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A New Multi-Objective Scenario-Based Robust Stochastic Programming for Recovery Planning Problem

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Abstract

Recovery of the key functions of an organization following disasters within the maximum tolerable period of disruption (MTPD) is addressed in this paper. We develop a new multi-objective scenario-based robust optimization model for recovery planning problem under uncertainty based on a set of scenarios with regards to disaster likelihood and impact probability. By selecting the most effective sub-plans after disasters, the proposed model aims to maximize total value of recovery capability and recovery completeness of the key functions while minimizing total cost of recovery. The weighted augmented ε-constraint method is applied to generate efficient solutions. A numerical example is provided to illustrate capability and applicability of the proposed approach.

Keywords: Business Continuity planning, Recovery planning, Weighted Augmented ε-constraint Method, Multi Objective Scenario-based Robust Stochastic Programming

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1. Introduction

Over the past 30 years, business continuity has become increasingly a critical subject for managers in all kinds of organizations. Natural disasters (e.g., earthquakes and hurricanes), human threats (e.g., terrorist attacks and fire), failure of utilities (e.g., power fluctuations and failure) or disruption in supply chains (e.g., in supply or demand side) are just some examples of major disasters that can result in widespread disruptions and failure of organizations (Greg Livingston, 2011). Business Continuity Management (BCM) is a management approach for helping the company managers to ensure the continuation of their key functions and supply chains at all times and under any circumstances. BCM is not just about dealing with incidents when they happen, it also establishes strategic and operational instructions to implement proactive operations (Staff, 2007). BCM system includes different tools for identifying, assessing, and prioritizing potential disasters systematically such as risk management tools (Hubbard, 2009). Dorfman (2007) introduces four options within the BCM framework to deal with disasters' effects involving *accept*, *mitigate*, *stop* and *plan*. Since the *plan* option is the most challenging issue for BCM managers, in this paper, we address the selection problem to deal with the plan option in the BCM context.

Generally speaking, each organization has to accept some level of residual risks (Elliot, 1999). Business Continuity Planning (BCP) was developed to deal with the negative outcomes of residual risks. Greg Livingston (2011) defines the BCP as a continuous process of identifying an organization's exposure to internal and external disasters' effects and their impacts on the key business functions. BCP provides strategies with the aim to fully restoring key functions as quickly as possible considering resource limitations when a disruptive effect occurs. Even occurrence of very improbable disruptive disasters with significant impacts on organizations' key functions could justify the need to a BCP (Greg Livingston, 2011).

Devargas (1999) proposes a step-wise approach to implement an effective BCP. Losada et al. (2012) argue that organizations with a comprehensive BCP were able to recover and restore to their normal situation within days after a disaster occurs, while the operations of many businesses without a BCP were shut down permanently.

The BCP is divided into two main stages: urgent response to incidents and Recovery Planning (RP). The consequences of lacking an effective RP are usually significant. Wheatman (2001) argues that two out of five organizations that experienced a disruption had to shut their business within five years. RP is a subset of BCP consisting of required sub-plans to return the organization's key functions to normal conditions following a disruption. In the context of RP, two concepts are considered for each key function as *Recovery Time Objective (RTO)* and *Recovery Point Objective (RPO)*. Cervone (2006) defines the RTO and RPO according to Claunch's (2004) definition as: *"the desired amount of time to recovery"* and *"the time interval between the last restoration point and the current point in time"*, respectively.

MTPD is defined as duration after which an organization's viability will be irrevocably threatened if key function cannot be resumed. Notably, the RTO of each key function should be less than the MTPD. Various factors including the occurrence of natural or man-made disasters and government regulations in certain businesses such as banking has resulted in intense interest in development of appropriate tools for RP. Jenkins (2000) classifies the potential disasters into a number of categories with approximately identical impact so as to assign an estimation of damage to each category. An integer program is then formulated that accounts for the similarity between all the disasters and a subset of disasters by which the suitable scenarios might be developed. Bryson et al. (2002) present a mixed-integer mathematical decision model for selecting sub-plans in RP and demonstrates the feasibility of using MS/OR tools in this area. Cerullo and Cerullo (2004) try to mitigate major business interruptions and provide insights into the current status of BCP, including perceptions about information security threats. Al-Badi et al. (2009) aim to draw the learned lessons from the experiences raised by Cyclone Gonu in Oman, to explore the issues of IT disaster recovery planning and BCP.

