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Abstract Title: **Multi-Period Sourcing Strategy Under Demand and Supply Uncertainty With Two Suppliers**

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1 Introduction

In the current competitive market, the companies have tried to gain a competitive advantage by managing their supply chain as effectively as possible. The job of managing the supply chain effectively becomes more challenging when uncertainty comes into the picture. Uncertainty may come into the supply chain due to many factors, like, the expansion of product variety, short product life cycle, globalization of businesses etc. On the other hand, continuous advancements in the field of information technology have supplied the companies the weapon to deal with these uncertainties.

The uncertainties in the supply chain can be broadly categorized into two groups, namely, demand uncertainty [12] and supply uncertainty [18]. The literatures in the field of supply chain management have linked the demand uncertainties with the product characteristics (functional or innovative products) and supply uncertainties with the supply characteristics (stable or evolving supply chain). The uncertainty in the supply chain is not only due to the fluctuations in the demand but also due to inefficient production processes and unreliable supply sources [22,27]. Finally, it is argued that, to achieve an efficient supply chain, a company has to correctly choose its supply chain strategies depending on its inherent demand and supply uncertainties. These strategies include whether the company should go for an efficient or risk hedging or responsive or agile supply chain.

The strategies chosen by the companies have a direct implication on their inventory holding pattern. For example, when the supply of a certain product is uncertain and it has a high stockout cost, then the concerned company will go for a risk hedging supply chain. This means that the company will prefer a high amount inventory holding in its own premises. So, we can conclude that uncertainty in demand as well as supply can alter the preferable amount of inventory (or, the safety stock) held. The literature in inventory holding concentrates highly on the effect of uncertainty in demand on the safety stock [17]. It has also been shown that attaining a smaller lead time will ensure lower uncertainty, and eventually a lower safety stock and lower inventory holding cost [18]. The effect of supply uncertainties on the inventory problem is discussed in relatively limited occasions [7].

However, supply chain literature has convincingly argued that in most of the cases the use of multiple supply sources reduces overall inventory holding and distribution cost [14,23]. In this article, the supply perspective of the uncertainty and its effect on the inventory will be discussed.

The current era of globalization also has a huge effect on the supply chain strategies of the firms. Globalization has not only given the firms an open access to a larger market, but also allowed them to attain resources at relatively lower prices. However, the price they are paying for this cheaper resource is through a longer lead time. Also, the reliability (or, quality) of the resources are different from the local resources. As, outsourcing the supply currently has become a common phenomenon for the global firms, how much to outsource has become a very important question for the firms. In this paper, we shall discuss a dual supplier scenario where one of the supplier is local and the other is global. The suppliers are identified by the following characteristics namely, quoted cost, reliability and the lead time for replenishment. We shall try to find out the optimal sourcing policy for the receiving firm depending on the parameters mentioned above. Also, we shall try to investigate the conditions in which outsourcing the entire supply would be more feasible and the conditions in which outsourcing is infeasible.

2 Literature Review

In this paper, we mainly concentrate upon the concept of multiple sourcing, which can be local or global. Literature suggests that one of the ways to decrease the amount of uncertainty present in the supply chain for a firm is via gathering its supply from multiple sources [21]. Elmaghraby [10] and Minner [23] provided a comprehensive review of extant researches done in the field of multiple sourcing. Also, the issue of single versus multiple supplier selection is extensively studied by Van der Vorst and Beulens [27] and Burke et al [8].

The dual sourcing problem with uncertain lead time was addressed by Ramasesh et al. [26]. They considered the demand to be constant and the lead time to be stochastic and compared the dual-sourcing solution with the single sourcing one. The results indicated that with high lead time uncertainty and low ordering cost, dual sourcing is more cost-effective than single sourcing.

Bassok and Akella [2] addressed the problem of identifying the optimal ordering quantity and production levels of a critical component and its parent finished good. The analysis was carried out for a single period considering uncertainty in both demand and supply. On similar lines, Gurnani, Akella and Lehoczky [16] developed modeling frameworks to study the order quantities and the production levels for a system formed by two components which were supplied by two different suppliers with random yield under stochastic demand. The basic outcome of this stream of research is that in case of uncertain supply, supplier diversification is a desired strategy.

Also, Kouvelis and Li [20] analyzed the replenishment process in the scenario where the demand environment was stable but the supply lead time was uncertain. Their findings suggested that the existence of a second supplier as an emergency response back up was potentially beneficial.

The uncertainty dimension of the supplier side has been often quantified as random yield problem. From the historical data of the supplier, the proportion of times when the supplier had been able to fulfill the demand of the buyers can be calculated. This proportion is termed as the supplier yield. In random yield problem, the yield of the suppliers is assumed to be random. Yano and Lee [30] provided an extensive review of the existing literature in this domain. In the literature of supplier uncertainty, the issue of random yield has been examined in the context of both single and multiple supply sources.

