The catastrophe and control model of emergency logistics capacity system

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Abstract: Taking the catastrophe theory as a foundation, the paper built a swallowtail catastrophe model of ELCS(emergency logistics capacity system) with ELC(emergency logistics capacity) as state variable, logistic flow change rate, time change rate and cost change rate as three control variables. The balance surface, singularities set and bifurcation set are ascertained by potential function. After this, the break points and its stability of ELCS are discussed, based on which, the nonlinear programming model in order to control and improve logistics capability are established. At last an example is given to test the effectiveness of the models. Three conclusions are educed. Firstly, the relevant data of three control variables can be measured through surveys, and the singularity set and bifurcation set of system can be determined, also to determine the stability and mutation trends of current system state can be ascertained. Secondly, according to the characteristics of emergency logistics needs, the nature of control variables and the relevant external force, we manage control variables effectively, which will enhance ELC instantaneously and significantly. Lastly, to enhance the stability of ELCS is a system engineering. As the control elements have the deviate active direction, we manage and control the main elements with Pareto thought, the non-inferior solutions for the relative balance of time-effectiveness, flow-effectiveness and cost-effectiveness can be obtained.

Key Words: ELCS(emergency logistics capacity system); swallowtail catastrophe model; singularity; control model

1 Introduction

In recent years, catastrophic events such as earthquakes, floods, mudslides, typhoons, tsunamis, infectious diseases (e.g. SARS, H1N1) happened frequently, which brought tragic loss of property and personnel. ELC(emergency logistics capacity) is one of the important factors influencing the loss degree of the disaster. ELCS(emergency logistics capacity system) refers to a comprehensive reflection of timeliness and reliability meeting the needs in response time, response speed, logistics cost, and other aspects in entire process of analysis and confirmation from the emergency needs, emergency supplies purchasing, sorting, transportation and delivery to emergency customer[1]. Emergency supplies and rescue workers brought into, affected people withdrawed, the dead buried, sanitation and epidemic prevention all depend on emergency response from a strong ,flexible and efficient ELCS. The 8.8 magnitude earthquake in Chile took only a few hundred lives, and led to more than 200,000 people be killed in Haiti's 7.0 magnitude earthquake, one of the difference in which is the strength of ELCS. The 2008 earthquake rescue process in Wenchuan of China shows us the strong response logistics capability than before posed by trained emergency rescue teams, emergency logistics equipment (e.g. reserve vehicles) and emergency logistics command system. ELCS construction in China is still in the early stages ,and to study it in-depth will have the theoretical and practical significance to improve ELC and respond to emergencies by the state.

Scholars have concerned about ELC issues and evaluated the ELC from different angles by different methods[2-5]. Many scholars have studied unexpected events by catastrophe model, such as established catastrophe model to study issues of the public space congestion, coastal water pollution, infectious disease outbreaks and other public emergencies [6-9]. Zhang Yaping, Tang Tieqiao et al.[10-11], respectively, predicted the traffic flow with cusp or swallowtail catastrophe theory. The former took traffic speed as state variable, traffic flow and traffic density for the control variables to establish cusp catastrophe model. Based on the former's study, the latter added time factor as the control variable. In both papers the authors achieved good results. Chen tao et al.[12] used cusp catastrophe theory to establish congestion control model, using the potential function and the kinetic energy function to describe the relationship between variables in crowded space transportation system, determining the critical mutation points, stable set points and the bifurcation points in the crowded space, and suggested the traffic flow congested control model. It is a good foundation and inspiration to research on issues of emergency and traffic flow with mutation model for this study. The applicability of the catastrophe model and the characteristics of ELCS all indicate that the catastrophe model is one of the effective tools to study dynamic ELCS. In this paper, swallowtail catastrophe model with emergency requirements of supplies, time and costs as three control elements will be established to study the ELCS. Based on the analysis of the potential function, the balance curved surface

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and the singularities set of ELCS, critical points will be solved and its stability and change direction will be discussed, joint with constituting system control model, ELC can be predicted, controlled and enhanced.

2 Swallowtail catastrophe model of ELCS

2.1 Catastrophe theory and ELCS

In 1960s and 1970s, the French mathematician Thom put forward catastrophe theory based on singularity theory and structure stability theory. The theory reckons that there are multiple steady states in the state space through the process of mutation system evolution, and also there are instable points or areas. While the control parameters changing and crossing certain line, the system will switch from one stable state to another stable one, that is mutated. There are five indications of mutation system: multi-mode, unreachable, divergence and hysteresis. Depending on the mutation indications, the number of state variables and control variables, Thom and others have set up seven elementary catastrophe functions: fold, cusp, swallowtail, butterfly, oval navel, hyperbolic and parabolic umbilical[14]. Catastrophe theory and models scattered in applications of mathematics, mechanics, physics, chemistry, biology, sociology currently. According to catastrophe theory, it can be used to analyze relative system issues if the system shows two or more mutation indications. Based on the number of state variables and control variables, we can select one of the appropriate elementary mutation functions as a system evolutionary model to analyze the evolution characteristics and patterns of mutation system.