Although the level of organizations' awareness about the RP has been growing in recent, the mathematical modeling of relevant issues from the operations management point of view has remained unexplored in the literature. This paper aim to address this gap by developing a mathematical model for RP that ensures efficient functionality of an organization with an acceptable reliability after occurrence of a disaster.

2. Problem description and formulation

In this section, a new mixed-integer linear programming model for recovery planning problem is developed. The business continuity of each organization typically dependson a set of key functions. Every disaster has a number of effects that disrupt normal functions of organizations. It is therefore essential to have an effective recovery plan to restore the organization to a normal condition at the earliest possible time. To this end, probable incidents need to be studied and their effects fully understood. Unfortunately, all types of disasters both natural and man-made are largely unpredictable. Hence, disaster likelihood and disaster impact probability are the two main uncertainties in recovery planning. Based on these ambiguities, a set of possible scenarios need to be generated to select the most effective recovery plan.

2.1. Assumptions

The main characteristics and assumptions used in the problem formulation are as follow:

- several disasters can occur simultaneously;
- each disaster has a set of possible effects that may disrupt the organization's key functions;
- to restore a key function, all of its effects should be responded;
- organization's viability will be irrevocably threatened if each key function cannot be resumed during the MTPD;
- a sub-plan is selected from a set of available sub-plans to respond to effects;
- each sub-plan consists of a number of processes and operations and require some limited resources;
- operation times and resource consumption rates are dependent to sub-plans.

***Indices and sets*:**

d Index of disasters, $d=1,2,\ldots,D$

e Index of effects, $e=1,2,\ldots,E$

f Index of key functions, $f=1,2,\ldots,F$

s Index of sub-plans, s=1,2,…,S

o Index of operations, o=1,2,…,O

r Index of resources, r=1,2,…,R

Ω Set of potential scenarios $\theta \in \Omega$

***Parameters*:**

ϕ_d The likelihood of disaster *d*,

ϕ^θ Disaster likelihood of scenario θ

χ_{de} 1: If effect *e* of disaster *d* is taken place,

ξ_{ef} 1: if key function *f* is affected by effect *e*,

I_f Relative importance of key function *f;* $\sum_f \mathrm{I}_f = 1$,

ζ_f The MTPD of key function *f*,

η_{es} 1: if sub-plan *s* is available for effect *e*

M_r Maximum capacity of resource *r*

π^θ Probability of scenario θ

$\omega_{es}^{r\theta}$ The amount of required resource type *r* for responding to effect *e* by sub-plan *s* at scenario θ

τ_{es}^θ The required time for sub-plan *s* to respond to effect *e* at scenario θ

C_r^θ Cost of required resource type *r* at scenario θ

β_d^θ 1: if disaster *d* is happened under scenario θ

M A big number

B_{fe} A binary auxiliary variable

***Variables*:**

X_{es} 1, if sub-plan *s* is selected for responding to effect *e*, 0, otherwise

Y_f 1, if key function *f* is completely restored, 0, otherwise

ψ_e 1, if effect *e* disrupts the organization, 0, otherwise

2.2. Problem formulation

Based on the above explanations and notations, a scenario-based stochastic optimization model is developed for the RP problem as follows. It should be noted that each scenario is characterized by the amount of required resources and time for implementing each sub-plan for responding to each effect.

$$\text{Max} \quad \sum_{e=1}^{E}\sum_{s=1}^{S} X_{es}.(\sum_{f=1}^{F} \mathrm{I}_f.\xi_{ef}) + \sum_{f \in F} \mathrm{I}_f Y_f \tag{1}$$

$$\text{Min} \sum_{e=1}^{E}\sum_{s=1}^{S}\sum_{r=1}^{R} X_{es}.C_r^{\theta}.\omega_{es}^{r} \tag{2}$$

