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**Coordinating a two-stage supply chain with stock-and
price-dependent demand**

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Abstract: We consider a two-stage supply chain consisting of a supplier and a retailer with stock-and price-dependent random demand. The PO contract is considered as a benchmark case, and then the RS contract will be researched. The impact of price-sensitivity factors and stock-dependent values on the RS contract and channel performance will be investigated in this paper.

1. Introduction

Nowadays, more and more researchers are focusing on the significance of effective supply chain management, such as Cachon (2003), Raju and Zhang (2005) and Li and Liu (2006) have investigated the supply chain coordination problems between suppliers (manufacturers) and buyers (retailers) in a supply chain, which assumed that the market demand was constant, price sensitive or effort-dependent. However, Levin et al (1972) and Silver and Peterson (1985) have observed that the sales quantity of company is proportional to its displayed product, such as supermarket. Although many researchers have observed inventory-dependent demand models, few of them discussed coordination issues of the supply chain with inventory-level-dependent demand. Zhou et. al (2008) dealt with coordination issues of such a two-stage supply chain, where they considered demand is dependent on

instantaneous stock level. However, this paper copes with coordination problems of such a two-stage supply chain with initial stock-dependent demand.

There are many related literature reviews about that demand is dependent on stock level. The demand can be classified as two distinct streams of research: demand is a function of: (1) the initial stock level and (2) the instantaneous stock level. When demand is assumed as a function of the initial stock level, "initial stock" is replaced by the word "stock", which can be considered as "order quantity" in a single period newsvendor model (Qin et al. 2011). Balakrishnan et al. (2008) generalized the newsvendor model to stock-dependent random demand. They use an inverse fractile function to capture the stimulating impact of stock on demand in a general random demand model. Then, Stavroulaki (2011) extended the model of Balakrishnan et al. (2008), which considered a retailer's inventory policy for two substitutable products, and assumes that inventory-dependent demand, thus the higher inventory level, the more products will be sold. Parthasarathi et al. (2011) captured the stock-dependent phenomenon and investigated the role of quantity discounts and returns policies in the coordination of a supply chain. Ma and Wang (2011) investigated the coordination in a two-stage supply chain with stock-dependent demand, and derived that the buyback contract can not only coordinate the supply chain, but also attain the win-win situation.

There are also some related literature reviews about our paper. Yao et. al (2008A) considered impact of price-sensitivity factors on the returns policy and its performance in coordinating the supply chain. Then Yao et. al (2008B) extended Yao

et. al (2008A) to the case with retail competition, they analyzed the impact of demand variability on optimal retail price, order quantity and profit sharing between the manufacturer and the retailers. Different from Yao et. al (2008A, B), this paper considers that the demand is dependent on both retail price and initial stock level.

In this paper, we will consider a newsvendor problem with a manufacturer and a retailer in a two-stage supply chain. Both the centralized supply chain and decentralized supply chain with price-only contract (PO) and revenue sharing contract (RS) will be investigated, respectively. In the centralized case, we assume that the manufacturer and the retailer are under the control of one leader. In the decentralized case, we consider the price-only contracts (PO) as a benchmark case where the manufacturer sells to the retailer as much as the retailer wants at the posted price, then the retailer resells the products and keeps all the revenue but is solely responsible for salvaging unsold products (Lariviere and Porteus, 2001), and then introduce the revenue sharing contract (RS) into the two-stage supply chain to study the supply chain coordination under such case, in which the manufacturer charges wholesale price from the retailer and the retailer shares with the manufacturer a percentage of its revenue (Cachon, 2003). According to the results of many researchers, PO contracts cannot coordinate the supply chain, thus many researchers try to find other contracts to coordinate the supply chain, such as revenue sharing contracts. The manufacturer uses the revenue sharing contracts to encourage the retailer to order more quantity than without it.

The main purpose of this paper is to study the order quantity decision and retail

price decision in the two-stage supply chain with stock-and price-dependent demand.

We will investigate the performance of two-stage supply chain with PO contracts and RS contracts, respectively. We will also consider how the values of stock dependency affect the coordination in the two-stage supply chain.

The paper is organized as follows. Section 2 gives model description, the centralized supply chain model, PO contracts model and RS contracts model. Section 3 gives some numerical examples to illustrate our results and insights. Section 4 concludes the paper.