ELCS is a complex problem described and controlled by a number of parameters. On the one hand, because of the community's concerning to unexpected events occurring, the first aid mentality makes ELC very stronger under the green channel of ideology, organization and access. On the other hand, some unexpected events such as earthquakes and floods will damage or block certain channels and logistics facilities, coupled with the occurrence of associated events such as poor communication, second nature disasters, traffic congestion caused by people gathering etc., will weaken ELC enormously. Under varies conditions and factors, this complex system has multi-mode (two basic stable states of strong or weak capacity, other stronger or weaker instability states), jumps (emergency supply materials all over flock to the rescue point, beyond the capacity of it, ELCS will stall or even collapse from strong to weak state), divergence (sudden jump occurs, the system blockage or severe secondary disasters, self-organization was unable to change the status quo), unreachable (the system in a variety of unstable region to continue to adjust until steady-state motion, instability is not up) and other features. This makes ELC mutate between strong capacity (to meet the needs of emergency logistics), weak capacity (can not meet the demand of emergency logistics). The paper intends to use one state variable, three control variables to describe ELCS. According to Thom's theory, swallowtail catastrophe model will be used to analyze mutation structure and character of ELCS. Unexpected event occurs, ELCS is disturbed and the control variables show different changes, so that ELCS continue to show states: strong capacity to weak capacity, to strong capacity again and to weak capacity again, which means ELCS jumps from one stable state to another one.

2.2 Control variables and state variables

The influence of emergency to ELCS represents the changes of control variables firstly, thereby causes the state variable changes in a number of state space systems as well as systems catastrophe. ELCS is in pursuit of maximum capacity to meet the demand of emergency logistics, mainly in aspects of mass material flow, short time-consuming, and lower cost etc.. So in essence, the flow, the time and the cost are three control elements of ELCS. Taking it into account that the actual values of three factors relative to meet the emergency needs impact on the state variable and system function more dynamicly, also taking the consistency of control variables dimension and dynamic model into account, in this paper, the control variables are amended and changed in use of flow change rate, time change rate and cost change rate of emergency logistics. Their definitions are as follows.

Definition 1. Emergency logistics flow change rate $\varphi(q)$, the first control variable, indicates the change rate of demand satisfaction degree of supply material flux compared to demand quantity from rescue points, that is,

$$\varphi(q) = \frac{\sum q_i(s) - \sum q_j(d)}{\sum q_j(d)} \times 100\,\% \text{ , } (i = 1,2,\cdots,I; j = 1,2,\cdots,J) \quad (1)$$

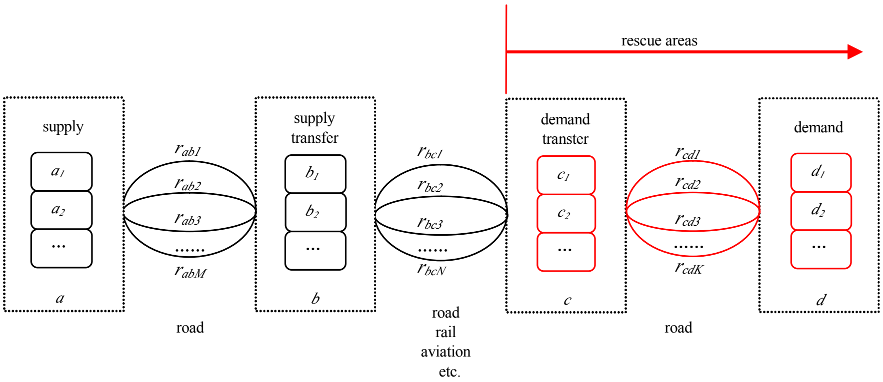

Fig.1. Emergency logistics path block diagram

Assuming adequate supply, the material flow (supply) only refers to those reaching the rescue points in the constraint conditions, that is total flow $\sum q_i(s)$ through arcs r_{cdk} ($k = 1,2,\cdots,K$) in the time window $[0,t_w]$in Fig.1. Time window is the effective time interval (hours) of the rescue points in need of emergency relief supplies. ELCS required to be effective in the material supply to rescue points within a time interval. In Fig.1., arcs r_{abm} ($m = 1,2,\cdots,M$), r_{cdk} are the road transportation modes， and arc r_{bcn} ($n = 1,2,\cdots,N$) are mixed modes of road, rail, aviation and others. Arcs r_{cdk} are the road transportation section to rescue points directly, according to the disaster situation, there may be various models of road transportation, such as trucks, carts and so on. Within the time window $[0,t_w]$, the total flow through section r_{cd} by V modes of transportation is,

$$\sum q_i(s) = \int_0^{t_w} \sum_{k=1}^{K} \sum_{v=1}^{V} q_{kv}^{cd}(e)dt \tag{2}$$

