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Managing a Supply Chain with Joint Replenishment, Channel Coordination, Deteriorating items, Exponential Demand[1,2]

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Abstract: This paper addresses a new research problem: managing a two-echelon supply chain with multiple deteriorating products and exponentially decaying demand. This problem extends the research problem addressed by Chen&Chen (2005) in which demand of the goods were assumed to follow a linear function. New exact solutions and their properties studied.

Keywords: Supply chain management, Inventory management, Channel Coordination, Joint replenishment

1. Introduction:

A Supply chain is a complex system consisting of many entities like supplier, manufacturer and retailer to deliver product(s) or service(s) to customers. Managing a supply chain even with one product is complex because it calls for co-ordinated actions in each and every stage of the supply chain which the conventional ways of managing the supply chain lacks (Kohli and Park (1994)).

The scenario is further complicated when one looks at a multi-item supply chain. Various studies have looked at how a co-ordinated supply chain can reap benefits in terms of minimizing the costs for multi-echelons like manufacturer and retailer (Li and Wang (2007).The studies highlight the fact that the savings is higher for a multiple product supply chain because of the resulting cost structures in case of co-ordination mechanisms.

Recently, Chen and Chen (2005) studied the effects of joint replenishment and channel coordination for managing a multiple deteriorating products in a supply chain. The study assumes a linearly proportional demand for the items but with exponential deterioration. The study is supposed to be the first to integrate the joint effects of both replenishment and co-ordination. The results of their experiments also indicate that the savings are higher in the case of joint replenishment and channel co-ordination.

This paper addresses the problem of managing a supply chain with multiple deteriorating items under exponential demand. Inventory with deterioration is a well-addressed concept. In this paper, we assume the inventories are of exponential demand. Inventory management with deteriorating items is well-researched and these type of products are prevalent in agricultural sector, high-tech products and fashion goods industry (Wang (2008)).

The paper is organized as follows. In section 2, we define the research problem. Section 3 outlines the underlying assumptions of the problem and in section 4, we propose mathematical models to address the research problem. Section 5 summarise the findings of the paper and provides scope for future work.

2. The research problem:

A two-echelon supply chain in which the entities are manufacturer and retailer is considered. The manufacturer is the supplier and the retailer is the buyer. The retailer is stocking multiple items to sell in the marketplace. The demand is assumed to be exponential and the goods are assumed to deteriorate exponentially. Apart from this, the assumptions mentioned in Chen and Chen (2005) also hold for this problem. The objective of the problem is to obtain minimum cost structures for the retailer and manufacturer in four scenarios: (1) Individual replenishment (IRP), No co-ordination(NCORD), (2) Joint Replenishment (JRP), NCORD, (3) IRP, CORD, (4) Joint Replenishment (JRP), CORD. These four scenarios exhaust the possibilities of replenishment and coordination. It is also desired to obtain the length of replenishment cycle for each case.

3. Assumptions and Notations:

Before presenting the notations used for the problem we list the main assumptions of the model:

(a) A two-stage supply chain is considered

(b) One manufacturer and one supplier is assumed

(c) Multiple items with exponential deterioration

(d) Demand for the end item is assumed exponential

(e) Both individual, joint replenishment is considered separately

(f) Both no co-ordination, co-ordination is considered separately

(g) There are two setup costs namely Major and minor

(h) Major setup costs assumed to be fixed and relate to costs like transportation cost

(i) Minor setup costs are item dependent and include order-processing cost, material handling cost etc.

T_i –The individual replenishment cycle of item '*I*' for the retailer, i=1,2, ….,n, are decision variables

T- the common replenishment cycle of finished items for the retailer.

$t_{m,i}$– the manufacturer's starting production time for item 'i'

FC – the major setup cost per order for the retailer

a_i- the minor setup cost for adding item 'i' in the replenishment for the retailer

B- the major setup cost per lot for the manufacturer

b_i- the minor setup cost for adding item '*i*' in the production schedule for the manufacturer

$C_{r,i}$ ($C_{m,i}$) –The cost of item 'i' per unit for the retailer (manufacturer, respectively)

p_i –production rate of item '*i*'

Θ_i-the deterioration rate of item 'i' facing both the retailer and the manufacturer

A_i- initial demand

λ_i- is a constant governing the decreasing rate of demand

$f_{r,i}$, ($f_{m,i}$)- the inventory holding cost as a fraction of the inventory cost of item 'I' for the retailer