The random yield problem in the context of single supplier was studied by Henig and Gerchak [19]. They used a general cost structure to prove the existence of an order point independent of the random yield value for a single period without any specification of the yield model. The problem of random supplier yield for single supplier was also addressed by Gallego and Moon [13] in the newsvendor framework. In this article, the objective was to minimize the upper limit of cost with known mean and variance but unknown distribution of demand. As an extension to the base case supplier yield was assumed to follow a binomial distribution, which led to higher purchasing quantity and cost as compared with the base case.

Gerchak and Parlar [15] addressed the dual sourcing problem in the context of deterministic demand and uncertain supply. They assumed the ordering costs for the two suppliers to be the same but different fixed cost of ordering from both the suppliers as compared to that from any one of them. The results indicated that the order quantities from each of the supplier was dependent on the mean and standard deviation of the yield for the respective supplier and the decision of supplier diversification depended on the fixed cost associated with each of the supplier. Later Parlar and Wang [25] extended this work to the context of unequal ordering cost for dual supplier and by using a profit maximization framework they examined a single-period model for supplier diversification for both EOQ (known demand) and newsvendor scenario (random demand). They obtained optimum order quantities by minimizing the cost in the EOQ setting, but used an approximate solution technique to solve the newsvendor scenario.

Anupindi and Akella [1] compared single sourcing strategy with dual sourcing strategy by evaluating for different allocations of the order quantity between the two respective sources when both supply and demand are uncertain. They showed that for the special case where the demand is exponentially distributed and the supplier yield is normally distributed, the order quantities to each of the suppliers can be a function of only the parameters of the yield distributions, regardless of their respective sourcing costs.

The sourcing decision making process in the context of stochastic demand and supply has been studied most recently by Burke, Carrillo and Vakharia [7]. They have maximized the buying firm's profit function to find out the optimum number of suppliers as well as the optimum order quantity to each of the suppliers. They observed that in the unreliable supplier setting, the supplier selection process is driven mainly by the costs associated with the suppliers, where as the quantity allocation is majorly driven by the reliability parameters associated with the respective suppliers. They have also identified different parametric conditions where single sourcing becomes more cost-effective strategy than multiple sourcing.

In the extant literature sourcing decisions in both uncertain demand and supply environment have mainly been investigated in a single period context. The issues of multi-period scenario have been largely omitted. However, the single-period newsvendor framework can act as a building block for modeling the multi-period framework for uncertain demand and supply [9]. One of the most practical features of multi-period inventory models is the amount of inventory held during each period and the cost associated with it. The inventory level at the end of each period plays a vital role in a multi-period inventory model, because it determines the amount of order to be placed at the beginning of next period [11].

Another specific feature of a multi-period inventory model is the presence of safety-stock to reduce the effects of uncertainty in supply chain. For example, safety-stock acts as an important parameter in the classic multi-period models, namely, the (Q,r) model and the (S,s) model. Also, in the extant literature, the issue of uncertainty is dealt commonly by means of addition of some safety stock or safety lead time [24,3,6]. This issue was empirically tested by Whybark and Williams [29] by using a simulation model. Their findings suggested that safety lead time can help the companies to counter supply or demand timing uncertainty, whereas, the quantity uncertainty associated with supply or demand can be countered best by using safety stock. This methodology was efficient for made-to-stock organizations, where the internal procedures are commonly traditional, hence traditional method of safety stock calculation could help the organizations to alleviate the uncertainty due to quantity and timing variations faced by them [4,24,5]. Wacker [28] provided a methodology of calculating dynamic safety stock to counter supply uncertainties for made-to-order organizations. However, the amount of safety stock required to maintain a given service level can affect the order size in any period [3].

In this article, we shall try to find out the optimal supply policy for a company when they have one local and one global supplier under the environment of both supply and demand uncertainty. We shall also investigate different conditions which can influence the optimal sourcing policies for the firm.

3 Mathematical Modeling

3.1 Framework

In this article, we have tried to formulate a framework for the well-rehearsed dilemma of whether to outsource the supply to the local suppliers or to the global suppliers. In our framework, we incorporated two suppliers, one local and one global. The two suppliers are differentiated by their lead times taken for the replenishment of the products, their costs quoted for each unit of supply and their quality of supply. The quality issue is formulated by using 'random yield problem', i.e., the historical distribution of the supply for each of the supplier is assumed to be known by the buyer firm and $g_i(.)$ denotes the ith supplier's continuous probability distribution function (p.d.f.) associated with its proportional random yield r_i. To comply with the reality, the proportional yield for any supplier is assumed to be always lesser than unity.

In this framework, the buying firm has x units of products at the beginning of the period. At that point of time, the buyer places an order of size q_1 to the local supplier and an order of size q_2 to the global supplier. During the formulation of this framework, the following assumptions have been made.

- The ordering window is open for access only at the beginning of each period.
- The local supplier is assumed to be delivering the supplies instantaneously, hence supplier lead time for the local supplier is assumed to be zero.
- Any unmet demand is backlogged, hence there is no chance of lost sales.
- Only single-product scenario has been analyzed here, i.e. no substitutional effect has been considered.
- The demand distributions for different periods are assumed to be iids.
- No quantity discount or any other type of contract has taken place between the parties involved.