Definition 2: Emergency logistics time change rate $\varphi(t)$, the second control variable, indicates the change rate of emergency logistics time ($\sum t_A(e)$) compared to time window ($T(e)$). (Taking normal logistics time $\sum t_A(z)$ as a reference, $T(e) = \sum t_A(z)$)

$$\varphi(t) = \frac{\sum t_A(e) - T(e)}{T(e)} \times 100\,\% \tag{3}$$

Where $\sum t_A(e)$, $\sum t_A(z)$ is total time spent in all nodes, lines and networks when in emergency or in normal respectively. The total time in the normal composes of road transportation time and the node operational time, i.e., $\sum t_A(z) = \sum t_A^{\ 1}(z) + \sum t_A^{\ 2}(z)$ .When unexpected event occurring, because of green channel of organization and operation, the time efficiency has improved in different degrees in arcs r_{ab} , r_{bc} and at nodes, the logistics time will be shortened. But due to external disturbance, the occurrence probability of road disruption, blocking and repairing is larger, there will increase the abnormal waiting time in section r_{cd} .So emergency response time composes of transportation time , node processing time and impedance response time, namely,

$$\sum t_A(e) = \sum t_A^1(e) + \sum t_A^2(e) + \sum t_A^3(e)$$

$$= \sum_{r=1}^{R}\sum_{v=1}^{V} t_{rv}^1(e) \times p_{rv}^1(e) + \sum_{u=1}^{U}\sum_{v=1}^{V} t_{uv}^2(e) + \sum_{h=1}^{H} t_h^3(e) \times p_h^3(e) \quad (4)$$

Respectively, where $t_{rv}^1(e)$, $p_{rv}^1(e)$ are the transportation time, the share of the transportation task undertaken by transportation mode v in section r , $p_{rv}^1(e) = \frac{q_v(s)}{\sum q_v(s)} \times 100\%$ .And $t_{uv}^2(e)$ is operational time at node u by transportation mode v, and the time consuming at the first node includes materials organization delay. $t_h^3(e)$, $p_h^3(e)$ denote impedance response time, the occurrence probability of the *h*-th impedance respectively.

Definition 3: Emergency logistics costs change rate $\varphi(c)$, the third control variable, refers to the change rate of an emergency logistics costs ($\sum c_A(e)$) compared to normal logistics costs ($\sum c_A(z)$).

$$\varphi(c) = \frac{\sum c_A(e) - \sum c_A(z)}{\sum c_A(z)} \times 100\,\% \quad (5)$$

The total emergency logistics costs consist of transportation costs, transit costs and impedance response costs mainly, that is,

$$\sum c_A(e) = \sum c_A^1(e) + \sum c_A^2(e) + \sum c_A^3(e)$$

$$= \sum_{r=1}^{R}\sum_{v=1}^{V} c_{rv}^1(e) \times p_{rv}^1(e) + \sum_{u=1}^{U}\sum_{v=1}^{V} c_{uv}^2(e) p_{uv}^2(e) + \sum_{h=1}^{H} c_h^3(e) \times p_h^3(e) \quad (6)$$

where $c_{rv}^1(e)$, $p_{rv}^1(e)$ mean the transportation costs or the transportation task share by transportation mode v in section r . $p_{rv}^1(e) = \frac{q_v(s)}{\sum q_v(s)} \times 100\%$. $c_{uv}^2(e)$, $p_{uv}^2(e)$ are the transfer costs, the share of transfer costs in transit node u by mode v . $c_h^3(e)$, $p_h^3(e)$ mean anti-impedance costs or the occurrance probability of *h*-th impedance respectively. Transportation costs and transit costs are associated with traffic flow, and transportation costs include basic freight (e.g. basic tariff of roads, rail, aviation) and mileage freight (freight associated with road length, weight of materials, fuel, labor and wasting and so on).

