Stochastic Single Machine Family Scheduling To Minimize the Number of Risky Jobs

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Abstract

In this paper, a stochastic single machine family and job sequencing problem is studied. The problem is observed in several manufacturing systems such as jewelry, frozen food, medical device and blood sugar. Jobs have individual deterministic due dates and probabilistic processing times. Due to jobs' having individual due dates, family splitting is allowed. A Stochastic Non-Linear Mathematical Model is developed and various problems are solved with Lingo software. The objective is to minimize the number of risky jobs while keeping the number of tardy jobs minimum. As a result of the experimentation, the number of early jobs increased and the number of tardy jobs stayed the same. This study provides scheduler to view a probabilistic gantt chart of optimal schedule. Based on the optimal schedule and probability of tardiness values, scheduler can also modify due dates to decrease the probability of tardiness to a desired level.

Key words: Stochastic scheduling, family and job sequencing, non-linear modeling, probability of tardiness

1. Introduction

Machine scheduling has been always one of the most challenging tasks in shop floor control. In a typical scheduling problem, there are jobs to be processed on a resource (e.g. machine) with processing time and due date information. The scheduler is expected to schedule jobs on resource(s) such that the performance measure is optimized. When jobs have due date information available, the performance measure used is usually due-date oriented. Minimizing the number of tardy jobs is one of the most commonly used due-date oriented performance measures in machine scheduling literature. A job is called tardy if the completion time is greater than its due date, otherwise it is called early. In certain production systems such as shoe, medical device manufacturing, jobs are grouped as families due to such differences as material, size or process requirements. In this case, the scheduling problem involves the family scheduling as well. Job families can have common due dates or each job of a family can have its own due date. If the processing time and due dates are known and constant, the scheduling problem is classified as deterministic. However, if any information is uncertain, the problem belongs to stochastic scheduling. Since the problem studied in this paper has two domains namely, group scheduling and stochastic scheduling, the literature is divided into two.

a) Group Scheduling Literature

Family and job scheduling problem is addressed in several works in machine scheduling literature. Majority of works consider the processing times and due dates as deterministic. Pan and Wu (1998) worked on single machine group scheduling problem to minimize the mean flow time subject to due date constraints (Pan & Wu, 1998). Gupta et al. (2008) studied single machine family group scheduling problem with setup times (Gupta & Chantaravarapan, 2008). A mixed integer linear programming model is used for the small-sized problems and simulated annealing model is developed and experimented with larger problems. Schaller and Gupta (2008) proposed optimal branch and bound algorithms to minimize total earliness and tardiness on a single machine scheduling problem with family setup times (Schaller & Gupta, 2008). Süer, Mese and Eğilmez (2011) proposed a mixed integer linear programming model and a genetic algorithms to minimize total tardiness in a cellular manufacturing system where jobs have individual due dates along with family setup times (Süer, Mese, & Eğilmez, 2011).

b) Stochastic Scheduling Literature

In addition to works that consider group scheduling (family and job scheduling), several stochastic scheduling problems and solution approaches have been addressed in literature. One of the commonly preferred objectives is minimizing the expected number of tardy jobs as the performance measure. As few examples, Balut (1973) worked on a single machine scheduling problem with normally distributed processing times and different due dates in order to minimize the expected number of tardy jobs (Balut, 1973). Soroush and Fredenhall (1994) studied single machine stochastic scheduling in order to analyze the impact of varying processing times for earliness-tardiness costs (Soroush & Fredendall, 1994). Pinedo (1983) studied single machine stochastic scheduling problem where processing times are exponentially distributed (Pinedo, 1983). Jang et al. (2002) worked on single machine stochastic scheduling to minimize the expected number of tardy jobs where jobs might arrive to single machine randomly and dynamically (Jang & Klein, 2002). For more works, see, ForsT (1995) (Forst, 1995), Lin and Lee (1995) (Lin & Lee, 1995), Seo, Klein and Jang (2005)(Seo, Klein, & Jang, 2005).

In a recent work, Eğilmez and Süer (2011) addressed stochastic family and job scheduling problem in a cellular manufacturing environment (Eğilmez & Süer, 2011) which considers both domains as stochastic and group scheduling. Same problem is studied on a single machine to minimize the number of risky jobs. Each job has probabilistic processing time and deterministic due date. In a probabilistic environment where processing times are uncertain, each job of a family has probability of being tardy from a scale of 0 to 1. Therefore, the probability of tardiness is included in the performance measure. The overall objective of this study is to minimize the number of tardy jobs and the total probability of tardiness. A stochastic non-linear mathematical model is developed to solve this complex problem. Finally, different versions of the problem are discussed and the experimental results are also reported.

