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Finding the Best Cost-Efficient Food Assortment for a Non-for-Profit Firm

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Abstract

Consider a non-profit firm that provides food to consumers where operational costs are highly affected by the assortment to offer. The assortment demand is uncertain and influenced by the presence of other items within the assortment. A mathematical formulation was developed to find the most cost-efficient assortment.

1. Introduction

Consider a non-profit firm that provides food to its consumers, e.g. school cafeterias operated by governmental agencies. United States' schools cafeterias aims to provide each student with a food tray composed of a food assortment (or menu) that complies with or exceeds daily nutritional requirements set by the USDA (United States Department of Agriculture)[1]. In this example, as is the case of Puerto Rico's Department of Education, the menu or food

[1] http://www.fns.usda.gov/cnd/lunch/

assortment decision is made considering two factors: students' preference for each food item and the nutritional composition of the daily menu. None of these factors consider direct or indirect costs of providing such food items. Hence, one can argue that the menu offered considering only these two factors will likely require high operational costs compared to other acceptable menus, i.e. menus that also comply with nutritional requirements while keeping a suitable level of consumer demand. This work presents a cost minimization model that aims to find a cost efficient menu that meets nutritional requirements and maintains an acceptable demand level.

Gonzalez-Morales (2011) models the 'menu' (or assortment) as a combination of food items, e.g. lasagna and bread, which is offered by the firm on any particular day. Each food item in this menu will have its own unit purchasing and cooking costs. Furthermore, the combination of items influences demand. The firms observe daily uncertain demand which is naturally influenced by the menu (or assortment) offered that day. In this scenario, there is a particular demand characteristic that poses a challenge and it is that consumer's preferences on a particular item are influenced by the presence of other items offered. For example, spaghetti does not have the same demand influence when paired with chicken than when paired with beef.

In this work we highlight the relevance of the model in Gonzalez-Morales (2011) using a numerical study inspired in the Puerto Rico School Meal Program (PRSMP), which provides food to public and private schools in the island.

We used the data of a single school within the PRSMP with a reduced number of items. To verify the model, we used a subset of nine food items of five different food categories from one hundred eighty-six different food items that has been used by the firm during 2010 and we obtained an optimal assortment. With this simplified scenario, using real and estimated parameters, we have found an optimal assortment composed by an item of each family. Using the same scenario, we structured a sample data for ten food items using it as a base model where we had two items per food category with the same parameters values. Changing the parameter values we were able to describe the structure of the optimal assortment in different instances and illustrate how the firm should take the assortment planning decisions taking into consideration the food items characteristics.

In the next section we provide a brief overview of the related literature. In Section 3, we provide the problem and model backgrounds. The motivational example is described in Section 4. Results of our model are presented in Section 5, using a numerical study. Finally, we conclude in Section 6.

2. Review of Related Literature

This work has relevance in different research areas. This section will be organized in two general subsections, which we labeled Food Management and Assortment and Inventory Decision. Food Management includes work related to the research of the optimum menu planning given different restrictions such as nutritional constraints. Assortment and Inventory Decisions focuses on finding the optimum assortment and inventory level that maximizes profits. Next we highlight the contributions of our work to both of the above mentioned areas.

2.1 Food Management

There are several studies made in the area of food management that uses mathematical modeling for the problem of optimizing a menu given a nutritional objective (e.g. Balintfy, (1975), Balintfy et al. (1978)). More recent literature has introduced the cost of the menu as the primary objective (to be minimized) and the nutritional constraint play then as constraints, (e.g. Sklan & Dariel (1993), Ford, (2006), Ferguson et al. (2006) and Garille & Gass, 2001).

The aforementioned literature deals with menu planning but only considering the unit purchasing cost of the food item on the market. We contribute to this literature by taking into consideration additional costs, such as: unit cooking and inventory holding. Moreover, our work models consumer demand whereas previous work only offered the cost-effective menu planning for one consumer at a time.

2.2 Assortment and Inventory Decisions

As mentioned in Kök et al. (2008), most of the literature in assortment planning focuses on single category or subcategory of product at a given time. In our model, we are focusing the cost minimization model for a single period considering more than one category. These previous works that are focused in single category and differ from ours in several ways, (e.g. van Ryzin & Mahajan (1999), Bish & Maddah (2004), Gaur & Honhon, (2006) and Kök & Fisher, (2007)). In contrast to the mentioned work, we consider multiple product categories and unequal operational costs, studying its effect in the optimal assortment, instead of the price effect. Like them, we are working with stochastic demand and single period plan horizon, but we are adding to our model a demand model that acknowledges the demand influence between items and restrictions in the assortment composition.