For our calculation purpose, we have assumed the yield of the suppliers to be normally distributed with means $\bar{r_i}$ and standard deviation σ_i. Also, we have divided each period into two parts, first part being the lead time portion, which denotes the part of the period when the firm receives the order from the local supplier, but has not yet received the order from the global supplier. Obviously, the length of this part is same as the lead time taken by the global supplier to replenish the order. The second part covers the rest of the period, which we call as the non-lead time portion. For our calculation, we assume the demand for the lead time portion to be uniformly distributed over the interval $[a_1, b_1]$ and that for the non-lead time period to be uniformly distributed over the interval $[a_2, b_2]$. The descriptions of the variables used in this article are shown in Table 1.

So, the expected profit function for the given scenario is,

$$E(\Pi) = \int_0^1 dr_1 g_1(r_1) \int_0^1 dr_2 g_2(r_2) f(q_1, q_2) \tag{1}$$

where,

$$\begin{aligned} f(q_1, q_2) &= -\sum_i c_i r_i q_i + \int_0^{y_1} \Big[r z_1 - h_1(y_1 - \frac{z_1}{2}) + \int_0^{y_2} \big[r z_2 - h_2(y_2 - \frac{z_2}{2}) \big] \phi_2(z_2) dz_2 \\ &+ \int_{y_2}^{\infty} \big[r y_2 + \alpha r (z_2 - y_2) - h_2 \frac{y_2}{2} - p_2(z_2 - y_2) \big] \phi_2(z_2) dz_2 \\ &\quad + \alpha \int_0^{\infty} f(y_2 - z_2) \phi_2(z_2) dz_2 \Big] \phi_1(z_1) dz_1 \\ &+ \int_{y_1}^{\infty} \Big[r y_1 - h_1 \frac{y_1}{2} - p_1(z_1 - y_1) + \int_0^{y_2} \big[r z_2 - h_2(y_2 - \frac{z_2}{2}) \big] \phi_2(z_2) dz_2 \\ &+ \int_{y_2}^{\infty} \big[r y_2 + \alpha r (z_2 - y_2) - h_2 \frac{y_2}{2} - p_2(z_2 - y_2) \big] \phi_2(z_2) dz_2 \\ &\quad + \alpha \int_0^{\infty} f(y_2 - z_2) \phi_2(z_2) dz_2 \Big] \phi_1(z_1) dz_1 \end{aligned} \tag{2}$$

with $y_1 = x + r_1 q_1$ and $y_2 = x + r_1 q_1 - z_1 + r_2 q_2$.

Now,

$$\alpha \int_0^{\infty} f(y_2 - z_2) \phi_2(z_2) dz_2 = \alpha c_1 \big(y_2 - \frac{a_2 + b_2}{2} \big) \tag{3}$$

Table 1: Descriptions of variables used in the article.

Variable	Description
q_i	Order quantity placed to supplier i
c_i	Per unit cost quoted by supplier i
x	Amount of inventory held by the buyer at the beginning of the period
h_1	Unit inventory holding cost for the lead time period
h_2	Unit inventory holding cost for the non-lead time period
r_i	Proportion yield calculated from the historical data of supplier i
$g_i(.)$	P.D.F. associated with the yield for supplier i
$\bar{r}_i$	Mean yield for supplier i
σ_i	Standard deviation in yield for supplier i
z	Random variable denoting demand
$\phi(.)$	P.D.F. associated with demand distribution
r	Selling price for the buyer firm per unit
p_1	Unit underage cost faced by the buyer firm for each unit of lost sales during the lead time period
p_2	Unit underage cost faced by the buyer firm for each unit of lost sales during the non-lead time period
a_1	Maximum demand during lead time (uniform demand)
b_1	Minimum demand during lead time (uniform demand)
a_2	Maximum demand during non-lead time (uniform demand)
b_2	Minimum demand during non-lead time (uniform demand)
Π	Profit function for the buyer firm

So, the expression for $f(q_1, q_2)$ becomes,

$$
\begin{aligned}
f(q_1,q_2) &= -\sum_i c_i r_i q_i + \int_0^{y_1} \Big[r z_1 - h_1(y_1 - \frac{z_1}{2}) \Big] \phi_1(z_1) dz_1 \\
&+ \int_{y_1}^{\infty} \Big[r y_1 - h_1 \frac{y_1}{2} - p_1(z_1 - y_1) \Big] \phi_1(z_1) dz_1 + \int_0^{\infty} \Big[\int_0^{y_2} \big[r z_2 - h_2(y_2 - \frac{z_2}{2}) \big] \phi_2(z_2) dz_2 \\
&+ \int_{y_2}^{\infty} \big[r y_2 + \alpha r(z_2 - y_2) - h_2 \frac{y_2}{2} - p_2(z_2 - y_2) \big] \phi_2(z_2) dz_2 + \alpha c_1 \big(y_2 - \frac{a_2 + b_2}{2} \big) \Big] \phi_1(z_1) dz_1
\end{aligned}
\tag{4}
$$

We assume the demand during the lead-time period to be uniformly distributed with range $[a_1, b_1]$ as well as during the non-lead time period with range $[a_2, b_2]$.

