 is produced by a different vector of inspection times $\mathbf{T}_m^*$.

If the system must meet a designated reliability p, there are two scenarios that can occur. The minimum of $\theta_\beta(t, \mathbf{T}_{m^*}^*)$ could meet the reliability level p, as in figure 1. Alternatively,

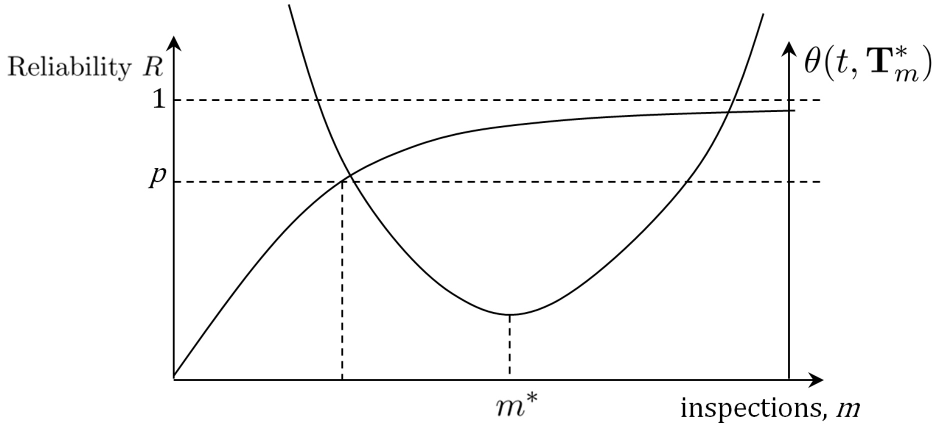

Figure 1: Minimal cost point exceeds minimal reliability point

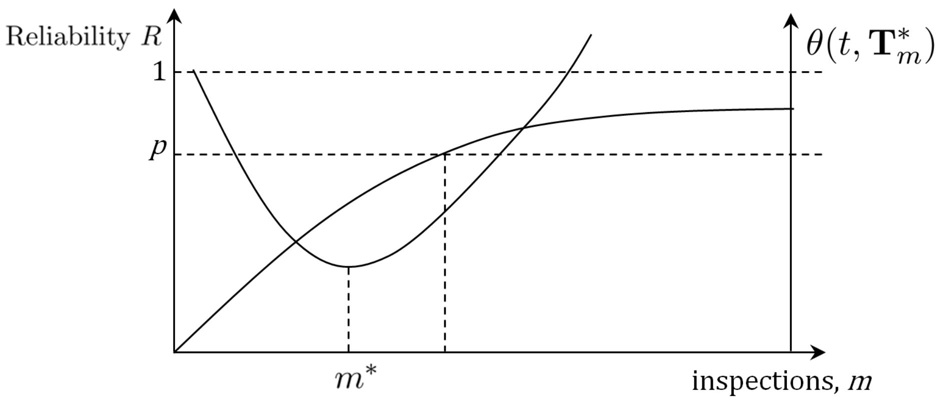

Figure 2: Minimal cost point does not meet minimal reliability point

$\theta_\beta(t, \mathbf{T}_{m^*}^*)$ may not meet p, as in figure 2. In the former case, the optimal solution is the minimum of $\theta_\beta(t, \mathbf{T}_m^*)$. The system will have reliability that is higher than required, but the overall maintenance cost is minimal. In the latter case, the optimal solution would be the point at which the reliability function R meets the required level p. While this is not the minimum of $\theta_\beta(t, \mathbf{T}_m^*)$, it is the least cost point that satisfies the reliability level.

4 Solution algorithm

Determining the optimal solution for this model is twofold: to find the optimal number of inspections m^*, and to optimally schedule the inspections at times $(t_1^*, t_2^*, \ldots, t_{m^*}^*)$.

Using an iterative approach, we can optimize the inspection schedule for m inspections, and increment m until $\theta_\beta(t, \mathbf{T}_{m+1}^*) > \theta_\beta(t, \mathbf{T}_m^*)$. Within each iteration, we initially schedule all m

inspections periodically, with $t_{i+1} - t_i = \frac{t}{(m+1)}$ as described in [4]. Then for $i = 1, \ldots, m$ find value $t_i \in [t_{i-1}, t_{i+1}]$ with other instances of inspections fixed, that minimizes $(\theta_\beta(t, \mathbf{T}_m))_j$, where j is the number of iterations at m inspections. Since optimality is measured at one inspection interval at a time, interval optimality may not imply optimality over the planning horizon $[0, t]$. Incrementing j, $\mathbf{T}_m$ is re-calculated until $(\theta_\beta(t, \mathbf{T}_m))_j - (\theta_\beta(t, \mathbf{T}_m))_{j+1} < \varepsilon$, at which point $(\mathbf{T}_m)_j$ is set to $\mathbf{T}_m^*$, m is incremented to $m+1$, and new periodic inspections are scheduled for the $m+1$ inspections.