2. Problem Statement

Prior to introducing the single machine stochastic family and job sequencing problem, the generalized minimizing the number of expected tardy jobs on a single machine problem is explained.

2.1 Minimizing the expected number of tardy jobs on a single machine

Let Π contain all possible sequences of n jobs. In a sequence $\varepsilon \in \Pi$ represented by [1],[2],.,[k],.,[n]), let [k] indicate the job occupying the k^{th} position in that sequence. The completion time of the k^{th} job, $c_{[k]}$, is shown in Equation 1.

$$c_{[k]} = \sum_{i=1}^{k} p_{[i]} \tag{1}$$

Following function, T(ε), which represents the expected number of tardy jobs of a sequence ε, is expressed by Equation 2.

$$\mathrm{T}(\varepsilon) = \mathrm{T}([1],[2],\dots,[\mathrm{k}],\dots,[\mathrm{n}]) = \sum_{k=1}^{n} \Pr\{c_{[k]} > d_{[k]}\} = \sum_{k=1}^{n} \Pr\left\{\sum_{i=1}^{k} p_{[i]} > d_{[k]}\right\} \tag{2}$$

The objective is to find the sequence $\varepsilon \in \Pi$ that minimizes T(ε). It is impossible to obtain an optimal sequence that satisfies the condition in polynomial time. This problem is only traceable with exponentially distributed processing times (“Scheduling Theory, Algorithms, and Systems by Michael Pinedo 1995, Hardcover | eBay,” n.d.). Therefore, there is a dire need to develop efficient solution approaches to use in industrial applications.

2.2 Single machine stochastic family and job sequencing

The problem studied in this paper is to allocate *n* jobs of m families to single machine as shown in Figure 1. There are m families and each family has positive number of jobs (e.g. family 1 has n jobs). In this problem, each job of a family has its individual due date and can be processed before or after another job from a different family.

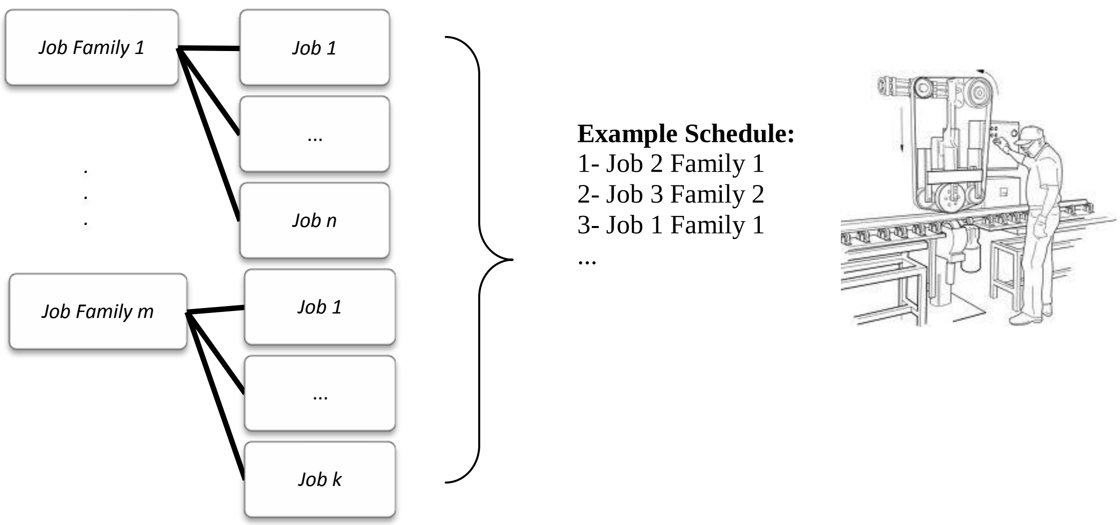

Figure 1. Problem Illustration (“Single Machine Example,” n.d.)

In this problem, preemption and splitting of jobs are not allowed and all jobs are assumed to be available prior to processing. Each job has random processing time, p_i , which follows the normal distribution N (μ_i, σ_i^2). Processing times of jobs are independent of each other and no relationship between the mean and the variance of processing times is imposed. In addition to normally distributed processing time, due date is assumed to be deterministic for each job, which is denoted as d_i. The objective is to minimize the number of risky jobs while keeping the number of tardy jobs minimum. In the following section, the proposed solution methodology is explained.