Subject to:

$$\sum_{s \in S} X_{es} \leq 1 \qquad \forall e \in E \tag{3}$$

$$X_{es} \leq \eta_{es} \qquad \forall e, s \tag{4}$$

$$\sum_{s \in S} X_{es} \leq \psi_e \qquad \forall e \tag{5}$$

$$\sum_{e\in E} X_{es}.\omega_{es}^{r\theta} \le M_r \qquad \forall r, s, \theta \tag{6}$$

$$\tau_{es}^{\theta}.X_{es}.\xi_{ef} \le \zeta_f \qquad \forall e, f, s, \theta \tag{7}$$

$$\sum_{e\in E} \xi_{ef}.\sum_{s\in S} X_{es} + (1 - Y_f) \le \sum_{e\in E} \psi_e.\xi_{ef} \qquad \forall f \tag{8}$$

$$\sum_{e\in E} \mathrm{B}_{fe} \le \sum_{e\in E} \xi_{ef}.\sum_{s\in S} X_{es} \qquad \forall f \tag{9}$$

$$\psi_e + Y_f \ge 2.\mathrm{B}_{fe} \qquad \forall e, f \tag{10}$$

$$\psi_e + Y_f - 2 \le \mathrm{B}_{fe} \qquad \forall e, f \tag{11}$$

$$\psi_e \le \sum_{d\in D} \beta_d^{\theta} \chi_{de} \qquad \forall e, \theta \tag{12}$$

$$\psi_e.M \ge \sum_{d\in D} \beta_d^{\theta} \chi_{de} \qquad \forall e, \theta \tag{13}$$

$$X_{es}, Y_f \in \{0,1\} \qquad \forall e, s, f \tag{14}$$

The first objective function includes two terms as recovery capability of the chosen sub-plans and recovery completeness of the key functions. The second objective function aims to recovery of key functions with minimum resource consumption. Constraints (3) represent that for each effect, at most one sub-plan can be selected. Constraint (4) guarantees that sub-plans are chosen from the set of available sub-plans. Constraint (5) allows that sub-plans are to be selected only for possible disruptive effects. Constraints (6) stipulate that the required resources for the selected sub-plans must not exceed the specified maximum limits. Constraint (7) ensures that the

RTO is less than the MTPD for each key function. Constraints (8)-(11) enforce complete restoration of key functions and response to all effects. Constraints (12) and (13) determine possible disaster effects after any incidents. Constraints (14) specify type of the binary variables.

3. Scenario generation

Each scenario is generated based on two main elements, i.e., the likelihood of each disaster and the disaster impact probability. We define *d* type/class of disasters such that the disasters belonging to each class have identical effects on the key functions. By taking into account criteria such as organization situation and its environmental conditions, structure of the organization and type of products or services, the likelihood of each disaster can be estimated. As it was mentioned before, one or more disaster might occur simultaneously. Therefore, the likelihood of the disaster for each scenario is obtained by equation (15).

$$\phi^{\theta} = \prod_{\substack{d' \in D^{\theta} \\ d'' \in D'^{\theta}}} \phi_{d'}.(1 - \phi_{d''}) \tag{15}$$

Where D^{θ} is the set of occurred disasters under scenario θ and D'^{θ} is the complement set of D^{θ}.

To alleviate complexity of the proposed approach, three types of impacts were considered as minor, moderate and major for each disaster. The disaster impact probability $\left[\rho_d^{I}, \rho_d^{II}, \rho_d^{III}\right]$, like disaster likelihood depends on the disaster and organizational characteristics. In this manner, $3^{|D^{\theta}|} \times 2^{d}$ (with $|D^{\theta}|$ representing the cardinality of set D^{θ}) scenarios will need to be generated. The probability of each scenario is obtained by equation (16).

$$\pi^{\theta} = \prod_{\substack{d' \in D^{\theta} \\ d'' \in D'^{\theta}}} (\phi_{d'}.[\rho_{d'}^{I}, \rho_{d'}^{II}, \rho_{d'}^{III}].\gamma_{d'}).(1-\phi_{d''}) \quad (16)$$

Where $\gamma_{d'}$ is a 3×1 matrix showing the disaster impact for $d' \in D^{\theta}$.