There have been some works that considered different operational costs similar to our model, for example Li (2007) and Kök & Xu (2010).

Smith & Agrawal (2000), Honhon et al. (2006) assumed a stochastic demand and unequal operational costs and prices as we do. However, all previously cited literature was focused on single product category, contrary to ours. There are few works that consider multiple categories as we do. Cachon & Kök (2007) took into consideration multiple products categories with unequal costs like ours, but with different focus because we do not consider the competition between categories and they do. Their purpose was to demonstrate that category management (CM) never finds the optimal solution and how it affects prices, which lead to poor decisions contrary to centralized management, which is focused on how the optimal assortment takes all the categories into consideration at the same time. Another work that considers multiple categories is Rodríguez & Aydin (2011) who study the assortment selection and pricing for configurable products under demand uncertainty. They find that the optimal prices are the ones where all variants of a component share the same effective margin. As we do, they consider the demand influence by combinations of items, but contrary as our work their utility contribution of an item is independent of which item is included in the assortment (with which item it is matched). In our work, the items' demand contribution for the assortment is influenced by which is the combination of items. Also, Rodríguez & Aydin (2011) do not have restrictions on assortment composition, as we have.

To the best of our knowledge, there is no literature that focuses on which is the optimal assortment that minimizes costs for a non-profit firm, assuming a given stochastic demand that is influenced by the presence of other items within the assortment. Adding unequal operational costs parameters (like unit holding costs and unit purchasing and cooking costs), funding revenues and products salvage values with a given inventory level, subject to the availability of the entire batch that satisfied a type-1 service level and assortment composition constraints.

3. Problem Description

We consider a single period problem where a firm offers a daily assortment composed of a combination of items. Each item, denoted with *i*, can belongs to one or more food categories *k*, e.g. chicken tenders and rice with beans, where the former belongs to the meats category and the latter belongs to the cereals and grains food categories. Let X_i be our binary decision variable, and define as

$$X_i = \begin{cases} 1, & \textit{if item i should be offered in the assortment} \\ 0, & \textit{otherwise.} \end{cases}$$

Consider a two-echelon supply chain scenario that consists of a distribution center and a firm, which have the objective in fulfilling a consumer request. In both echelons (or parties) inventory is stored. Furthermore, demand is observed at the lower echelon and inventory is received periodically, i.e. inventory is received every fix number of periods (e.g. each month). (For the sake of model tractability, in this work we model only the costs at the lower echelon level of the supply chain, e.g. school cafeteria).

At the lower echelon there are several events that take place. Figure 1 has a flowchart representation of those events.