Now, assuming the inventory holding cost to be significantly lower than the penalty cost for any unmet demand, we can find out three sets of solutions, namely,

- Ideal Solution: $a_1 \leq y_1 \leq b_1$ and $a_2 \leq y_2 \leq b_2$,
- Trial Solution 1: $a_2 \leq y_2 \leq b_2$ but $y_1 > b_1$,
- Trial Solution 2: $a_1 \leq y_1 \leq b_1$ but $y_2 > b_2$.

We term the first solution as ideal solution as this solution denotes the ideal condition where the local (high cost) supplier will supply for the lead time period demand, whereas, the global (low cost) supplier will supply for the demand of rest of the period. However, if the cost differential between the suppliers (as well as other factors, like the stock-out cost of the product, or the reliability of the suppliers) becomes such that it becomes beneficial for the retailer to place more order to the local supplier, we have to consider trial solution 1. On the other hand, for the condition where the scenario becomes just reversed, we have to follow the trial solution 2.

3.2 Ideal Solution: $a_1 \leq y_1 \leq b_1$ and $a_2 \leq y_2 \leq b_2$

Let us assume that the ideal solution for this problem will be (q_{11}^*, q_{12}^*). Then it can be shown that the expected value of the profit function under the given condition is,

$$
E(\Pi_1) = A_1(\bar{r_1}^2 + \sigma_1^2) q_{11}^2 + B_1(\bar{r_2}^2 + \sigma_2^2) q_{12}^2 + C_1 \bar{r_1} q_{11} + D_1 \bar{r_2} q_{12} + E_1 \bar{r_1} \bar{r_2} q_{11} q_{12} + F_1 \tag{5}
$$

where,

$$
\begin{aligned}
A_1 &= \frac{-r - \frac{h_1}{2} - p_1}{2(b_1 - a_1)} + \xi \\
B_1 &= \xi \\
C_1 &= \frac{-rx - \frac{h_1 x}{2} + h_1 a_1 - p_1 x + r b_1 + p_1 b_1 - h_1 b_1}{b_1 - a_1} \\
&\quad -2\xi \frac{b_1 + a_1}{2} + 2\xi x + \omega - c_1 \\
D_1 &= -2\xi \frac{b_1 + a_1}{2} + 2\xi x + \omega - c_2 \\
E_1 &= 2\xi \\
F_1 &= \frac{1}{2(b_1 - a_1)}\big(- rx^2 - \frac{h_1 x^2}{2} - r a_1^2 - \frac{h_1 a_1^2}{2} - p_1 b_1^2 - p_1 x^2 \\
&\quad +2 r b_1 x + 2 p_1 b_1 x + 2 h_1 a_1 x - h_1 b_1 x\big) + \xi \frac{b_1^2 + b_1 a_1 + a_1^2}{3} \\
&\quad -(2\xi x + \omega)\frac{b_1 + a_1}{2} + \xi x^2 + \omega x + \epsilon
\end{aligned}
$$

$$
\begin{aligned}
\xi &= -\frac{r(1-\alpha) + \frac{h_2}{2} + p_2}{2(b_2 - a_2)} \\
\omega &= \frac{a_2[r(1-\alpha) + \frac{h_2}{2} + p_2]}{b_2 - a_2} + r(1-\alpha) - \frac{h_2}{2} + p_2 + \alpha c_1 \\
\epsilon &= \frac{(\alpha r - p_2) b_2^2 - (r + \frac{h_2}{2}) a_2^2}{2(b_2 - a_2)} - \alpha c_1 \frac{b_2 + a_2}{2}
\end{aligned}
$$

Now, to find the optimum order quantities, we need to differentiate this expression with q_{11} and q_{12} and equate them to zero:

$$2A_1(\bar{r_1}^2 + \sigma_1^2) q_{11} + E_1 \bar{r_1}\bar{r_2} q_{12} + C_1 \bar{r_1} = 0$$

$$E_1 \bar{r_1}\bar{r_2} q_{11} + 2B_1(\bar{r_2}^2 + \sigma_2^2) q_{12} + D_1 \bar{r_2} = 0$$

Note that, the second order differential of the expected profit function are $2A_1(\bar{r_1}^2 + \sigma_1^2)$ $2B_1(\bar{r_2}^2 + \sigma_2^2)$ respectively and they are negative in value. Hence we can conclude that the profit function is concave in nature with respect to the order quantities.

Now, solving for the order quantities, we can obtain,

$$q_{11}^* = \frac{-2C_1 B_1 \bar{r_1}(\bar{r_2}^2 + \sigma_2^2) + D_1 E_1 \bar{r_1}\bar{r_2}^2}{4A_1 B_1(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_1^2 \bar{r_1}^2 \bar{r_2}^2} \tag{6}$$

$$q_{12}^* = \frac{-2A_1 D_1 \bar{r_2}(\bar{r_1}^2 + \sigma_1^2) + E_1 C_1 \bar{r_1}^2 \bar{r_2}}{4A_1 B_1(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_1^2 \bar{r_1}^2 \bar{r_2}^2} \tag{7}$$

And these are the optimum order quantities to be placed to each of the suppliers under ideal conditions.