Definition 4. Emergency logistics capacity l ,the system state variable, refers to the ELCS offering rescue points satisfaction degree of mass-flux, short-time, low-cost when sudden incident happened. It results in system mutations directly when control variables changes in different degrees under various disturbances outside the world. The control variables being stable or not, are the direct affecting factors of the system would mutate or not, which can also be said ELC is the independent variable and ELCS is dependent variable in one system. ELC's small perturbation can cause a complete change of ELCS state, i.e. mutations. If it can meet the number, time, cost emergency needs when emergency occurring and sudden demand increasing, ELCS is in strong state, otherwise in weak state. As the inconsistency among control variables change direction and extent, resulting in ELC changing constantly, thus ELCS will change between the strong state and the weak one, or among several unstable states.

2.3 Swallowtail catastrophe model of ELCS

According to catastrophe theory and the characteristics of this study, taking ELC as the state variable, and taking flow change rate, time change rate and costs change rate as control variables, the paper constructs the potential function of the swallowtail catastrophe model of ELCS,

$$f(l;\varphi(q),\varphi(t),\varphi(c)) = l^5 + \varphi(q)l^3 + \varphi(t)l^2 + \varphi(c)l \quad (7)$$

Where l is ELC, $\varphi(q)$ is flow change rate, $\varphi(t)$ is time change rate, $\varphi(c)$ is the costs change rate.

To get the equilibrium surface equation by $f'(l) = 0$,

$$5l^4 + 3\varphi(q)l^2 + 2\varphi(t)l + \varphi(c) = 0 \quad (8)$$

The singular point set is obtained by $f''(l) = 0$,

$$10l^3 + 3\varphi(q)l + \varphi(t) = 0 \quad (9)$$

3 Singularity and its stability of ELCS

3.1 Bifurcation curve and potential function curve of ELCS

The bifurcation set of swallowtail model is obtained by eliminating l with (8) and (9) ,

$$4096\varphi^6(q) - 46629\varphi^4(t) + 4096\varphi^3(c) = 0 \quad (10)$$

The bifurcation set of ELCS is a space surface in a multi-dimensional space $(\varphi(q),\varphi(t),\varphi(c))$, and the points on the surface that would lead ELCS to mutate are singularities. When $\varphi(q) > 0$ or $\varphi(q) \le 0$, the curve images of $\varphi(t) - \varphi(c)$ section are different. According to the characteristics of this study, the article only discusses the case when $\varphi(q) \le 0$. When the other control parameters remain unchanging, the process of $\varphi(q)$ tending to zero is that of ELC enhancing. With (8), (9) and (10), we draw the diverging curve when $\varphi(q) \le 0$ (see Fig.2.), where small pictures show the corresponding potential function space curve. The curve divides the surface of potential function space into different areas, the cut lines of different regions are different shapes, different number of extreme points, different nature of potential function, and different steady-state of ELCS.

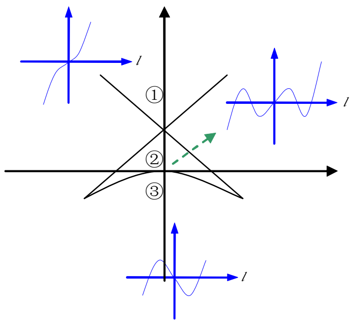

Fig.2. Bifurcation set when $\varphi(q)$ is constant($\varphi(q) \le 0$) and potential function curve of each region[13]

3.1.2 Singularities and their stability of ELCS when $\varphi(q) \le 0$

From symmetry characteristic of bifurcation set, to maintain control variable $\varphi(q)$ a constant, division of the area is discussed when $\varphi(q) \le 0$ in flat $\varphi(t) - \varphi(c)$. As the bifurcation set is the number of equilibrium position and the boundary of system nature change, only to find a point can be represented[14-15]. Considering to meet the emergency time requirements, that is the case when $\varphi(t) = 0$, the solutions of the equilibrium surface are found from (8),

$$l^2 = \frac{1}{10}(-3\varphi(q) \pm \sqrt{\Delta}) \quad (11)$$

Where $\Delta = 9\varphi^2(q) - 20\varphi(c)$, followed the same. (11) into (9) is,

$$f''(l) = \left\{ \pm \left[\frac{1}{10}(-3\varphi(q) \pm \sqrt{\Delta}) \right]^{1/2} \right\} \bullet \left\{ 20 \times \left[\frac{1}{10}(-3\varphi(q) \pm \sqrt{\Delta}) \right] + 6\varphi(q) \right\} \quad (12)$$

When $f''(l) > 0$, the system potential function has a minimum point l, that is a stable singularity, where ELCS is stable and not to mutate. When $f''(l) < 0$, the system has a maximum point l, that is unstable singularity point, where ELCS is unstable and the mutation will occur. When $f''(l) = 0$, the system only has turning point, not to mutation. According to (9) to (12) and Fig.2., analysis results show that the system has three states when $\varphi(t) = 0$,

State 1: When $\varphi(q) \le 0$ and $\varphi(c) > \frac{9}{20}\varphi^2(q)$, the system potential function has no singularities, ELCS is stable and not to mutate.