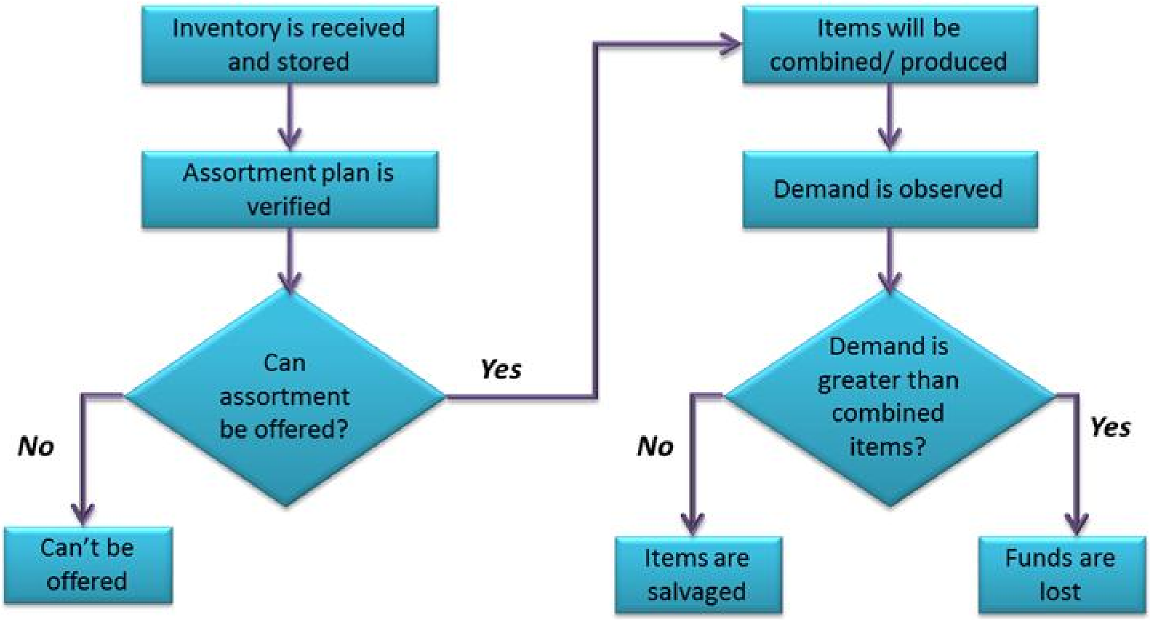

Figure 1. Chronological events at lower echelon level of the supply chain

Observe from Figure 1 that after the food items are received at the lower echelon, the firm stores the item-level inventory until the assortment planning indicates that it needs to be offered. Before items are combined (or produced), the firm has to determine if the inventory meets or exceeds the necessary quantity to satisfy a type-1 service level, which is the probability that the firm complies with the demand of the food item i on any given period. If there are enough raw materials, then the items will be combined as the total cost of all individual units purchasing and cooking costs of the items included in the assortment. Then, demand is observed. At the end of the period the firm receives revenues from external funds per consumer who received certain quantity of food items[2]. Alternatively, if there are leftover items they will be salvaged at a predetermined value.

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[2] An established minimum quantity of items must be selected by the customer in order for the firm to get the external funds. This requirement (constraint) is based on the case were federal agencies reimburse schools only if the students get a minimum quantity set by the agency.

For the sake of exposition, we next provide a brief description of the general cost minimization model presented in Gonzalez-Morales (2011). In the next section we describe how the model was adapted to our motivational example.

Notation:

Subscripts:
i: product
k: food category
S: assortment

Parameters:
β_i: regression's coefficient for variable X_i.
β_{ij}: regression's coefficient for variable X_{ij}.
Y_s: expected demand for assortment S where $Y_s = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} X_i X_j$
γ_i: assortment demand at which the food item i is requested by the consumers with an expected rate of μ_{γ_i} and standard deviation of σ_{γ_i}
D_i: demand of food item i, $D_i = Y_s \gamma_i$
μ_i: expected demand for food item i assuming normal distribution, $\mu_i = Y_s \mu_{\gamma_i}$
σ_i: standard deviation of the food item i assuming normal distribution, $\sigma_i = Y_s \sigma_{\gamma_i}$
α: in-stock rate
Q_i: item i's quantity (measured in number of consumers) to offer, where $Q_i = Z_{Q_i} \sigma_i + \mu_i$
Z_{Q_i}: standard normal random variable
Φ: standard normal cumulative distribution function
$\lambda_{i,k}$: amount per serving per food category k of food item i
U_i: total amount per serving per food item i that must be offered, $U_i = \sum_k \lambda_{i,k} \quad \forall\, i$
c_{p_i}: unit purchasing cost of food item i
c_{c_i}: unit cooking cost of food item i
C_i: unit purchasing and cooking cost for food item i, $C_i = c_{p_i} + c_{c_i}$
I_i: inventory level of food item i
i_R: interest rate per period
h_i: inventory holding costs or opportunity cost of having an unit of food item i in stock, $h_i = i_R * c_{p_i}$
g_i: revenue per quantity of excess of food item i
$E(Q_i - D_i)^+$: expected excess demand for cooked food item i (maximum between zero and $Q_i - D_i$), $E(Q_i - D_i)^+ = \int_{-\infty}^{Q_i} (Q_i - D_i) F(D_i)\, dD_i$.
f: fund received per consumer satisfied
l: number of items to find and choose
$P(l \geq m)$: probability that a consumer find and choose m or more food items
R_k: standard amount per serving per food category k that must be served in each period
$F_{i,k}$: binary parameter that has a value of one if $\lambda_{i,k}$ is positive ($\lambda_{i,k} > 0$) or zero otherwise

The objective function of the model is formulated as

$$\min_{X_i} \sum_i C_i\, U_i Q_i X_i + \sum_i h_i (I_i - Q_i U_i) X_i - \ f Y_s P(l \geq m) - \sum_i g_i U_i E(Q_i - D_i)^+ X_i. \tag{1}$$

The first term in (1) accounts for the purchasing and cooking cost. The second term accounts for the holding cost associated with carrying item i. The third and fourth terms in (1) are sources of revenue for the PRSMP supply chain. The third represents the federal refund obtained if a consumer selects at least m items from the available items in the menu. The fourth and last term accounts for the fact that excess prepared food has a salvage value, mainly because it is sold to farmers.