3.3 Trial Solution 1: $a_2 \leq y_2 \leq b_2$ but $y_1 > b_1$

In the previous section, we have calculated the optimum order quantities for each of the supplier under ideal conditions where $a_1 \leq y_1 \leq b_1$ and $a_2 \leq y_2 \leq b_2$. However, we can face scenarios where different parameters of the problem become be such that the optimum solution does not lie between the boundaries defined by our assumptions. So, in this section, we shall try to find out a solution for conditions where $a_2 \leq y_2 \leq b_2$ but $y_1 > b_1$. We assume that this solution will be denoted by q^*_{21}, q^*_{22}.

Now, using similar arguments like the previous section, we can express the expected profit function $E(\Pi_2)$ as,

$$E(\Pi_2) = A_2(\bar{r_1}^2 + \sigma_1^2)q_{21}^2 + B_2(\bar{r_2}^2 + \sigma_2^2)q_{22}^2 + C_2\bar{r_1}q_{21} + D_2\bar{r_2}q_{22} + E_2\bar{r_1}\bar{r_2}q_{21}q_{22} + F_2 \qquad (8)$$

where,

$$\begin{aligned}
A_2 &= \xi \\
B_2 &= \xi \\
C_2 &= -h_1 - 2\xi\frac{b_1 + a_1}{2} + 2\xi x + \omega - c_1 \\
D_2 &= -2\xi\frac{b_1 + a_1}{2} + 2\xi x + \omega - c_2 \\
E_2 &= 2\xi \\
F_2 &= \left(r + \frac{h_1}{2}\right)\frac{b_1 + a_1}{2} - h_1 x + \xi\frac{b_1^2 + b_1 a_1 + a_1^2}{3} \\
&\quad -(2\xi x + \omega)\frac{b_1 + a_1}{2} + \xi x^2 + \omega x + \epsilon
\end{aligned}$$

Also, by differentiating this expression with respect to q_{21} and q_{22} and equating them to zero, we can find the optimum order quantities for each of the suppliers.

Hence,

$$q_{21}^* = \frac{-2C_2B_2\bar{r_1}(\bar{r_2}^2+\sigma_2^2)+D_2E_2\bar{r_1}\bar{r_2}^2}{4A_2B_2(\bar{r_1}^2+\sigma_1^2)(\bar{r_2}^2+\sigma_2^2)-E_2^2\bar{r_1}^2\bar{r_2}^2} \tag{9}$$

$$\text{and } q_{22}^* = \frac{-2A_2D_2\bar{r_2}(\bar{r_1}^2+\sigma_1^2)+E_2C_2\bar{r_1}^2\bar{r_2}}{4A_2B_2(\bar{r_1}^2+\sigma_1^2)(\bar{r_2}^2+\sigma_2^2)-E_2^2\bar{r_1}^2\bar{r_2}^2} \tag{10}$$

Here also, we can observe that the second order differential of the expected profit function are $2A_2(\bar{r_1}^2+\sigma_1^2)$ $2B_2(\bar{r_2}^2+\sigma_2^2)$ respectively and again they are negative in value. Therefore, here also we have the profit function that is concave in nature with respect to the order quantities.

3.4 Trial Solution 2: $a_1 \leq y_1 \leq b_1$ but $y_2 > b_2$

In this section, we shall find the solution of the given problem at the opposite end of the continuum where y_1 lies within the demand range during the lead time period $[a_1, b_1]$ but y_2 is greater than the maximum demand during the non-lead time period b_2. Therefore, obviously, there will not be any case of shortage/back-order during the non-lead time period. We denote the optimum solution under such condition by (q_{31}, q_{32}).

Following similar steps like the previous sections, the expression for the expected profit function under these conditions would take the form:

$$E(\Pi_3) = A_3(\bar{r_1}^2+\sigma_1^2)q_{31}^2 + C_3\bar{r_1}q_{31} + D_3\bar{r_2}q_{32} + F_3 \tag{11}$$

where,

$$\begin{aligned}
A_3 &= \frac{-r-\frac{h_1}{2}-p_1}{2(b_1-a_1)} \\
C_3 &= \frac{-rx-\frac{h_1x}{2}+h_1a_1-p_1x+rb_1+p_1b_1-h_1b_1}{b_1-a_1} \\
&\quad -(h_2-\alpha c_1)-c_1 \\
D_3 &= -(h_2-\alpha c_1)-c_2 \\
F_3 &= \frac{1}{2(b_1-a_1)}\Big(-rx^2-\frac{h_1x^2}{2}-ra_1^2-\frac{h_1a_1^2}{2}-p_1b_1^2-p_1x^2 \\
&\quad +2rb_1x+2p_1b_1x+2h_1a_1x-h_1b_1x\Big)+\Big(r+\frac{h_2}{2}-\alpha c_1\Big)\frac{b_2+a_2}{2} \\
&\quad +(h_2-\alpha c_1)\frac{b_1+a_1}{2}-(h_2-\alpha c_1)x
\end{aligned}$$

