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Optimal Order of a Supply Chain under the Effect of Group Purchase

Abstract:

This article develops an optimal control model of a singel supply chain triggered by the demand produced by group purchases to find the optimal order. Assuming demand information being shared totally, we give the optimal order by using optimal control theory and study the effect of group purchase on it.

Keywords: Group Purchase, Optimal Control Theory, Supply Chain

Introduction

With the development of internet trading, group purchase takes more and more important role in marketing behaviors. By group purchase, consumers can buy products with a price lower than the market price while retailers can sell more products and make its products to be well-known.

Group purchase (or GP) has two characters: the first is amount discount and the second is the effect to encourage consumers to purchase. In the United States, a group purchasing organization (or GPO) is an entity that is created to leverage the purchasing power of a group of businesses to obtain discounts from vendors based on the collective buying power of the GPO members. In China, group purchase also has the effect to encourage customers to purchase more than under the situation with no group purchasing activities.

There are many Empirical researches on the group behaviors. Gwen R.B., *et.al* researched children's susceptibility to peer group purchase influence [1]. Donald G.N. assumed that individuals would purchase the product of interest over the specified time periods[2]. William O.B., et.al studied the perception of reference group influence on product and brand purchase decisions [3]. Gustav F., et.al reviewed some developments in dynamic optimal models of advertisement [4]. X.H. Zhang, L.P. Lv researched the manufacturer's optimal control policy on wholesale price and retailer's optimal control law on both inventory on price which adjust the supply chain at some time point related to the lifetime of goods and stock level [5]. Song and Zipkin showed that a state feedback (s, S) policy is optimal by viewing the production-inventory model as a dynamic program[6]. Sethi and Cheng found that a state feedback (s, S) policy is still optimal when they research a single location

discrete-time system with a general cost structure [7]. The optimal inventory policy of a period review inventory system for a perishable product under two replenishment modes was studied [9]. There are also many others literatures researching by dynamic programming methodology. However, very few literatures studied the case that demand depends on the some marketing behaviors of retailers. As an important marketing behavior, group purchase should be taken seriously.

It is worthwhile to note that the supply chain referred in our research is a centralized supply chain composed of a manufacturer and a retailer. Our work is concerned with modeling the state transition and the relation between system state and its profit. Taking the effect that group purchase could encourage customers buy more products in the future in to consideration, we introduce an optimal control model viewing the level of group purchase and orders as control various to study the optimal order policy.

Problem statement

A centralized supply chain composed of a manufacture and a retailer wants to make an order policy to maximize its profit in a finite horizon, in the case where demand follows some given rules. Facing the demand fluctuation, the retailer can launch group purchase programs to encourage customers to purchase more products. Because of the excitation effect, retailer can sell the products with a lower price while its future demand being improved. The problem is how to decide the level and the time for launching group purchase.

Considering the dynamic nature, we view investigate a discrete production-inventory planning problem by dynamic programming methodology, where retailers are like to take out some products for group purchase to stimulate the demand. The decision problem is a multi-stage decision making problem. At the beginning of each stage, retailer should decide whether or not they should launch group purchase and at what level the launching should be.

Model

Assumptions and notations

The following assumptions and notations will be used throughout this paper.

(1) $D_1(t)$: the customer demand at time t.

(2) $D_2(t)$: the scale of the group purchase at time t.

(3) $I(t)$: the inventory at time t.

(4) L: the lead time.

(5) $O(t-L)$: the order sent by retailer at time t-L.

(6) p_1: retail price

(7) p_2: group purchase price

(8) c: cost of one unit product.

(9) h: hold cost of unit product.

Mathematic Model

We define objective function as $\sum_{t=0}^{T}\left(x(t)^{T}\bullet A_{1}+u(t)^{T}\bullet A_{2}\right)$ which represents the total

profits obtained from initial stage to the T-stage. The value $x(t)^{T}\bullet A_{1}+u(t)^{T}\bullet A_{2}$ denotes the

profits obtained from time t to time t+1. Our goal is to maximize the total profits,i.e.

$$\max_{u(t)}\sum_{t=0}^{T}\left(x(t)^{T}\bullet A_{1}+u(t)^{T}\bullet A_{2}\right)$$

Where

$$A_{1}=\begin{pmatrix}p_{1}-c\\-h\end{pmatrix}, A_{2}=\begin{pmatrix}p_{2}-c\\0\end{pmatrix}$$

$$x(t)=\begin{pmatrix}D_{1}(t)\\I(t)\end{pmatrix}, u(t)=\begin{pmatrix}D_{2}(t)\\O(t-L)\end{pmatrix}$$

State transition equation:

$$x(t)=B_{1}x(t-1)+B_{2}u(t-1)$$

Subjected to

$$x(t)\geq0,\quad u(t)\geq0$$

$$x(0)=\begin{pmatrix}D_{1}(0)\\I(0)\end{pmatrix},\quad I(T)=0$$

Where

$$B_{1}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}, B_{2}=\begin{pmatrix}k&0\\-1&1\end{pmatrix}$$

Viewing $D_{2}(t)$ and $O(t-L)$ as controllable input various, $D_{1}(t)$ and $I(t)$ as

state various. Denoted $u(t)$ as $\begin{pmatrix}D_{2}(t)\\O(t-L)\end{pmatrix}$, $x(t)$ as $\begin{pmatrix}D_{1}(t)\\I(t)\end{pmatrix}$. Our goal is to find an

optimal order policy to maximize the value of object function.

Algorithm

Considering the complexity, we adopt an approximate situation to replace the one

in the model. We apply dynamic programming methodology to solve the problem. Define edge as one of the choices satisfying the subjections of u(t). Its weight is the value of the object function in phase t. We reproduce the process in computer environment as follows:

(1) Weighting the edges representing the route of state transform.

V={x0}, E=0, i=1, j=1, w is a function define on set E.

A1. U= U(i-1)

A2. If $U = \varnothing$, then $E_i = E$, do A3; else choose one point from U, denote this point $u_{i-1,j}$, $E = E \bigcup \{e_{i-1,j}\}$, $w(e_{i-1,j}) = x(i-1)^T \bullet A_1 + u_{i-1,j}^{\ T} \bullet A_2$, $U = U / \{u_{i-1,j}\}$, $x_{i-1,j} = B_1^{-1} x(i-1) + B_1^{-1} B_2 u_{i-1,j}$, j=j+1, Do A2.

A3. If i-1<T, then i=i+1, j=1, E=0, do A1; else B1.

(2) Appling Bellman principle to calculate the optimal paths.

B1.

$for\ i \leftarrow T\ downto\ 1$
$\quad for\ m \leftarrow |E_{i-1}|\ downto\ 1$
$\quad\quad for\ n \leftarrow |E_T|\ downto\ 1$
$\quad\quad\quad set\ p_{mn}\ as\ the\ optimal\ path\ between\ x_{i-1,m}\ and\ x_{T,n}$
$\quad output\ p_{mn}$

Conclusion

To research the effect of group purchase on the optimal order policy, we turn to experiment with computer. It is worthwhile to present that our model assumes continuous order quantity, while decisions are made at some integer points in quantity. We derive the trends of optimal policy while we control the change of the vector sequence $u(t)$ based on the contrast of different $u(t)$.

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