There are a few constraints that need to be satisfied by the assortment offered. Some of these constraints are due to the requirement set for the firm, e.g. nutritional requirements constraint, and others are for modeling purposes. We next provide the constraints for our model.

- ✓ $\sum_i X_i \lambda_{i,k} \geq \ R_k \quad \forall\, k$ (2)

- ✓ $\sum_i X_i F_{i,k} \geq 1 \quad \forall\, k$ (3)

- ✓ $X_i (I_i - Q_i U_i) \geq 0 \quad \forall\, i$ (4)

✓ $X_i \in \{0,1\}$ (5)

The first set of constraints is established to ensure that nutritional requirements per each food category *k,* established by the USDA agency are met. The second set of constraints guarantees that the quantity of food items offered per each food category k is greater or equal than one. Therefore, this constraint guarantees that the firm will offer at least one item of each food category *k.* The third set of constraints guarantees that if a food item i is offered, the firm has the necessary quantity to satisfy the desired type-1 service level in stock, otherwise this constraint guarantees that the food item *i* will not be offered. Finally, the last constraint is the standard integrality constraint for the decision variable.

4. Motivational Example

To highlight the relevance of our model we next present an example of the application of our model using as scenario the Puerto Rico School Meal Program (PRSMP). In this section we describe the scenario studied.

The PRSMP is operated by Puerto Rico's Department of Education. PRSMP provide food service to public and private schools' cafeterias in Puerto Rico. They offer nutritional lunches and breakfasts services to children between kindergarten to high schools (primarily children between 5 to 18 years old) during the fall, spring and in some schools during summer.

The supply chain of the PRSMP consists of six distribution centers where food and equipment is stored (see Figure 2). This food is purchased by the government and then it is transported to each distribution center by the supplier. Deliveries to schools are made by the PRSMP by trucks once a month in the case of frozen and dry food (no fresh food). Fresh food items, such as milk, are delivered by a third party on a more frequent basis. Each distribution center has to make deliveries to an average of 220 schools that are at different locations in the school district area. Once the food items arrive at each school they are stored and served.