2. Model description

We assume that the retailer faces a stock-and price-dependent random demand

$$X(Q) = a - bp + cQ + \varepsilon \quad (1)$$

Where $a > 0$, $b > 0$, $0 < c < 1$ and ε is a random variable which is support on the range $[0, \infty)$. Define $F(\cdot)$ and $f(\cdot)$ as its cumulative distribution function (CDF) and probability density function (PDF), respectively. A style or seasonal product with a short life cycle is assumed. The manufacturer acts a Stackelberg leader, offers the contract terms as a take-it-or-leave-it to the retailer. The retailer will accept this contract if its profit is non-negative profit. We also assume the retail price p is given, v is the salvage value, and m is the manufacturer's production cost. Both the retail price and the demand distribution are known to the manufacturer and the retailer. The manufacturer decides the wholesale price w and the retailer decides the ordering quantity Q , respectively. The retailer has only one opportunity of replenishment in a selling period.

2.1. Centralized supply chain model

We assume the centralized supply chain model as the benchmark case, and then the total profit of whole supply chain is as follows:

$$\pi_c = \begin{cases} pX - mQ + v(Q - X) & X \leq Q \\ (p - m)Q & X > Q \end{cases} \quad (2)$$

Define $z = Q - (a - bp + cQ)$, and then we can derive the expected profit of whole supply chain as

$$\begin{aligned} \Pi_c(z, p) &= pE \min(X, Q) - mQ + vE[(Q - X)^+] \\ &= (p - m) \cdot \frac{z + a - bp}{1 - c} - (p - v) \int_0^z F(\varepsilon) d\varepsilon \end{aligned} \quad (3)$$

The optimal decision is derived by maximizing expected profit (3) with respect to z and p :

$$\underset{z, p}{\text{Maximize}} \Pi_c(z, p) \quad (4)$$

After taking the first and second partial derives of (3) with respect to z and p , we can derive that

$$\frac{\partial \Pi_c(z, p)}{\partial z} = \frac{p - m}{1 - c} - (p - v)F(z) \quad (5)$$

$$\frac{\partial^2 \Pi_c(z, p)}{\partial z^2} = -(p - v)f(z) \quad (6)$$

$$\frac{\partial \Pi_c(z, p)}{\partial p} = \frac{z + a - bp}{1 - c} - \frac{b(p - m)}{1 - c} - \int_0^z F(\varepsilon) d\varepsilon \quad (7)$$

$$\frac{\partial^2 \Pi_c(z, p)}{\partial p^2} = -\frac{2b}{1 - c} < 0 \quad (8)$$

From (7), we can derive the optimal solution is given by

$$p^* = p(z) = \frac{z + a + bm - (1 - c) \int_0^z F(\varepsilon) d\varepsilon}{2b} \quad (9)$$

After substituting (9) into (4), the optimal problem becomes

$$\underset{z}{\text{Maximize}} \Pi_c(z, p(z)) \quad (10)$$

2.2. Price-only contracts model

2.2.1. The retailer's problem

Under the wholesale price only contract, the manufacturer sells to the retailer at a wholesale price w_p per unit purchased, and then the retailer resells the items at the retail price p to its customers and takes all risk of keeping any unsold items. Then, we can derive the retailer's profit as follows

$$\pi_R = \begin{cases} pX - w_p Q + v(Q - X) & X \leq Q \\ (p - w_p)Q & X > Q \end{cases} \quad (11)$$

Define $z = Q - (a - bp + cQ)$, and then we can drive that the retailer's expected as

$$\begin{aligned} \Pi_R(z, p) &= pE \min(X, Q) - w_p Q + vE[(Q - X)^+] \\ &= (p - w_p) \cdot \frac{z + a - bp}{1 - c} - (p - v) \int_0^z F(\varepsilon) d\varepsilon \end{aligned} \quad (12)$$

Thus, the retailer's problem becomes

$$\underset{z, p}{\text{Maximize}} \Pi_R(z, p) \quad (13)$$

In order to solve this problem, after taking the first and second partial derives of (12) with respect to z and p , we can derive that

$$\frac{\partial \Pi_R(z, p)}{\partial z} = \frac{p - w_p}{1 - c} - (p - v)F(z) \quad (14)$$

$$\frac{\partial^2 \Pi_R(z, p)}{\partial z^2} = -(p - v)f(z) \quad (15)$$

$$\frac{\partial \Pi_R(z, p)}{\partial p} = \frac{z + a - bp}{1 - c} - \frac{b(p - w_p)}{1 - c} - \int_0^z F(\varepsilon) d\varepsilon \quad (16)$$

$$\frac{\partial^2 \Pi_R(z, p)}{\partial p^2} = -\frac{2b}{1 - c} < 0 \quad (17)$$

From (17), we know $\Pi_R(z, p)$ is concave in p for a given z . From (16), we can derive that

$$p^* = p(z) = \frac{z + a + bw_p - (1-c) \int_0^z F(\varepsilon) d\varepsilon}{2b} \quad (18)$$