3. Methodology

In this section, the proposed methodology is explained. The proposed approach is a modification of the deterministic model (Süer & Mese, 2011). Jobs are classified as early, risky or tardy since the probability of tardiness is considered in the objective function. Jobs are classified "early" if probability of tardiness is 0. If probability of tardiness is between 0 and 1, the corresponding job is classified as "risky" job. If the probability of tardiness is 1, the job is called "tardy". The objective function is to minimize the number of risky jobs (nR) while keeping the number of tardy jobs (nT) minimum as shown in Equation 1.

Each job can be assigned only once as given in Equation (2). Equation (3) guarantees that each position is filled with at most one job. Equation (4) enforces jobs to be assigned consecutively. Equation 5.a. calculates the expected completion time of a job that is assigned to first position. Equation 5.b. calculates remaining jobs' expected completion times based on the expected completion time of the job on the first position. Equation (6) calculates the expected tardiness value of a job. Equation (7) calculates the probability of tardiness (PrT) for a job. Finally, Equation (8) counts each job if tardiness occurs.

Notation:

Indices:

i *Family index*

j *Job index*

k *Position index*

Parameters:

n	*Total number of jobs*
n_i	*Number of jobs in family i*
f	*Number of families*
μP_{ij}	*Mean process time of job j from family i*
σPij^2	*Variance of process time of job j from family i*
D_{ij}	*Due date of job j from family i*
R	*Big number*

Decision Variables:

X_{ijk}	*1 if job j from family i is assigned to the k^{th} position, 0 otherwise.*
C_k	*Expected completion time of the job in k^{th} position*
T_k	*Expected tardiness value of the job in k^{th} position*
nT_k	*1 if job in k^{th} position in cell m is tardy, 0 otherwise*
PrT_k	*Probability of tardiness value of the job in k^{th} position*

Objective Function:

$$\min Z = \sum_{k=1}^{n} (nT_k + PrT_k) \tag{1}$$

Subject to:

$$\sum_{k=1}^{n} X_{ijk} = 1 \qquad for\ i = 1..f\ and\ \ j = 1,,n_i \tag{2}$$

$$\sum_{i=1}^{f} \sum_{j=1}^{n_i} X_{ijk} \ \leq 1 \ \ for\ k = 1,,n \tag{3}$$

$$\sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ijk} \geq \sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ij(k+1)} \quad \text{for} \quad k = 1,,n-1 \tag{4}$$

$$C_1 = \sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ij1} * \mu P_{ij} \tag{5.a}$$

$$C_k = C_{(k-1)} + \sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ijk} * \mu P_{ij} = 0 \quad for\ k = 2,,n \tag{5.b}$$

$$C_k - \sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ijk} * D_{ij} \leq T_k \quad \text{for } k = 1,,n \tag{6}$$

$$PrT_k = p\left(Z_{mk} \leq \frac{\left(C_{mk} - \sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ijmk} * D_{ij} \right)}{\sqrt{\sum_{i=1}^{f}\sum_{j=1}^{n_i} X_{ijmk} * \sigma_{p_{ij}}{}^2}} \right) \quad \text{for} \quad k = 1,,n \tag{7}$$

$$T_k \leq M * nT_k \quad \text{for } k = 1,,n \tag{8}$$

Definition of Variables:

$X_{ijk} \in \{0,1\},\ nT_k \in \{0,1\},\ \ C_k \geq 0,\ T_k \geq 0,\ 0 \leq PrT_k \leq 1$

4. Experimentation and Results

Experimentation is performed with two datasets, namely: 1) 10-job and 2 family, 2) 20-job and 2-family. To present the benefit of the proposed approach, the datasets are also experimented with the deterministic approach and results are compared.

4.1. Experiment with 10-Job Dataset

In this experiment, the data given in Table 1 is used. There are 10 jobs and 2 families where each family has 5 jobs as shown in Table 1.

Table 1. 10-Job Dataset

Family	Job	Mean Pro Time	Variance of Processing Time	Due Date
1	1	198	392.04	218
1	2	135	182.25	1961
1	3	214	457.96	683
1	4	214	457.96	933
1	5	216	466.56	673
2	6	141	198.81	2835
2	7	126	158.76	752
2	8	126	158.76	1003
2	9	104	108.16	1243
2	10	234	547.56	905

The 10-job problem is solved with both deterministic and the proposed stochastic approach. The results of deterministic model is shown in Table 2 and illustrated in Figure 2. According to the deterministic model results (Table 1), there are 2 tardy jobs, 4 risky jobs and 2 early jobs obtained. And the risky jobs are observed in the beginning and middle parts of the schedule.