State 2: When $\varphi(q) \le 0$ and $0 < \varphi(c) \le \frac{9}{20}\varphi^2(q)$, the system potential function has four singularities, $l_{1,2,3,4} = \pm\left[\frac{1}{10}(-3\varphi(q)) \pm \sqrt{\Delta}\right]^{1/2}$,where $l_2 = -\left[\frac{1}{10}(-3\varphi(q)) + \sqrt{\Delta}\right]^{1/2}$, $l_3 = \left[\frac{1}{10}(-3\varphi(q)) - \sqrt{\Delta}\right]^{1/2}$ are unstable singularities that will bring ELCS to mutate from one stable state to another.

State 3: When $\varphi(q) \le 0$ and $\varphi(c) \le 0$, the system potential function has two singularities $l_{1,2} = \pm\left[\frac{1}{10}(-3\varphi(q)) + \sqrt{\Delta}\right]^{1/2}$, the same unstable ones (negative solution) would make ELCS mutate.

When $\varphi(t) = 0$, system has unstable singularities in state 2 or in state 3. When the unstable singularities cross the bifurcation set, the system will mutate, or change from one stable state to another (e.g., from state 2 to state 3 or state 3 to state 2), or stable state disappear (such as from state 2 to state 1 or from state 3 to state 1). The more unstable singularities are in the region, the more the system is active and prone to mutate.

4 ELCS control and optimization

In mutation model, the greater the value of potential function is, the stronger ELCS'performance is. However, according to Maxwell agreed, when potential function reaches a maximum value, it will move quickly to a minimum point direction until equilibrium stability in another area of bifurcation set. Namely, the system is mutated. At this point, only to rely on external force, the potential function can move back to maximum direction. Or changing values of control variables when potential function in the maximum state, leveling off control points move around the maximum point, the system will not mutate as long as the control point does not cross the bifurcation set. Because of multiple control variables have inconsistency control direction, the system changes uncertainly and complexly. In case of road congestion, we control logistics flow traffic can regain flux, but the supply reduced. Or in case of road block, we use large logistics facilities and equipments so that the road can re-access, and traffic flux increased, waiting time reduced, but costs increased. Catastrophe model provides us with a system status and change direction, while controlling the control parameters can make the system in good condition. The direction of the control variables are inconsistent, so it is difficult to find a solution to makes the multi-objective optimal for each target. With Pareto optimal thinking, the paper will establish non-linear planning and control models to solve non-inferior solution,

$$\max\ f = l^5 + \varphi(q)l^3 + \varphi(t)l^2 + \varphi(c)l \tag{13}$$

$$\text{s.t. } 4096\varphi^6(q) - 46629\varphi^4(t) + 4096\varphi^3(c) = 0 \tag{14}$$

Models represent that the system potential function seeks the maximum while maintaining the control point in the corresponding area of bifurcation region. ELCS can be maintained in high level by the control point control. After the incident, facing to needs of great materials, short-term and low-cost from aid-points, ELCS strives to enhance the balance of the three to get the great ELC and to achieve the best rescue effect. That is the change process of $\varphi(q)$ varying in the positive direction, $\varphi(t)$ and $\varphi(c)$ to the negative direction at the same time, also it is the process of ELCS becoming stronger. This article will transform the above model ((13) and (14)) into the non-linear goal programming model to obtain the system's maximum flow and minimum cost when $\varphi(t) = 0$ (for emergency time management and to meet the emergency time needs), that is,

$$\max\ z = \left\{\sum q(s), 1/\sum c(e)\right\} \tag{15}$$

$$\text{s.t.} \quad q_{l_{a_i c_i}}(s) = \sum_j q_{l_{c_i d_j}}(s), (i = 1,2,\cdots I) \tag{16}$$

$$q_i(s) \le \overline{C_i},\ (i = 1,2,\cdots I) \tag{17}$$

$$\varphi^6(q) + \varphi^3(c) = 0 \tag{18}$$

$$\varphi(t) = 0 \tag{19}$$

$$q_i(s) \ge 0, t_A(e) \ge 0, c_A(e) \ge 0\ (i = 1,2,\cdots I) \tag{20}$$

Where (15) is the objective function, for the maximum emergency flow and the minimum emergency cost. (16) and (17) are the flow (supply) constraints, inflows and outflows of transit node to be equal, but also to meet the capacity limit $\overline{C_i}$ of the arc. (18) is the balance constraints of the flow change rate and the cost change rate（obtained by equation (10) when $\varphi(t) = 0$）. (19) is the time constraint. (20) is the non-negative constraints.