Differentiating the expected profit function equation with respect to q_{31} and equating it to zero, we can find out the optimum order quantity to be placed to the first supplier:

$$q_{31}^* = \frac{-C_3 \bar{r_1}}{2A_3(\bar{r_1}^2 + \sigma_1^2)} \tag{12}$$

Now, we can observe that the expected profit function $E(\Pi_3)$ is no longer concave with respect to the order quantity placed to the second supplier. In fact, the expected profit function is actually a linear function of the order quantity q_{32}. The reason behind this is that, in this profit function, the value of the order quantity to the second supplier is no longer bounded by demand. Hence, if the order quantity increases, the inventory holding cost increases and the penalty cost and the ordering cost for the next period decreases. Since, all of these relationships are linear in nature, the relationship between q_{32} and the profit function is also linear in nature. This error is occurring due to the myopic nature of the profit function. The logic is that, although the value of q_{32} for this period is no longer bounded by the demand for this period, it should be bounded by the demand for the next period. To deal with this problem, we translate the cycle in the forward direction by the lead time amount. That means, at zero time the company will receive q_{32} amount of order from the long distance supplier. After the non-lead time amount is exhausted, the company receives q'_{31} amount of orders from the local supplier. We retain the constraint that $a_1 \leq y_1 \leq b_1$ but $y_2 > b_2$. We refer to this solution as a long term solution under this conditions.

Let us also note that, the inventory in hand at the beginning of the period (zero time) will no longer be the same as before (which is, x). Since, the firm has a uniform demand $[a_1, b_1]$ during the lead time period and has already received an order q_{31} from the local supplier, the inventory in hand at zero time will be the expected value of inventory in hand at the end of the lead time period using these variables:

$$x' = x + \bar{r_1} q_{31} - \frac{b_1 + a_1}{2} \tag{13}$$

In the similar way as we have done in the previous sections, the expected profit function under these new conditions can be expressed as,

$$E(\Pi_3') = A_3'(\bar{r_1}^2 + \sigma_1^2)q_{31}'^2 + B_3'(\bar{r_2}^2 + \sigma_2^2)q_{32}'^2 + C_3'\bar{r_1}q_{31}' + D_3'\bar{r_2}q_{32}' + E_3'\bar{r_1}\bar{r_2}q_{31}'q_{32}' + F_3' \quad (14)$$

where,

$$\begin{aligned}
A_3' &= \xi' \\
B_3' &= \xi' \\
C_3' &= -2\xi'\frac{b_2 + a_2}{2} + 2\xi' x' + \omega' - c_1 \\
D_3' &= -h_2 - 2\xi'\frac{b_2 + a_2}{2} + 2\xi' x' + \omega' - c_2 \\
E_3' &= 2\xi' \\
F_3' &= (r + \frac{h_2}{2})\frac{b_2 + a_2}{2} - h_2 x' + \xi'\frac{b_2^2 + b_2 a_2 + a_2^2}{3} \\
&\quad -(2\xi' x' + \omega')\frac{b_2 + a_2}{2} + \xi' x'^2 + \omega' x' + \epsilon' \\
\xi' &= -\frac{r(1-\alpha) + \frac{h_1}{2} + p_1}{2(b_1 - a_1)} \\
\omega' &= \frac{a_1[r(1-\alpha) + \frac{h_1}{2} + p_1]}{b_1 - a_1} + r(1-\alpha) - \frac{h_1}{2} + p_1 + \alpha c_2 \\
\epsilon' &= \frac{(\alpha r - p_1)b_1^2 - (r + \frac{h_1}{2})a_1^2}{2(b_1 - a_1)} - \alpha c_2\frac{b_1 + a_1}{2}
\end{aligned}$$

Using Lagrangian optimization technique as before, the optimum order quantities for each of the suppliers can be expressed by the following method:

$$q_{31}'^* = \frac{-2C_3'B_3'\bar{r_1}(\bar{r_2}^2 + \sigma_2^2) + D_3'E_3'\bar{r_1}\bar{r_2}^2}{4A_3'B_3'(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_3'^2\bar{r_1}^2\bar{r_2}^2} \quad (15)$$

$$\text{and } q_{32}'^* = \frac{-2A_3'D_3'\bar{r_2}(\bar{r_1}^2 + \sigma_1^2) + E_3'C_3'\bar{r_1}^2\bar{r_2}}{4A_3'B_3'(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_3'^2\bar{r_1}^2\bar{r_2}^2} \quad (16)$$

Again, it can be observed that the second order differential of the expected profit function, $2A_3'(\bar{r_1}^2 + \sigma_1^2)$ $2B_3'(\bar{r_2}^2 + \sigma_2^2)$ respectively, are negative in value. Hence, the profit function is concave in nature with respect to the order quantities.

Also, by putting $q_{32} = q_{32}'^*$ in equation (11), we can obtain the expected value of the profit function Π_3. Thus, we obtain three sets of solution and three expected values of the profit functions. Now, by comparing these expected values, we can conclude which solution will be the optimum one for the firm. To elaborate, if $E(\Pi_2) > E(\Pi_1)$, then the firm should consider the second solution (Trial Solution 1) as its optimum solution. Similarly, if $E(\Pi_3) > E(\Pi_1)$, then the firm should consider the third solution (Trial Solution 2) as its optimum solution. And if, $E(\Pi_1)$ is greater than both $E(\Pi_2)$ and $E(\Pi_2)$, then the firm should consider the ideal solution as its optimum solution.