$I_{r,i}(t)$, ($I_{m,i}(t)$)- the inventory level of item '*i*' at time 't' for the retailer (manufacturer, respectively)

$TC_{r,i}(t)$, ($TC_{r,i}(t)$)- total cost for the retailer (manufacturer respectively) per unit time under policy I,

i.e. I (I,II,III,IV)

TC_i-total cost for the supply chain per unit time under policy i.e. I (I,II,III,IV)

4. The Models:

4.1 Scenario 1 - The Individual Replenishment, No Coordination scenario

This is the classical case in which the retailer is trying to optimize his cost structure for individual items. Also, no coordination happens between the retailer and manufacturer. This scenario is primarily developed for comparing the cost structures in the other scenarios.

Under this scenario, the retailer faces the problem of determining individual replenishment cycle for each item separately. The demand rate is assumed to be:

$$Dt = A_i e^{-\lambda it} \qquad (1)$$

The change in the inventory level can then be represented by the following function:

$$\frac{dIr,i}{dt} + \theta i I_{r,i}(t) = -A_i e^{-\lambda it} \text{ for } 0 \leq t \leq T_i \qquad (2)$$

Solving for $I_{r,i}(t)$ with the boundary condition $I_{r,i}(t)=0$ yields:

$$I_{r,i}(t) = \frac{Ai}{(\theta i - \lambda i)} e^{-\lambda it}\left[e^{(\theta i - \lambda i)(Ti - t)} - 1\right] \text{for } 0 \leq t \leq T_i \tag{3}$$

The inventory holding cost of an item *i* per unit time for the retailer is given by:

$$H_{r,i} = f_{ri} C_{ri} / Ti \int_0^{Ti} Ir, i\ (t) dt \tag{4}$$

Simplifying the above equation we get:

$$H_{r,i} = \frac{fr,iCr,iAie^{-\lambda iTi}}{Ti\theta i(\theta i - \lambda i)} [e^{\theta iTi} - 1 - \frac{\theta i}{\lambda i}\left(e^{\lambda iTi} - 1\right)] \tag{5}$$

The total cost structure in scenario 1 reflects the fact that Major and minor setup costs are incurred for each item separately. The equation is given by:

$$TC_{r,I} = \sum_{i=1}^{n} \{\frac{FCi + ai}{Ti} + \frac{fr,iCr,iAie^{-\lambda iTi}}{Ti\theta i(\theta i - \lambda i)} [e^{\theta iTi} - 1 - \frac{\theta i}{\lambda i}\left(e^{\lambda iTi} - 1\right)]\} \tag{6}$$

We get the optimum replenishment cycle for each item by differentiating the equation above and equating to zero which is:

$$Ti^* = \frac{[\frac{(FCi + ai)\theta i(\theta i - \lambda i)e^{\lambda iTi}}{fr,iCr,iAi} + e^{\theta iTi} - 1 - \frac{\theta i}{\lambda i}\left(e^{-\lambda iTi} - 1\right)]}{\left[\theta i\left(e^{\theta iTi} - e^{\lambda iTi}\right) - \lambda i(e^{\theta iTi} - 1 - \frac{\theta i}{\lambda i}\left(e^{\lambda iTi} - 1\right)\right]} \tag{7}$$

The derivation of the cost model for the manufacturer is similar to the cost model derived above. The inventory level change now includes both production and deterioration instead of demand and deterioration:

$$\frac{\mathrm{dImi(t)}}{\mathrm{dt}} = pi - Imi(t)\theta i \quad \text{for } t_{m,i} \leq t \leq Ti \tag{8}$$

The derivation of starting time of manufacturing is given by:

$$t_{mi} = Ti - Ti\sqrt{\frac{Aie^{-\lambda iTi}}{pi}} \tag{9}$$

With initial condition of $I_{mi}(t) = 0$ when $t = t_{m,i}$

$$I_{mi}(t)=\frac{\mathrm{pi}}{\theta i}\left[1-e^{\theta i(\mathrm{tm,i}-t)}\right] \text{ for } t_{m,i} \leq t \leq Ti \tag{10}$$

Then the total cost structure for the manufacturer is given by:

$$TC_{m,I}=\sum_{i=1}^{n}\left[\frac{B+bi}{Ti}+\frac{fmiCmipi}{Ti\theta i}\left(\frac{e^{-\theta iTi\sqrt{\frac{Aie^{-\lambda iTi}}{pi}}}}{\theta i}-\frac{1}{\theta i}+\frac{T_i}{\theta_i}\sqrt{\frac{Aie^{-\lambda iTi}}{pi}}\right)\right] \tag{11}$$