4. Solution methodology

To cope with uncertainty and multiple objectives of the developed model, a two phase approach is proposed. First, the weighted augmented ε-constraint method (Esmaili et al., 2011) is applied to convert the original multi-objective model into its equivalent single objective problem. Then, the robust optimization model of Yu and Li (2000) is used to find the most preferred compromise solution.

4.1. Phase 1

In Multi-Objective Programming (MOP), the concept of optimality is replaced with the notion of efficiency in order to find the most preferred solution (Mavrotas, 2009). For this research, we have used the weighted augmented ε-constraint method (Esmaili et al., 2011) which is briefly explained as follows. Consider the following MOP:

$$Max\ f_1(x) + \delta r_1(\frac{\alpha_2 s_2}{r_2} + ... + \frac{\alpha_p s_p}{r_p}) \tag{17}$$

$$Subject\ to:$$

$$f_2(x) = e_2 + s_2,$$

$$\vdots$$

$$f_p(x) = e_p + s_p,$$

$$x \in S,$$

$$s_i \in R^+, i = 1,...,p.$$

Where there are *p* objective functions, x is the vector of decision variables and *S* is the feasible solution space. Moreover, r_i denotes the range of the *i*th objective function, $(w_1, w_2, ..., w_p)$ is the weight vector and $\alpha_p = \frac{w_p}{w_1}, \forall p \neq 1$. The slack variables are scaled to the main objective function f_1 by using r_i values, in order to avoid scaling problems. In this manner, the single objective formulation of the proposed model is as follows:

Objective function:

$$\text{Max} \quad \sum_{e=1}^{E}\sum_{s=1}^{S} X_{es}.(\sum_{f=1}^{F} \mathrm{I}_f.\xi_{ef}) + \sum_{f \in F} \mathrm{I}_f Y_f + \delta.\frac{w_2.r_1}{w_1.r_2} S_2^{\theta} \tag{18}$$

Constraints:

$$\sum_{e=1}^{E}\sum_{s=1}^{S}\sum_{r=1}^{R} X_{es}.C_r^{\theta}.\omega_{es}^{r} = \varepsilon_2 + S_2^{\theta} \tag{19}$$

equations (3)-(12)

4.2 Phase 2: the robust formulation

The imprecise nature of some critical parameters such as disaster likelihood, required resources and recovery times are dealt with using a robust counterpart of the aforementioned model. In this paper, we apply the scenario-based stochastic robust optimization model proposed by Yu and Li (2000). In this way, the original problem can be reformulated as follows:

Objective function:

$$\text{Max} \quad w_1' \left[\sum_{\theta\in\Omega} \pi^\theta F^\theta - \Theta \sum_{\theta\in\Omega} \pi^\theta ((F^\theta - \sum_{\theta\in\Omega} \pi^\theta F^\theta) + 2\Xi^\theta) \right] - w_2' . \sum_{\theta\in\Omega} \pi^\theta \Delta^\theta \qquad (20)$$

Constraints:

$$F^\theta = \sum_{e=1}^{E} \sum_{s=1}^{S} X_{es} . (\sum_{f=1}^{F} \mathrm{I}_f . \xi_{ef}) + \sum_{f\in F} \mathrm{I}_f Y_f + \delta . \frac{w_2 . r_1}{w_1 . r_2} S_2^\theta \qquad \forall \theta \qquad (21)$$

$$F^\theta - \sum_{\theta\in\Omega} \pi^\theta F^\theta + \Xi^\theta \geq 0 \qquad \forall \theta \qquad (22)$$

$$\sum_{e\in E} X_{es} . \omega_{es}^{r\theta} - \Delta_1^\theta = M_r \qquad \forall r, s, \theta \qquad (23)$$

$$\tau_{es}^\theta . X_{es} . \xi_{ef} - \Delta_2^\theta = \zeta_f \qquad \forall e, f, s, \theta \qquad (24)$$

Constraints: (3)-(5), (8)-(14) and (19).