3.5 Conditions for Single Supplier Solution

We can observe that, so far in this study, we have only considered dual solutions, i.e. both the suppliers have been considered. But, obviously, there could be scenario in which single supplier solution would be more preferable compared to dual supplier solution. In this section, we shall try to find out the conditions in which single supplier solution would lead to optimization of the profit function rather than the dual supplier solution.

3.5.1 Complete Supply to the Local Supplier

Here, we shall try to find out the condition in which the entire supply will be to the local supplier. Obviously, the solution to be considered here would be the second solution (Trial Solution 1), i.e. $a_2 \leq y_2 \leq b_2$ but $y_1 > b_1$. In this solution, the expression for the supply to the global supplier is,

$$q_{22}^* = \frac{-2A_2D_2\bar{r_2}(\bar{r_1}^2 + \sigma_1^2) + E_2C_2\bar{r_1}^2\bar{r_2}}{4A_2B_2(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_2^2\bar{r_1}^2\bar{r_2}^2}$$

Here, the denominator is,

$$\begin{aligned} & 4A_2B_2(\bar{r_1}^2 + \sigma_1^2)(\bar{r_2}^2 + \sigma_2^2) - E_2^2\bar{r_1}^2\bar{r_2}^2 \\ = \; & 4\xi^2(\bar{r_1}^2\sigma_2^2 + \bar{r_2}^2\sigma_1^2 + \sigma_1^2\sigma_2^2) \end{aligned}$$

Therefore, the denominator is always positive. Now, the value of q_{22} cannot be negative. Hence, if the numerator of the given expression becomes negative, that can serve as the condition in which the order size to the global supplier becomes zero and the entire supply is ordered to the local supplier. Thus, it can be shown that the order to the global supplier becomes zero if,

$$
\begin{aligned}
& -2A_2D_2\bar{r_2}(\bar{r_1}^2+\sigma_1^2)+E_2C_2\bar{r_1}^2\bar{r_2}\leq 0 \\
\Rightarrow \quad & \bar{r_1}^2[-h_1+X_1+\alpha c_1-c_1]\geq[X+\alpha c_1-c_2](\bar{r_1}^2+\sigma_1^2)
\end{aligned}
$$

where, $X_1=\frac{r(1-\alpha)+\frac{h_2}{2}+p_2}{b_2-a_2}\left[\frac{b_1+a_1}{2}-x+a_2\right]+r(1-\alpha)-\frac{h_2}{2}+p_2$. Further simplifying, we can obtain the condition, in terms of the cost quoted by the suppliers, in which the firm will order the entire amount from the local supplier:

$$
c_1\leq\frac{1}{1+V_1^2\alpha}[c_2(1+V_1^2)-V_1^2X_1-h_1] \tag{17}
$$

where, $V_1=\frac{\sigma_1}{\bar{r_1}}$ denotes the co-efficient of variation for the local supplier.

3.5.2 Complete Supply to the Global Supplier

In this section, we shall try to find out the condition in which the global supplier will get the entire order amount rather than the local supplier. It is obvious that, for this investigation, we need to consider the third solution (Trial solution 2), where $a_1\leq y_1\leq b_1$ but $y_2>b_2$, for our inspection. Now, we are talking about a scenario, where the firm has x amount of inventory in hand at the beginning of the period, it places an order size of q_{31} to the local supplier to take care of the demand during the lead-time period (which is uniform over $[a_1,b_1]$). So, although there might be a condition in which q_{31} can be zero, but that solution is not of our interest, because by common sense one can predict that q_{31} would be zero if the inventory in hand x itself can take care of the demand or the inventory holding cost for the lead-time period is unbelievably high compared to the stock-out cost (which is practically improbable). Hence, in this section, we shall investigate the condition in which the global supplier will get the entire order in the long run, i.e. the local supplier would not get any order in the next periods.

So the solution to be considered here is the optimized form of $E(\Pi_3')$. So, from equation (15), we have the expression for the amount of order placed to the local supplier in the long run. Now, by following the same logic as in the previous section, we can obtain the condition in which the entire order will be placed to the global supplier:

$$c_2 \leq \frac{1}{1+V_2^2\alpha}[c_1(1+V_2^2) - V_2^2 X_2 - h_2] \tag{18}$$

where, $X_2 = \frac{r(1-\alpha)+\frac{h_1}{2}+p_1}{b_1-a_1}\left[\frac{b_2+a_2}{2} - x' + a_1\right] + r(1-\alpha) - \frac{h_1}{2} + p_1$ and $V_1 = \frac{\sigma_1}{\bar{r_1}}$ denotes the co-efficient of variation for the local supplier.