After substituting (18) into (13), the retailer's problem become as follows:

$$\underset{z}{\text{Maximize}} \Pi_R(z, p) \quad (19)$$

2.2.2. The manufacturer's problem

The manufacturer's problem is to maximize its expected profit by setting the wholesale price w . The manufacturer's expected profit is as follows:

$$\Pi_M(w_p) = (w_p - m)Q = (w_p - m) \cdot \frac{z + a - bp}{1-c} \quad (20)$$

2.3. Revenue sharing contracts model

With the revenue sharing contracts, the manufacturer charges wholesale price w_r per unit purchased and the retailer shares the manufacturer a percentage of its revenue. The retailer keeps ϕ times channel revenue while the manufacturer keeps $1 - \phi$ times of the channel revenue. This contract form is an extension of the revenue sharing contract defined by Cachon and Lariviere (2005). We also assume the manufacturer is the Stackelberg leader, and the retailer is the follower.

2.3.1. The retailer's problem

In this case, we can drive that the retailer's expected profit as follow

$$\begin{aligned} \Pi_R(z, p) &= \phi p E \min(X, Q) - w_r Q + v E[(Q - X)^+] \\ &= (\phi p - w_r) \cdot \frac{z + a - bp}{1-c} - (\phi p - v) \int_0^z F(\varepsilon) d\varepsilon \end{aligned} \quad (21)$$

The retailer's optimal policies are similar to that of the price-only contracts case, except for some different parameters.

2.3.2. The manufacturer's problem

The manufacturer's problem is to maximize its expected profit by setting the wholesale price w_r . The manufacturer's expected profit is as follows:

$$\begin{aligned}\Pi_M(w_r, \phi) &= (w_r - m)Q + (1 - \phi)pE \min(X, Q) \\ &= (w_r - m) \cdot \frac{z + a - bp}{1 - c} + (1 - \phi)p \left[\frac{z + a - bp}{1 - c} - \int_0^z F(\varepsilon) d\varepsilon \right]\end{aligned}\quad (22)$$

3. Numerical study

In this section, we give a numerical example to illustrate the performance of the supply chain under the decentralized and the centralized supply chain case. The demand faced by the retailer is assumed to be stock-and price-dependent and is modeled as: $X(Q) = 100 - 10p + 0.2Q + \varepsilon$, ε is uniformly distributed in the range $[0, 10]$. The cost parameters are given as follows: $w = 10, m = 6, v = 4$.

Table 1 optimal value

	Decentralized supply chain (PO)	Centralized supply chain
z^*	2.8055	6.608
p^*	9.1066	8.2431
Q^*	14.6744	30.2213

Form **Table 1**, we can find that both the optimal stock level and ordering quantity under the decentralized supply chain are smaller than that of the centralized case while the optimal retail price under the decentralized supply chain is larger than that of centralized case.

We can measure the channel performance by $E_f = \Pi_D / \Pi_T$ or by Π_R / Π_M , where $\Pi_D = \Pi_R + \Pi_M$ is the total profit of the supply chain with wholesale price

only contract (Cachon and Lariviere, 2005).

Table 2 Optimal policies under different contract model settings

	z^*	p^*	Q^*	Π_R^*	Π_M^*	Π_R^* / Π_M^*	w^*
Centralized	6.608	8.2431	30.2213				
PO	2.8055	9.1066	14.6744	14.8483	45.5874	0.3257	7.9642
RS $\phi = 0.9$	5.1305	8.6449	23.3519	28.2163	23.5777	0.8276	6.1939

Form **Table 2**, we can find that the optimal wholesale price under PO is lager that RS contract with $\phi = 0.9$. The channel performance under RS with $\phi = 0.9$ is better that that of PO contracts.

4. Conclusions

This paper studies the order, pricing and stock level decisions of a two-stage supply chain with demand uncertainty and dependent on initial stock level (the retailer's ordering quantity) and the retail price. We discuss the centralized supply chain can the decentralized supply chain, respectively. The PO contracts and the RS contracts model are investigated in the decentralized supply chain. In the end, we give some numerical examples under both the PO contracts and the RS contracts. We derive the following results: The optimal stock level and ordering quantity under the decentralized supply chain are smaller than that of the centralized case while the optimal retail price under the decentralized supply chain is larger than that of centralized case. We can also find that the optimal wholesale price under PO is lager that RS contract with $\phi = 0.9$, and The channel performance under RS with $\phi = 0.9$ is better that that of PO contracts.

For future research, we can extend this case to the supply chain which consists of two competitive retailers.

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