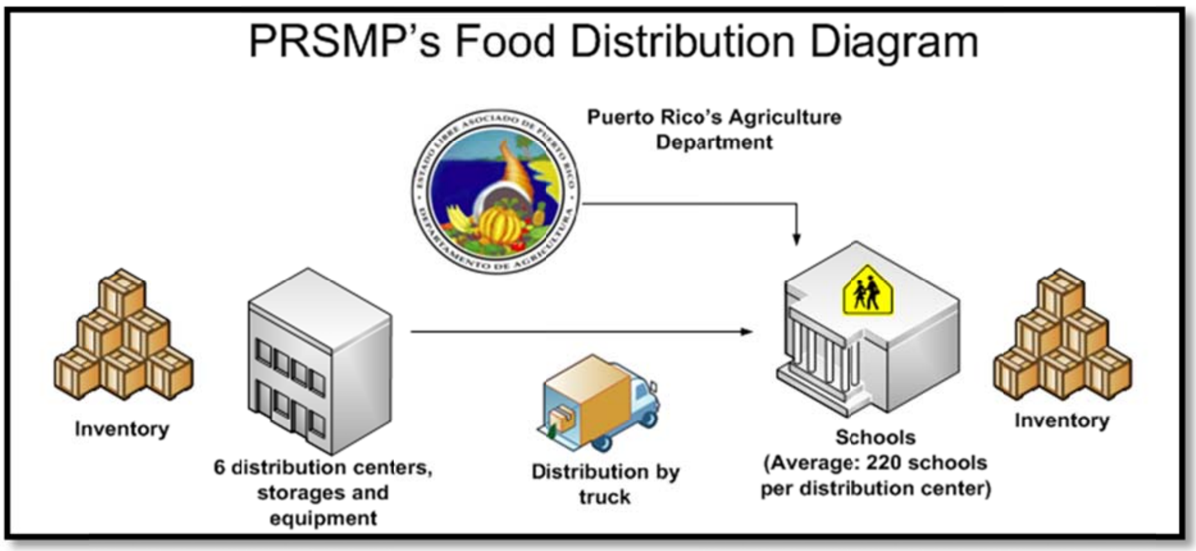

Figure 2. PRSMP Supply Chain

After deliveries are made, the schools have to serve the meals as scheduled by the PRSMP. These meals must offer all the nutrients required to ensure the "good health" on every child. These nutrients are divided into six different food categories, which are: meats, cereals, grains, fruit, vegetables and milk (these last one will not be considered in our study because it must be offered every day). The amounts of nutrients that they must satisfy are more specific compared with the nutritional constraints that we presented in the more general model (model in Section 3). However, the constraints presented in the general model were modified in order to model the PRSMP. The general model's nutritional constraints in (2) apply for the food categories k equal to meats ($k=1$) and cereals ($k=2$). As for the remaining categories on the PRSMP case, the amount per serving is measured together to comply with only one standard nutritional quantity. Then, letting ρ as the standard amount per total servings of the remaining food categories k, where the food categories k are equal to vegetables ($k=3$), grains ($k=4$) and fruits ($k=5$), the next set of constraints was established

$$\sum_{i}\sum_{k=3}^{5} X_i \lambda_{i,k} \geq \rho \,. \tag{6}$$

Furthermore, the PRSMP has a minimum requirement for the meats, cereals and fruit categories ($k=1$, 2 & 5) in which they must offer one of each. As for the vegetable category ($k=3$) they can offer more than one food items i. Contrary to the previous categories, a maximum of one food item i of grains category ($k=4$) can be offered. Therefore, the next three constraints substitute the set of constraints in (3),

$$\sum_{i} X_i F_{i,k} = 1 \quad \forall\, k = 1, 2 \,\&\, 5, \tag{7}$$

$$\sum_{i} X_i F_{i,k} \geq 1 \quad \forall\, k = 3, \tag{8}$$

and

$$\sum_{i} X_i F_{i,k} \leq 1 \quad \forall\, k = 4. \tag{9}$$

The numbers of students that assist daily to these cafeterias is uncertain. Although, the school receives a good estimate by asking each day during the morning how many students wants to participate from the program and with this estimate the school decides the quantities to cook. After the firm offered all the food services, PRSMP receives federal funds for every student that received three or more food items from the assortment offered. Then, for this scenario the amount of students that is expected to receive three or more food items will be given by the product of the expected assortment demand, Y_s, and the probability that the student find and choose three or more items. If we define the probability of find three or more items, $P(l \geq m)$, as the multiplication of the probability of find three or more items by the probability of choose three or more, we have

$$P(l \geq m) = \omega\eta \tag{10}$$

where ω is the probability of find and η is the probability of choose, m or more food items. Since the firm meets a type-1 service level, for this scenario we can model the probability of finding three *(m=3)* or more food items as

$$\omega = \sum_{l=3}^{t} [\alpha^{l}(1-\alpha)^{t-l}], \tag{11}$$

where, α is the in-stock rate established by the firm, l is the number of items to find and t is the total number of food items offered in the assortment $(t = \sum_i X_i)$. The PRSMP has the characteristic that they offer a maximum of six items per tray including milk. Therefore, considering this characteristic the total number of food items offered in the assortment is less or equal than five $(t \leq 5)$ in this example. On the other hand for this scenario, we also can calculate the probability that a consumer choose three or more food items from the assortment, which is given by

$$\eta = 1 - P(l < 3) = 1 - P(l = 1) - P(l = 2). \quad (12)$$

Recall that we defined in Section 3 the expected rate of the expected assortment demand at which the food item i is requested by the consumers, denoted by μ_{γ_i}. Then, with this rate we can calculate, using formulation in (12), the probability that a consumer choose three or more food items from the assortment as

$$\eta = 1 - \sum_i (\mu_{\gamma_i} X_i \prod_{j \neq i} (1 - \mu_{\gamma_j} X_j)) - \sum_i \sum_{j \geq i+1} (\mu_{\gamma_i} \mu_{\gamma_j} X_i X_j \prod_{n \neq i \neq j} (1 - \mu_{\gamma_n} X_n). \quad (13)$$

Then substituting (11) and (13) in (10), the expected funding that the PRSMP will received for their service will be given by,

$$Expected\ funding\ revenue = f Y_s \sum_{l=3}^{t} [\alpha^l (1 - \alpha)^{t-l}] \left(1 - \sum_i (\mu_{\gamma_i} X_i \prod_{j \neq i} (1 - \right. \quad (14)$$