5. A numerical example

To demonstrate validity of the proposed model and the proposed solution method, a numerical example is provided here. Notably, the research in the context of RP is scarce and there is not a standard test problem for validating relevant models. Therefore, we have generated a hypothetical problem in this paper which is explained as follows:

Assume that three types of disasters, i.e., earthquake, fire and flood are likely to happen. Each disaster has three effects on an organization including: building destruction, temporary unavailability of the systems and critical data loss. In addition, three key functions are considered with the relative importance of 0.5, 0.2 and 0.3, respectively. The MTPD for each key function is generated according to uniform distribution *U (300,400)*. As mentioned in section 2, each disruption has a set of effects and each effect will disrupt a particular key function. Hence, the respective disruption-effect and effect-key function matrices are assumed as follow:

$$\chi = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \xi = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Table 1 shows the uniform distributions used for generating the disasters' likelihoods and impact probabilities based on which the scenarios were defined.

Table 1. Disasters' likelihoods and impact probabilities

Disaster	Disaster likelihood (ϕ_d)	Impact probability ($\left[\rho_d^I, \rho_d^{II}, \rho_d^{III}\right]$)
1	Uniform (0.1,0.25)	Uniform (0.2,0.3)
2	Uniform (0.01,0.15)	Uniform (0.4,0.6)
3	Uniform (0.15,0.30)	Uniform (0.2,0.3)

In this manner, with three possible disasters and three types of impact, 24 scenarios were generated as shown in Fig. 1. Table 2 represents the corresponding data for the generated scenarios by using equations (15) and (16).