Equation 6 and 11 together represents the total cost of the system which is:

$TC_I = TC_{r,I} + TC_{m,I}$

4.2 Scenario 2- Joint Replenishment and no co-ordination:

Under this scenario, the number of major setup costs is reduced to one because the items jointly replenished. So the cost structure needs to be optimized only for one cycle. The total cost per unit time for the retailer is:

$$TC_{r,II}=\frac{FC}{T}+\sum_{i=1}^{n}\left\{\frac{ai}{T}+\frac{fr,iCr,iAie^{-\lambda iT}}{T\theta i(\theta i-\lambda i)}\left[e^{\theta iT}-1-\frac{\theta i}{\lambda i}\left(e^{\lambda iT}-1\right)\right]\right\} \tag{12}$$

The common replenishment cycle length can be derived by setting the first derivative to zero:

$$T^*=\frac{FC+\sum_{i=1}^{n}ai}{\sum_{i=1}^{n}\left[\frac{fr,iCr,iAi}{\theta i(\theta i-\lambda i)}\left\{\begin{array}{c}-\lambda ie^{-\lambda iT*}\left(e^{\theta iT*}-1-\frac{\theta i}{\lambda i}(e^{\lambda iT*}-1)\right)+e^{-\lambda iT}\left(\theta ie^{\theta iT*}-\theta ie^{\lambda iT*}\right)\\-\frac{e^{-\lambda iT*}}{T}\left(e^{\theta iT*}-1-\frac{\theta i}{\lambda i}(e^{\lambda iT*}-1)\right)\end{array}\right\}\right]} \tag{13}$$

The total cost per unit time for the manufacturer and for the system are:

$$TC_{m,II}=\frac{B}{T*}+\sum_{i=1}^{n}\frac{fmiCmipi}{T*\theta i}\left(\frac{e^{-\theta iT*\sqrt{\frac{Aie^{-\lambda iT*}}{pi}}}}{\theta i}-\frac{1}{\theta i}+\frac{T}{\theta i}*\sqrt{\frac{Aie^{-\lambda iT*}}{pi}}\right) \tag{14}$$

And

$TC_{II} = TC_{r,II} + TC_{m,II}$

4.3 Scenario 3 – Individual replenishment, Co-ordination:

The scenario 3 and 4 includes the effect of co-ordination. In scenario 3, the manufacturer and the retailer replenishment cycle for each item is jointly determined so that the overall cost is minimized. So under this scenario, total cost is the sum of equations 6 and 11:

$TC_{III} = TC_{r,I} + TC_{m,I}$

$$=\sum_{i=1}^{n}\begin{bmatrix}\frac{FC+B+a_i+b_i}{T_i}+\frac{CR_i}{T_i}e^{-\lambda_i T_i}\left(e^{\theta_i T_i}-1-\frac{\theta_i}{\lambda_i}\left(e^{\lambda_i T_i}-1\right)\right)+\\ \frac{CM_i p_i}{T_i\theta_i}\left(T_i\sqrt{RAP_i e^{-\lambda_i T_i}}+\frac{1}{\theta_i}\left(e^{-\theta_i T_i}e^{\sqrt{RAP_i e^{-\lambda_i T_i}}}-1\right)\right)\end{bmatrix} \quad (15)$$

Where, $CR_i = \frac{f_{ri}C_{ri}A_i}{\theta_i(\theta_i-\lambda_i)}, CM_i = \frac{f_{mi}C_{mi}A_i}{\theta_i}, RAP_i = \frac{A_i}{p_i}$

The optimal replenishment cycle for each item is now found by differentiating the above equation and setting to zero:

$$T_i^{**} = \frac{\begin{bmatrix}FC+B+a_i+b_i+CR_i e^{-\lambda_i T_i}\left(e^{\theta_i T_i}-1-\frac{\theta_i}{\lambda_i}\left(e^{\lambda_i T_i}-1\right)\right)+\\ CM_i\left\{T_i\sqrt{RAP_i e^{-\lambda_i T_i}}+\frac{1}{\theta_i}\left(e^{\theta_i T_i\sqrt{RAP_i e^{-\lambda_i T_i}}}-1\right)\right\}\end{bmatrix}}{\begin{bmatrix}CR_i\left\{-\lambda_i e^{-\lambda_i T_i}\left(e^{\theta_i T_i}-1-\frac{\theta_i}{\lambda_i}\left(e^{\lambda_i T_i}-1\right)\right)+e^{-\lambda T_i}\left(\theta_i e^{\theta_i T_i}-\theta_i e^{\lambda_i T_i}\right)\right\}+\\ CM_i\left\{\sqrt{RAP_i e^{-\lambda_i T_i}}+\frac{T_i}{2}\left(RAP_i e^{-\lambda_i T_i}\right)^{-\frac{1}{2}}+\frac{1}{\theta_i}\begin{pmatrix}e^{-\theta_i T_i}\sqrt{RAP_i e^{-\lambda_i T_i}}e^{\sqrt{RAP_i e^{-\lambda_i T_i}}}-\\ \theta_i e^{-\theta_i T_i}e^{\sqrt{RAP_i e^{-\lambda_i T_i}}}\end{pmatrix}\right\}\end{bmatrix}} \quad (16)$$

4.4 Scenario 4 – Joint Replenishment and Coordination

The scenario 4 considers the ideal situation of both the joint replenishment and co-ordination. In this case, the replenishment cycle length is only one and also includes the effect of co-ordination. The total cost in this scenario per unit time is given by:

$TC_{IV} = TC_{r,II} + TC_{m,II}$

$$= \frac{FC+B}{T} + \sum_{i=1}^{n} \begin{bmatrix} \frac{a_i+b_i}{T} + \frac{CR_i}{T} e^{-\lambda_i T}\left(e^{\theta_i T} - 1 - \frac{\theta_i}{\lambda_i}\left(e^{\lambda_i T} - 1\right)\right) + \\ \frac{CM_i p_i}{T\theta_i}\left(T\sqrt{RAP_i e^{-\lambda_i T}} + \frac{1}{\theta_i}\left(e^{-\theta_i T} e^{\sqrt{RAP_i e^{-\lambda_i T}}} - 1\right)\right) \end{bmatrix} \tag{17}$$

The model is then optimised for common replenishment cycle as below:

$$\boldsymbol{T}^{**} = \frac{\begin{bmatrix} FC+B+\sum_{i=1}^{n}(a_i+b_i)+\sum_{i=1}^{n}(CR_i e^{-\lambda_i T}\left(e^{\theta_i T}-1-\frac{\theta_i}{\lambda_i}(e^{\lambda_i T}-1)\right)+CR_i e^{-\lambda_i T}\left(e^{\theta_i T}-1-\frac{\theta_i}{\lambda_i}(e^{\lambda_i T})\right) \\ +CM_i\left\{T\sqrt{RAP_i e^{-\lambda_i T}}+\frac{1}{\theta_i}\left(e^{\theta_i T_i \sqrt{RAP_i e^{-\lambda_i T}}}-1\right)\right\}) \end{bmatrix}}{\sum_{i=1}^{n}\begin{bmatrix} CR_i\left\{-\lambda_i e^{-\lambda_i T}\left(e^{\theta_i T}-1-\frac{\theta_i}{\lambda_i}(e^{\lambda_i T}-1)\right)+e^{-\lambda_i T}(\theta_i e^{\theta_i T}-\theta_i e^{\lambda_i T})\right\}+ \\ CM_i\left\{\sqrt{RAP_i e^{-\lambda_i T}}+\frac{T}{2}(RAP_i e^{-\lambda_i T})^{-\frac{1}{2}}+\frac{1}{\theta_i}\begin{pmatrix} e^{-\theta_i T}\sqrt{RAP_i e^{-\lambda T}}e^{\sqrt{RAP_i e^{-\lambda_i T}}} - \\ \theta_i e^{-\theta_i T} e^{\sqrt{RAP_i e^{-\lambda_i T}}} \end{pmatrix}\right\} \end{bmatrix}} \tag{18}$$

5. Conclusion and future work:

This paper addressed the problem of managing a supply chain with multiple deteriorating items with exponential demand, joint replenishment and channel co-ordination mechanisms. Although it appears that scenario 4 would yield best results based on just observation, the testing of models for base settings and a sensitivity analysis is under way. This will let us know the actual results and which combination of parameters would yield the maximum savings.

Also, in this paper we have considered only a single manufacturer and a single supplier. Thus the research direction would be to include multiple manufacturers or multiple sources of supply which takes the problem close to reality.

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