$\mu_{\gamma_j}X_j)) - \sum_i \sum_{j\geq i+1}(\mu_{\gamma_i}\mu_{\gamma_j}X_iX_j \prod_{n\neq i\neq j}\left(1-\mu_{\gamma_n}X_n\right)\Big).$

This scenario is an example that applies to our study because has the characteristics of the firm that is taken into account in our model. This non-profit program offers a nutritional food assortment to an uncertain demand of consumers, which have certain food preferences influenced by the combination of food items offered. Also, PRSMP incurred in all the operational costs taken into account in our model, like purchasing, cooking and inventory holding. The objective of this program is to provide each student with a nutritional food assortment, and as a public firm that is operated by the government, it will be useful the minimization of operational costs. On the other hand, as mentioned before, this firm has some sources of revenues that inspired the formulation of it in our model.

5. Results

5.1 Model Verification

To illustrate the results of the model presented, a numerical study was used inspired in our motivational example. One of the challenges with these types of public firms is the time it takes to get real input data for the model verification. Therefore, it was decided for purpose of verification of this model that we were going to use a combination of real and estimated data of a single school within the PRSMP with a reduced number of items.

The PRSMP has more than one hundred eighty-six different food items that belong to one or more food categories, from the five categories that we are going to consider. To verify our model we used a subset of nine food items, which are presented in the next table with its corresponding food categories (see Table 1).

Table 1. Subset of food items used in the verification of the model

i	**Food item**	**Food categories**
1	Turkey Stew	Meats
2	White Rice	Cereals
3	Pinto Beans	Grains
4	Carrots	Vegetables
5	Peaches	Fruits
6	Rice /w sausage	Cereals & Meats
7	Pink Beans	Grains
8	Green bean salad /w carrots	Vegetables
9	Pears	Fruits

For these nine food items, we have real and estimated data for the next parameters presented in the Table 2. For this one single period scenario, we are assuming that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level. Therefore, we are assuming for this illustration that

$$I_i = Q_i U_i \quad \forall\, i. \tag{15}$$

Table 2. Real and estimated input data

Real Data	***Estimated Data***
F	c_{pi}
$\lambda_{i,k}$	c_{ci}
U_i	C_i
$F_{i,k}$	h_i
R_k	g
P	α
	β_i, $\beta_{i,j}$
	μ_{yi}
	σ_{yi}
	Y_s
	μ_i
	σ_i
	Q_i

	Z_{qi}
	I_i
	ω
	η

The model presented was introduced in optimization software that has the solution technique needed to solve an integer non-linear programming, like our model. The software that we used is the eleventh version of LINGO©, which have the "Global Solver" technique to solve nonlinear models. The "Global Solver" technique employs branch-and-bound methods to break a model down into many convex sub-regions to find a number of locally optimal points and then it reports the global solutions to the non-convex model contrary to other nonlinear solvers that typically will converge to a local or sub-optimal point. After the model was introduced with the input data for the subset of nine food items the software was run and after forty-one seconds we obtained an optimal menu. The model for this subset of food items consists of sixty-two variables (with 40 nonlinear and 9 integers variables), seventy constraints (where 17 constrains are nonlinear) and the software makes 20,418 iterations to find the optimal solution. The optimal assortment for this subset consists of five food items, which are: Turkey Stew, White Rice, Carrots, Pinto Beans and Peaches. Some input and output data and related costs for this menu is presented in the next tables (see Tables 3 and 4).

Table 3. Input Data

Parameters	***Notation***	***Value***
Salvage value	g	$ 0.05
Funds	f	$ 3.50
In stock rate	α	0.90
Nutritional standards	R_1	2.00 oz
	R_2	1.50 oz
	ρ	6.00 oz

Table 4. Output Data

Output	*Notation*	*Value*
Expected menu demand	Y_s	233
Probability of find 3 or more items	ω	0.663
Probability of choose 3 or more items	η	0.975
Total purchasing and cooking costs		\$166.58
Total inventory costs		\$0.00
Total salvage value		\$18.92
Total funds		\$489.76
Objective function value		**\$ -342.11**

To verify that this is the optimal menu, we ran the model with all feasible menus that we can obtained with those nine food items and we can observed that our optimal menu is which obtained the minimal costs, as can be seen in Figure 3. Negative values in cost indicate a profit.