5 Simulation numerical analysis

5.1 Simulation case design

7.0 level Earthquake occurred in South area. After the disaster-hit, the demand of emergency supplies from four cities d_1, d_2, d_3, d_4 is at least 250*t* (*t* for ton, after the same) each. According to the material needs of four demand cities and the supplies from foreign urban, 900*t* goods are decided to transport from two cities a_1, a_2 around the disaster areas, and 80*t* short-supply goods are from the northwestern city a_3 by train, 20*t* shortage materials are from the northeast city a_4 by air. According to the disaster, relief supplies should reach relief points within 12*h* to 16*h* (*h* for hours, after the same). According to the operation data of a logistics company and situation analysis of this case, the total transportation costs are about 200,000 yuan under normal logistics state from four supply points to four demand points by 12 lines, where transportation unit cost is about 2.5 yuan/5*t•km* (*t•km* for ton•kilometer, after the same). The topology from supply points to rescue points is shown in Fig.3., where *S*, *T* are the virtual node of supply point set or demand point set respectively, in which section or notes flow, time and cost do not occur. Table 1 displays the relative data of each section, the node-related traffic, time and cost. According to the experience of the logistics company , emergency traffic capacity for the road with speed limit of 60*km/h*, 100*km/h*, 40*km/h* is 300*v/h*, 500*v/h*, 200*v/h* respectively (*v/h* for vehicles/hour, after the same), i.e., that is normal traffic capacity minus normal flow. The average service capacity of each node is 10*v/h* during emergency period. Rescue vehicles are with standard model 5*t*, assuming full load.

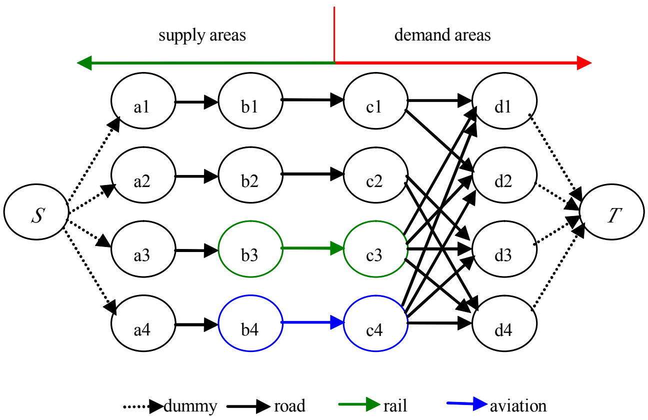

Fig.3. Network topology of emergency logistics

Table 1 Related data of sections, vehicles of emergency logistics

serial number	road section	distance	vehicle number	Node time-spent	node cost	normal transportation time	impedance probability (%)	anti-impedance time	unit cost of anti-impedance
1	r_{a1b1}	12	90	0.5	20	0.26	-	-	-
2	r_{a2b2}	15	90	0.5	20	0.33	-	-	-
3	r_{a3b3}	18	16	0.5	20	0.39	-	-	-
4	r_{a4b4}	9	4	0.5	20	0.20	-	-	-
5	r_{b1c1}	180	90	-	20	1.80	-	-	-
6	r_{b2c2}	160	90	-	20	1.60	-	-	-
7	r_{b3c3}	225	-	0.5	20	2.25	-	-	-
8	r_{b4c4}	400	-	1	20	1.20	-	-	-
9	r_{c1d1}	25	25	0.5	20	1.25	0.4	2	1000
10	r_{c1d2}	16	65	0.5	20	0.80	0.2	3	500
11	r_{c2d3}	8	35	0.5	20	0.40	0.8	2	1000
12	r_{c2d4}	18	55	0.5	20	0.90	0.5	5	500
13	r_{c3d1}	25	4	1	30	1.25	0.9	5	1000
14	r_{c3d2}	21	4	1	30	1.05	0.5	2	600
15	r_{c3d3}	20	4	1	30	1.00	0.4	3	300
16	r_{c3d4}	15	4	1	30	0.75	0.7	4	800
17	r_{c4d1}	25	1	1.5	50	1.25	0.2	3	500
18	r_{c4d2}	10	1	1.5	50	0.50	0.5	6	500
19	r_{c4d3}	16	1	1.5	50	0.80	0.5	7	500
20	r_{c4d4}	17	1	1.5	50	0.85	0.4	4	500

5.2 Operation results

The unilateral time window $t_w(e)$ (12*h*,13*h*,14*h*,15*h*,16*h)* is considered to analyze the flow changes and cost changes. Considering $\varphi(t)=0$, according to (1) to (6) , (15) to (20), the operation results are shown in Table 2.