5 Simplified Example

While the model in the current form does not lend itself to an analytical result, by selecting two simplifying parameters, the model reduces to a simple policy.

To remove the first complicating factor, set the probability of a successful inspection $\beta = 1$. Then equation 1 is reduced to the final term only, when $j = k$

$$\Lambda\left(t_{j-1}, t_j\right) = \int_{t_{j-1}}^{t_j} \lambda(\tau)\left[G(t_j - \tau) - G(t_{j-1} - \tau)\right] \mathrm{d}\tau.$$

Since $G(t)$ is a cumulative distribution function, $G(t) = 0$ when $t < 0$, hence eliminating the second term in the integral and leaving

$$\Lambda\left(t_{j-1}, t_j\right) = \int_{t_{j-1}}^{t_j} \lambda(\tau) G(t_j - \tau) \,\mathrm{d}\tau \qquad .$$

The second simplifying parameter is to adopt a homogeneous poisson process, with rate λ. Then the expected number of failures over period $[0, t]$ is

$$E(t, \mathbf{T}_m) = \lambda \sum_{j=1}^{m} \int_{t_{j-1}}^{t_j} G(t_j - \tau) \mathrm{d}\tau,$$

and the resulting cost function is

$$\theta(t, \mathbf{T}_m) = C_f \lambda \sum_{j=1}^{m} \int_{t_{j-1}}^{t_j} G(t_j - \tau) \mathrm{d}\tau + C_i m. \tag{4}$$

In order to minimize the cost, we must differentiate with respect to the inspection times $\mathbf{T}_m$. For the case of one inspection, only the timing of the inspection and the distribution of delay time must be considered.

By a change of variables, $\int_{t_{j-1}}^{t_j} G(t_j - \tau)\mathrm{d}\tau = \int_0^{\delta} G(u)\mathrm{du}$ where $\delta = t_j - t_{j-1}$. For the case of one inspection over the period $[0, t]$, the integral can be expressed as

$$\int_0^{\delta} G(u)\mathrm{du} + \int_0^{t-\delta} G(u)\mathrm{du}. \tag{5}$$

The minimizing inspection time for one inspection can be found to be $\delta = \frac{t}{2}$, by differentiating equation 5 and equating to 0. Similarly, for two or more inspections, it can be shown that the cost-optimizing inspections will be periodic.

Since the inspections are periodic, all inspection intervals will have a common probability distribution, and equation 4 becomes

$$\theta(t, \mathbf{T}_m) = C_f \lambda t \frac{(m+1)}{t} \int_0^{\frac{t}{m+1}} G(u)\mathrm{du} + C_i m. \tag{6}$$

The integral $\dfrac{(m+1)}{t} \displaystyle\int_0^{\frac{t}{m+1}} G(u)\mathrm{du} \searrow 0$ as $m \nearrow \infty$, suggesting that equation 6 will tend to a linear function beyond some point m_0.

6 Concluding Remarks

In this paper, a delay-time model to minimize the cost of a system under imperfect inspections was developed. An analytical model was investigated with several simplifying parameters. Future work will include alternative solution algorithms, including a nonlinear program, and further considerations with $\beta < 1$ and $\lambda(t)$ a non-constant function of time.

References

[1] A. Jardine and A. Tsang, *Maintenance, Replacement and Reliability*. Boca Raton, USA: CRC Press, 2006.

[2] A. Christer, "Delay-time model of reliability of equipment subject to inspection monitoring," *The Journal of the Operational Research Society*, vol. 38, no. 4, pp. 329–334, 1987.

[3] K. Lai, "Practices of preventive maintenance and replacement for engines: A case study," *European Journal of Operational Research*, vol. 124, no. 2, pp. 294–306, 2000.

[4] J. Zhao, A. Chan, C. Roberts, and K. Madelin, "Reliability evaluation and optimisation of imperfect inspections for a component with multi-defects," *Reliability Engineering and System Safety*, vol. 92, no. 1, pp. 65–73, 2007.

[5] A. Christer and W. Wang, "A delay-time-based maintenance model of a mutli-component system," *IMA journal of Mathematics applied in Business and Industry*, vol. 6, no. 2, pp. 205–222, 1995.

[6] A. Akbaraov, A. Christer, and W. Wang, "Problem identification in maintenance modelling: a case study," *International journal of production research*, vol. 46, no. 4, pp. 1031–1046, 2008.

[7] W. Wang and A. Christer, "Solution algorithms for a nonhomogeneous multi-component inspection model," *Computers and Operations Research*, vol. 30, no. 1, pp. 19–34, 2003.