4 Conclusion and Scope for Further Research

Thus, in this paper, we have formulated a mathematical model for analyzing the sourcing policy of a firm under both demand and supply uncertainty. We have also incorporated the concept of outsourcing of supply in our framework. We have found out optimal ordering policy for the firm, as well as the conditions in which the firm should stick to a single supplier. However, in our analysis, we have assumed the suppliers to have normally distributed yield, which can be generalized to find a more comprehensive result. Also, we have assumed uniformly distributed demand for the lead time and non-lead time. As a drive for further research opportunity, one can also generalize the demand distribution. Moreover, we have not studied the effect of fixed cost on the sourcing policy. The inclusion of fixed cost in the model offers another opportunity to draw more insights in the given context.

5 References

[1] R. Anupindi, R. Akella, Diversification under supply uncertainty, Management Science 39(1993) 944-963.

[2] Y. Bassok, R. Akella, Ordering and production decisions with supply quality and demand uncertainty, Management Science 37(1991) 1556-1574.

[3] W.C. Benton, Safety Stock and Service Levels in Periodic Review Inventory Systems, The Journal of the Operational Research Society 42(1991) 1087-1095.

[4] W.L. Berry, Lot sizing procedure for requirement planning systems: a framework for analysis, Production and Inventory Management 13(1972) 19-39.

[5] W.L. Berry, D.C. Whybark, Research perspectives for material requirements planning systems, Production and Inventory Management 16(1975) 19-26.

[6] R. Bollapragada, U.S. Rao, J. Zhang, Managing Inventory and Supply Performance in Assembly Systems with Random Supply, Management Science 50(2004) 1729-1743.

[7] G.J. Burke, J. Carrillo, A.J. Vakharia, Sourcing decisions with stochastic supplier reliability and stochastic demand, Production and Operations Management 18(2009) 475-484.

[8] G.J. Burke, J. Carrillo, A.J. Vakharia, Single Vs. Multiple supplier sourcing strategies, European Journal of Operational Research 182(2007) 95112.

[9] M. Dada, N.C. Petruzzi, L.B. Schwarz, A newsvendor model with unreliable suppliers, Working Paper 03-0112 (2003), University of Illinois at Urbana-Champaign.

[10] W.J. Elmaghraby, Supply contract competition and sourcing policies, Manufacturing and Service Operations Management 2(2000) 350-371.

[11] R.M. Feldman, C. Valdez-Flores, Applied Probability and Stochastic Processes, 2nd Ed., Springer, Berlin, 2010.

[12] M. Fisher, What Is the Right Supply Chain for Your Product, Harvard Business Review 75(1997) l05-116.

[13] G. Gallego, I. Moon, The distribution free newsboy problem: Review and extensions, Journal of Operational Research Society 44(1993) 825-835.

[14] R. Ganeshan, Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model, International Journal of Production Economics 59(1999) 341-354.

[15] Y. Gerchak, M. Parlar, Yield randomness/cost tradeoffs and diversification in the EOQ model, Naval Research Logistics 37(1990) 341-354.

[16] H. Gurnani, R. Akella, J. Lehoczky, Supply management in assembly systems with random yield and random demand, IIE Transactions 32(2000) 701-715.

[17] G. Hadley, T.M. Whitin, A Family of Inventory Models, Management Science,

7(1961) 351-371.

[18] M.A. Hariga, A Stochastic Inventory Model with Lead Time and Lot Size Interaction, Production Planning and Control 10(1999) 434-438.

[19] M. Henig, Y. Gerchak, The structure of periodic review policies in the presence of random yield, Operations Research 38(1990) 634-643.

[20] P. Kouvelis, J. Li, Flexible backup supply and the management of lead-time uncertainty, Production and Operations Management 17(2008) 184-200.

[21] H.L. Lee, Aligning supply chain strategies with product uncertainties, California Management Review 44(2002) 105-19.

[22] H.L. Lee, C. Billington, Supply Chain Management: Pitfalls and Opportunities, Sloan Management Review 33(1992) 65-73.

[23] S. Minner, Multiple-supplier inventory models in supply chain management: A review, International Journal of Production Economics 81/82(2003) 265-279.

[24] J. Orlicky, Material Requirement Planning, McGraw-Hill Book Co., New York, 1975.

[25] M. Parlar, D. Wang, Diversification under yield randomness in inventory models, European Journal of Operational Research 66(1993) 52-64.

[26] R. Ramasesh, J.K. Ord, J.C. Hayya, A. Pan, Sole versus dual sourcing in stochastic lead-time (s, Q) inventory models, Management Science 37(1991) 428-443.

[27] J.G.A.J. Van der Vorst, A.J.M. Beulens, Identifying sources of uncertainty to generate supply chain redesign strategies, International Journal of Physical Distribution and Logistics Management 32(2002) 409-430.

[28] J.G. Wacker, A theory of material requirements planning (MRP): an empirical methodology to reduce uncertainty in MRP systems, International Journal of Production Research 23(1985) 807-824.

[29] D.C. Whybark, J.C. Williams, Materials requirement planning under uncertainty, Decision Sciences 7(1976) 595-606.

[30] C.A. Yano, H.L. Lee, Lot sizing with random yields: A review, Operations Research 43(1995) 311-334.