Table 2 **Each index value of ELCS**

time window	mutation data					control data			
	$\varphi(q)$	$\varphi(c)$	l	$f''(l)$	stable or unstable	$\max\sum q(s) / \min\sum c(e)$ $\alpha_0=0$	$\alpha_1=0.1$	$\alpha_2=0.3$	$\alpha_3=0.5$
$t_1(e)=12$ $\varphi(t)=0$	-0.816	0.034	1.596 -1.596 0.120 -0.120	36.734 -36.734 -0.277 0.277	stable unstable unstable stable	188 /150772	181 /161388	169 /180338	156 /193608
$t_2(e)=13$ $\varphi(t)=0$	-0.289	0.018	0.845 -0.845 0.155 -0.155	5.310 -5.310 -0.097 7.187	stable unstable unstable stable	711 /172077	765 /182654	875 /201297	984 /213665
$t_3(e)=14$ $\varphi(t)=0$	-0.184	0.001	0.768 -0.768 0.043 -0.043	4.109 -4.109 -0.023 0.023	stable unstable unstable stable	816 /191210	870 /201764	980 /220217	1089 /232026
$t_4(e)=15$ $\varphi(t)=0$	-0.084	-0.009	0.716 -0.716	3.489 -3.489	stable unstable	916 /210344	970 /220874	1080 /239136	1189 /250388
$t_5(e)=16$ $\varphi(t)=0$	0.000	-0.015	0.735 -0.735	3.972 -3.972	stable unstable	1016 /227031	1070 /232952	1180 /242104	1289 /245107

Notes: α_i is the reduce rate of impedance response time.

(1) From left six in Table 2 can be seen, as $t_w(e)$ is 12*h*, 13*h*, 14*h* respectively, the system potential function is

very unstable, there are two unstable singular points. With $\varphi(q)$ changing from negative to zero and $\varphi(c)$ from the positive to zero, the system mutation happens in the transition process of 14h to 15h, and the system transfers from four -singularity steady state into two-singularity steady state, and the value of $\varphi(c)$ changes from positive to negative. From changes of $\varphi(q)$, $\varphi(c)$, there is the larger magnitude of changes in the process of 12h to 13h. Because of the control point did not cross the critical point, so no mutation happens in the nature of the system, but varying in this stage promotes the mutation in next stage.

（2）According to the data of right four in Table 2, through the strengthening of resistance management, the average anti-impedance time is shortened by 10%, 30%, 50%. Within different time windows, the values of $\sum q(s)$ and $\sum c(e)$ change in different degree and the relative control points change accordingly. The flow data after controlling in the Table 2, compared to that before controlling, are changed little with 181t, 169t, 156t. However, in the phase of 12h to 13h, flow is obtained with 765t, 875t and 984t respectively, which increased by 321.90%, 417.91%, 529.32% compared to that one hour ago. From the extend of supply meet, the material requirements meet entirely within 16h when $\alpha_1 = 0.1$, and that all meet within 15h when $\alpha_1 = 0.3$, within 14h when $\alpha_1 = 0.5$. From a view of the cost change, the higher degree of management is, the higher costs are. The change rate of flow is greater than that of the costs when impedance response time shortened by 10%, 30%, and the latter is reduced much greater than the former by 50% time shortened. Similar situation to this case, if the time window is 13h when $\varphi(t) = 0$, the ideal state is that flow change rate is more substantial upgrade, while the cost rate does not change dramatically.

（3）The most important indicator to measure ELC is the satisfaction degree of the flow and the time requirement. In this case, compared to normal logistics, emergency response time increases mainly due to material organization abnormal at the first node and response delay or impedance in section r_{cd}. Because of the mass material flow, difficult supply organizations, and logistic risks, the road time is much longer from the supply cities bearing 90% of the material supply missions to rescue points, although there are not far from each other. Operation results show that, when $\varphi(q) = 0$, i.e., the flow is equal to the minimum requirement, the average time of relief supplies reaches the rescue points is 14.28h, then $\varphi(t) = 0.262$, $\varphi(c) = -0.021$. It belongs to state 3, two singularities (± 0.805), where the unstable singularity may make the system mutate. If strengthening the external force, response time can be shortened, costs can be flatted or be declined under the condition $\varphi(q) = 0$. Or to remain $t_w(e)$ unchanged when $\varphi(t) = 0$, if the flow increases, the costs are flat or decline, we can say ELC upgrade continuously.