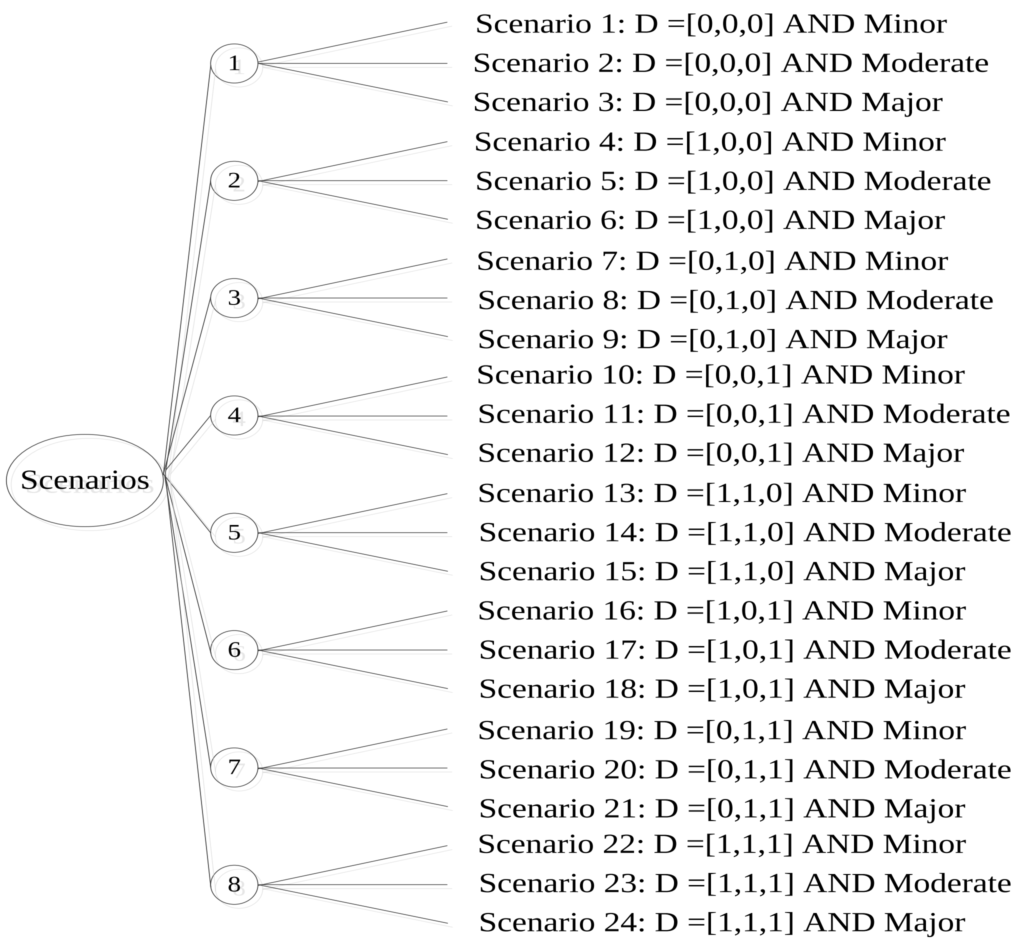

Figure 1. Generated scenarios

Table 2. Required data for the generated scenarios

Scenario	ϕ^{θ}	π^{θ}	**Scenario**	ϕ^{θ}	π^{θ}
1	0.623	0.187	13	0.008	0.002
2	0.623	0.311	14	0.008	0.004
3	0.623	0.125	15	0.008	0.002
4	0.085	0.025	16	0.025	0.007
5	0.085	0.043	17	0.025	0.012
6	0.085	0.017	18	0.025	0.006
7	0.054	0.016	19	0.016	0.005
8	0.054	0.027	20	0.016	0.008
9	0.054	0.011	21	0.016	0.003
10	0.186	0.056	22	0.003	0.001
11	0.186	0.093	23	0.003	0.002
12	0.186	0.037	24	0.003	0.001

To respond to each effect, three sub-plans were considered. Required resources and execution times of sub-plans are generated using the uniform distributions represented in Table 3.

Table 3. Sub-plans' related data

Sub-Plan	**Resource**			**Time(*h*)**	η_{es}		
	1	2	3		1	2	3
1	*U*(20,25)	*U*(10,15)	*U*(15,25)	*U*(150,250)	1	1	0
2	*U*(50,70)	*U*(35,60)	*U*(5,15)	*U*(50,100)	1	1	0
3	*U*(15,25)	*U*(5,15)	*U*(10,15)	*U*(5,30)	0	0	1
C_r	*U*(50,250)	*U*(50,250)	*U*(50,250)				
M_r	*U*(70,150)	*U*(50,95)	*U*(30,60)				

The above test problem was solved by LINGO 9 optimization software on a Pentium 4 with a 2 GHz CPU processor and 1 GB of RAM. It should be noted that by changing the value of ε_2, different efficient solutions could be found. According to the weighted augmented ε-constraint method, after finding the positive and negative ideal solutions for the second objective function, the range of second objective function is split into 3 point as follows:

$$\varepsilon_2^k = f_2^N + k\frac{R_2}{Q_2}, k = 0\, to\, Q_2 \tag{25}$$

Where R_2 is the range of the second objective function and is equal to the difference between the respective positive and negative ideal solution as follows:

$$R_2 = f_2^N - f_2^P \tag{26}$$

It should be noted that the second objective function is of minimization type and therefore the positive ideal solution is less than the negative ideal solution. Table 4 shows the three different efficient solutions found for the test problem.

Table 4.Computational results of the test problem

ε_2^k	X_{es}				Y_f		ψ_e		Objective functions	
		e=1	e=2	e=3					Recovery	cost
$\varepsilon_2^1 = 4025$	*s=1*	0	0	0	*f=1*	1	*e=1*	1	1	4025
	s=2	0	0	0	*f=2*	0	*e=2*	1		
	s=3	1	0	0	*f=3*	0	*e=3*	1		
$\varepsilon_2^2 = 15000$	*s=1*	0	1	0	*f=1*	1	*e=1*	1	1.6	11750
	s=2	0	0	0	*f=2*	1	*e=2*	1		
	s=3	1	0	0	*f=3*	0	*e=3*	1		
$\varepsilon_2^3 = 30000$	*s=1*	0	0	1	*f=1*	1	*e=1*	1	2	26130
	s=2	1	0	0	*f=2*	1	*e=2*	1		
	s=3	0	1	0	*f=3*	1	*e=3*	1		

6. Conclusion

In this paper, a new multi-objective robust model for recovery planning problem is proposed under uncertainty. A two phase approach was applied to solve the proposed model. In the first phase, the original problem is converted into an equivalent single objective scenario based stochastic mixed-integer linear model. Then, in the second phase, a robust formulation was used for finding a preferred compromise solution through an interactive approach between the decision maker and model analyzer. The numerical example show the applicability of the proposed model and its solution approach.

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