Table 1. The results of deterministic approach

Job	**J (1,1)**	**J (2,8)**	**J (2,7)**	**J (1,3)**	**J (1,4)**	**J (2,10)**	**J (2,9)**	**J (1,2)**	**J (1,5)**	**J (2,6)**
Mean Pro. Time	198	126	126	214	214	234	104	135	216	141
Due Date	218	1003	752	683	933	905	1243	1961	673	2835
Comp. Time	198	324	450	664	878	1112	1216	1351	1567	1708
Prob. of Tardiness	15.6%	0.0%	0.0%	28.9%	8.6%	100.0%	28.6%	0.0%	100.0%	0.0%
Tardy	0	0	0	0	0	1	0	0	1	0
Risky	1	0	0	1	1	0	1	0	0	0
Early	0	1	1	0	0	0	0	1	0	1

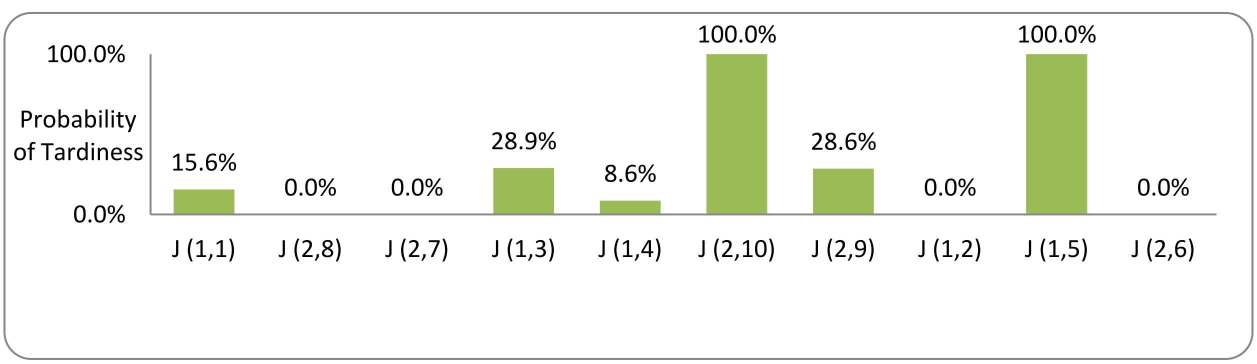

Figure 2. The probabilistic Gantt Chart of deterministic approach for 10-Job problem

The results obtained from the proposed stochastic approach are shown in Table 2 and illustrated as Gantt chart in Figure 3. The proposed approach takes the variance of processing time into account and minimizes the total probability of tardiness in addition to number of tardy jobs. There are 2 jobs classified as tardy and 8 jobs classified as early in the optimal schedule. The threshold of probability of tardiness is 0.001 which means any job has a probability of tardiness less than 0.001 is classified as early. The proposed approach modified the optimal schedule given by the deterministic approach such that remaining 8 jobs other than the 2 tardy jobs carry 0 risk of tardiness as shown in Figure 3.

Table 2. The results of the proposed stochastic approach

Job	J (1,3)	J (1,5)	J (2,7)	J (1,4)	J (2,8)	J (2,9)	J (1,2)	J (2,6)	J (1,1)	J (2,10)
Pro. Time	214	216	126	214	126	104	135	141	198	234
Due Date	683	673	752	933	1003	1243	1961	2835	218	905
Comp. Time	214	430	556	770	896	1000	1135	1276	1474	1708
Prob. of Tardiness	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
Tardy	0	0	0	0	0	0	0	0	1	1
Risky	0	0	0	0	0	0	0	0	0	0
Early	1	1	1	1	1	1	1	1	0	0

Figure 3. The probabilistic Gantt chart of the proposed stochastic approach for 10-Job problem

4.1. Experiment with 20-Job Dataset

The deterministic approach and the proposed stochastic approach also experimented with 20-Job dataset. The resulted Gantt chart obtained from the deterministic model is shown in Figure 2.