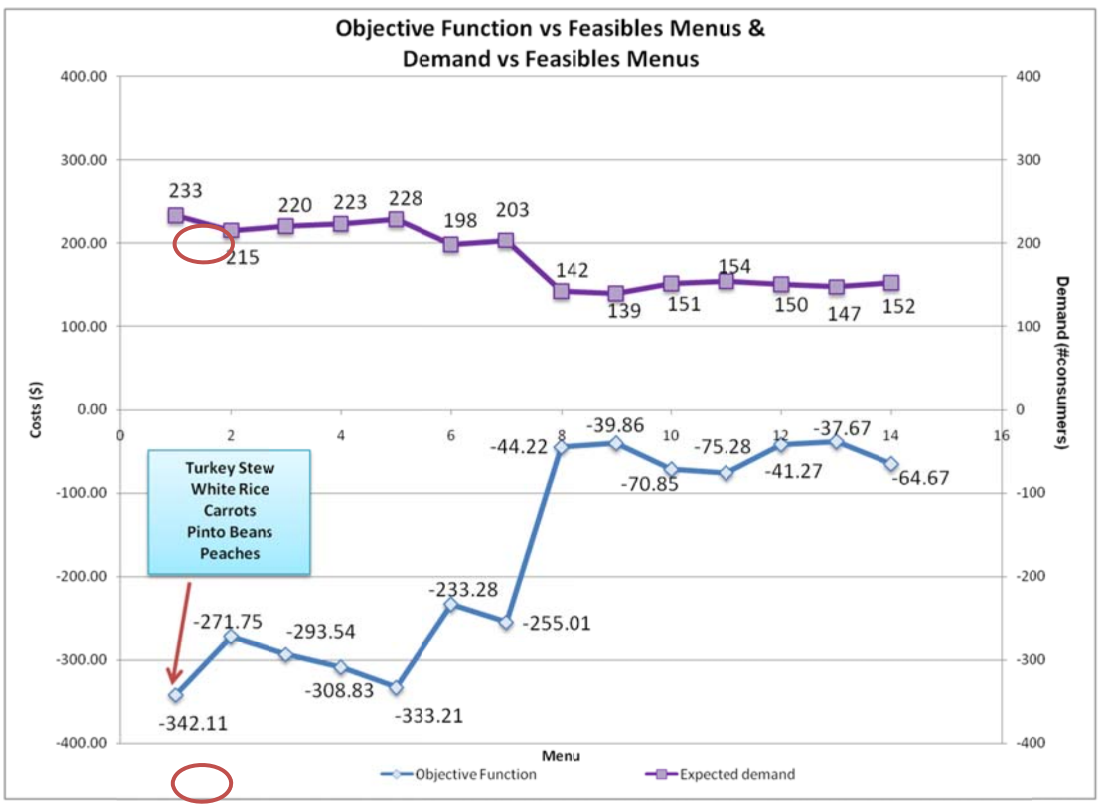

Figure 3. Objective function vs. feasible assortments and demands

Also, it is observed from Figure 3 that the optimal menu has the highest expected demand and its cost (profit) is approximately nine times lower than the menu with the higher costs that has lower expected demand, which strengthen the argument that the current menu planning technique could be carrying unnecessary higher costs.

To identify others characteristics of the model, all feasible menus were sorted from the lower cost to higher cost (see Table 5). We can observe that the menus with the lower costs contain five items, one more that the menus with the highest costs. The items that make these menus different are the Turkey Stew and White Rice (for the menus with lower costs) and Rice with sausage (the ones with higher costs). Although the menus with lower costs have more foods items, the two items that are different in these menus have the total purchasing and cooking cost per serving lower than the cost per serving for the Rice with sausage. Also, the menus that offer five food items have higher demands than the ones that offer four items. In addition, we can observe that the greater the number of foods to offer, lower is the probability of find three or more items but greater is the probability of choose three or more items. On the other hand, we can observe that the two menus that offer the lower costs (menus 1 and 5) differ in the food items that belong to the grains and fruits categories. These food items differ in their purchasing and cooking costs per serving, having Pinto Beans the lower cost comparing with Pink Beans, but Peaches being the one with higher cost comparing with Pears. The menu 1 has the lower total purchasing and cooking costs than menu 5, and the total funding being greater than the menu 5.