6. Conclusions

Based on swallowtail catastrophe theory, this paper has built the catastrophe model of ELCS with ELC as the state variable, the flow change rate, the time change rate and the cost change rate as three control variables. The potential function of the system is formed and its equilibrium surface, the singularity set and the bifurcation set are determined to discuss the critical points and its stability of ELC when meet the emergency time needs, based on which the ideas of controlling and enhancing ELC are advanced. Three conclusions are obtained: (1) According to the survey and the measured data ,we can obtain the values of three control variables and to determine the bifurcation set. The value of the control point determines the number and the nature of singularities in different bifurcation set, also determines the stability and the mutation direction of the system: an control point in regions with many unstable singular points is more active, will mobile to the regions with less unstable singularities, which result in a large possibility of the system mutation, whereas the possibility is small, even not a mutation.(2) According to the characteristics of emergency logistics demand, the nature of control variables and the external force, if we manage emergency logistics effectively, ELC will be enhanced instantaneously. (3)To enhance the stability of ELCS is a system engineering. As the control elements have the deviate active direction, we manage and control the main elements with Pareto thought, the non-inferior solutions for the relative balance of time-effectiveness, flow-effectiveness and cost-effectiveness can be obtained. As the information collection of disaster event is more difficult, the paper only use a simple numerical example to verify the feasibility of the model. For a more complicated example, the feasibility of the model needs further verification.

ACKNOWLEDGMENT

This research is supported by the National Natural Science Foundation project (71073079), Jiangsu Educational Science Fund project (08SJB7900029), Jiangsu Society Science Fund Project(11EYD045).

Reference

[1] Liu X Q,You X Z,Sun J Z, Sun Q Z. Conception and constitution research of emergency logistics capability [J]. Journal of Catastrophology, 2007,22(2) :123-127.

[2] Xu X H,Wang Z Q. Logistics Activeness: A criterion for instant response ability of emergency logistics system[J]. China Business and Market, 2009,(9):24-27.

[3] Mohamed Naim, Gilbert Aryee, Andrew Potter. Determining a logistics provider's flexibility capability Original Research Article[J].International Journal of Production Economics, 2010,127(9):39-45.

[4] Deng A M,Zhang F,Xiong J,Wu P F. The emergency logistics capability assessment based on Grey fuzzy comprehensive evaluation method[J]. Statistics and Decision, 2010,306(6):174-176.

[5] Lin Y M,Zhuo J X,Yang K. The study on emergency logistics capability based upon multilevel gray evaluation[J] .Logistics Sci-Tech, 2010,(10):57-60.)

[6] Dong H, Yang W B. The catastrophe models in the forecast of accident and disaster[J].Journal of Geological Hazards and Environment Preservation, 2003,14(3):39-44.)

[7] Wenjun Wang,Suling Liu,Shushen Zhang,Jingwen Chen. Assessment of a model of pollution disaster in near-shore coastal waters based on catastrophe theory[J]. Ecological Modelling,2010,222(6):307-312.

[8] Zheng Xiaoping,Sun Jiahui,Cheng Yuan. Analysis of crowd jam in public buildings based on cusp-catastrophe theory[J].Building and Environment,2010,45(9):1755– 1761.

[9] Chi-Kuo Mao,Cherng G.Ding,Hsiu-Yu Lee.Post-SARS tourist arrival recovery patterns:An analysis based on a catastrophe theory[J].Tourism Management, 2010, 31(7): 855–861.

[10] Zhang Y P, Zang Q S. The application of cusp catastrophe theory in the traffic flow forecast[J]. Journal of Systems Engineering, 2000, 15(3): 272-276.

[11] Tang T Q,Huang H J.The discussion of traffic flow forecast by using swallowtail catastrophe theory[J]. Journal of Mathematical Study,2005,38(1):112-116.

[12] Chen T, Chen S F. Research on models of congestion control based on catastrophe theory[J]. Journal of Systems Engineering, 2006,21(6):598-605.

[13] Wei X L, Zhao H Y, Liu G Z, Wu Y H. Analysis of pest population dynamic model using swallowtail catastrophe theory[J].Ecological Journal, 2009,29(10):5478-5484.)

[14] Ling F H.The theory and application of catastrophe theory[M].Shanghai:Shanghai Jiaotong University Press, 1987:103-106.

[15] Jiang L, Yu L Y. The application of primary catastrophe theory to social science [J]. System Engineering -Theory & Practice, 2002,(10):113-117.)