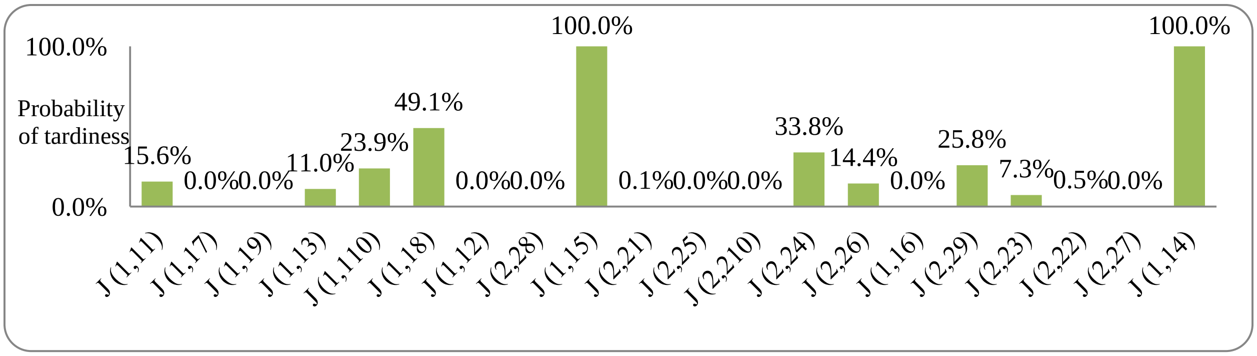

Figure 4. The probabilistic Gantt chart of deterministic approach for 20-Job problem

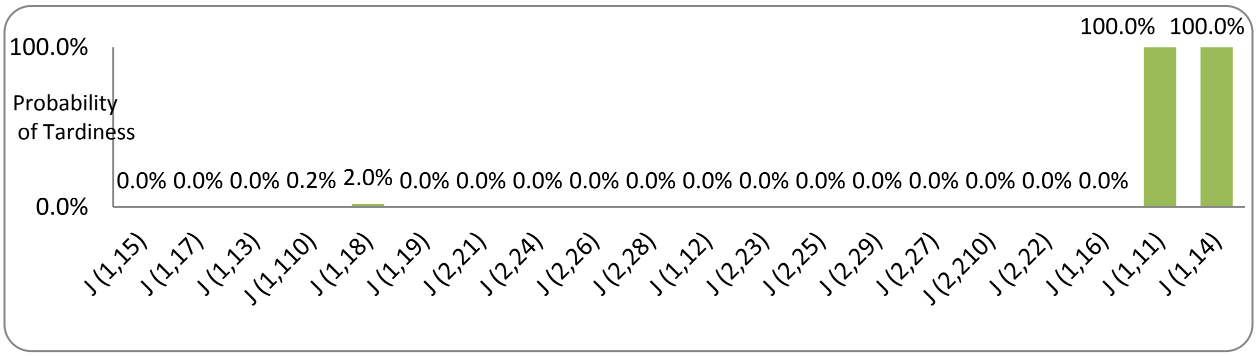

Figure 5. The probabilistic Gantt chart of the proposed stochastic approach with 20-Job problem

Similar pattern of behavior is observed for the 20-Job problem. The deterministic approach provides a schedule with 2 tardy jobs (jobs with 100% probability of tardiness) and 10 risky jobs and 8 early jobs as shown in Figure 4. According to the optimal schedule provided by the deterministic approach, half of the jobs in the schedule carry a positive risk or tardiness since it does not take the variance of processing time into account thus probability of tardiness. On the other hand, the proposed approach provides a very safe schedule in terms of probability of tardiness as shown in Figure 5. There are 2 tardy jobs, 16 early jobs and only 2 risky jobs with less than 2% risk of tardiness obtained.

5. Conclusion and Future Work

In this study, a stochastic single machine group scheduling problem with individual due dates is addressed. Each job has individual deterministic due date and normally distributed processing time. A stochastic non-linear mathematical model to minimize total probability of tardiness and the number of tardy jobs is developed and experimented with two datasets, namely: 10-Job and 20-Job. Once the optimal schedule is obtained, jobs are grouped into three: tardy, risky and early based on the obtained probability of tardiness. A job is called tardy if the probability of tardiness is 1 and early if it is 0. If the job is neither early nor tardy (the probability is between 0 and 1), it is classified as risky. The proposed approach decreased the number of risky jobs in both of the datasets and provided a "safer" schedule in terms of risk of tardiness. It is also important that if the scheduler knows the probability of tardiness, it will be a guide to modify due dates such that a safer schedule can be obtained. Due date modification algorithms, metaheuristics and family setup times are considered as future work of the current research.

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