Table 5. All feasible menus with their respective objective value (from lower to higher costs)

Menu	Objective Function	Expected demand	Assortment					ω	η
			Meats	Cereals	Grains	Vegetables	Fruits		
1	-342.11	233	Turkey Stew	White Rice	Pinto Beans	Carrots	Peaches	0.663	0.975
5	-333.21	228	Turkey Stew	White Rice	Pink Beans	Carrots	Pears	0.663	0.979
4	-308.83	223	Turkey Stew	White Rice	Pink Beans	Carrots	Peaches	0.663	0.985
3	-293.54	220	Turkey Stew	White Rice	Pinto Beans	Green bean salad /w carrots	Pears	0.663	0.968
2	-271.75	215	Turkey Stew	White Rice	Pinto Beans	Green bean salad /w carrots	Peaches	0.663	0.978
7	-255.01	203	Turkey Stew	White Rice	Pink Beans	Green bean salad /w carrots	Pears	0.663	0.981
6	-233.28	198	Turkey Stew	White Rice	Pink Beans	Green bean salad /w carrots	Peaches	0.663	0.987
11	-75.28	154	Rice /w sausage		Pink Beans	Carrots	Pears	0.729	0.854
10	-70.85	151	Rice /w sausage		Pink Beans	Carrots	Peaches	0.729	0.884
14	-64.67	152	Rice /w sausage		Pinto Beans	Carrots	Peaches	0.729	0.821
8	-44.22	142	Rice /w sausage		Pink Beans	Green bean salad /w carrots	Pears	0.729	0.868
12	-41.27	150	Rice /w sausage		Pinto Beans	Green bean salad /w carrots	Pears	0.729	0.808
9	-39.86	139	Rice /w sausage		Pink Beans	Green bean salad /w carrots	Peaches	0.729	0.896
13	-37.67	147	Rice /w sausage		Pinto Beans	Green bean salad /w carrots	Peaches	0.729	0.838

Note: Recall that ω is the probability of find m=3 or more food items and η is the probability of choose m=3 or more food items.

To understand and evaluate in more detail the previous results we performed a numerical study in order to answer questions about the characteristics of the optimal assortment. The following section outlines which are the characteristics that possess an item to be attractive to belong to an assortment.

5.2 Numerical Study

To perform the numerical analysis we first ran our model for different number of items in order to select the number of food items to be used in the study to obtain an accurate and promptly results. The next graph (see Figure 4 and Table 6) shows how running time increases as the number of items in the model increases.

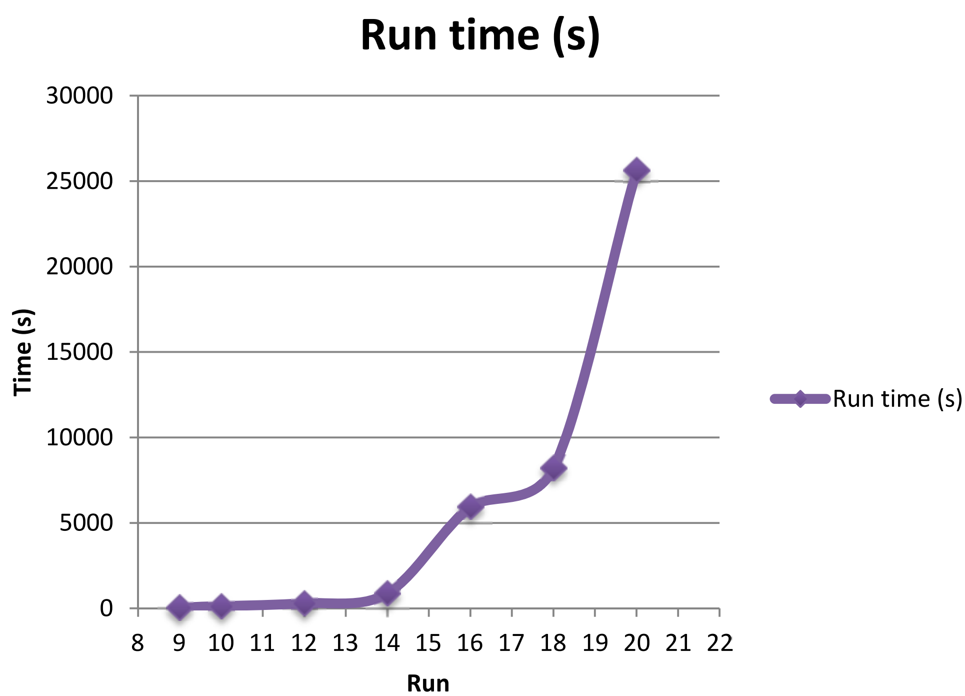

Figure 4. Run time vs. Number of items

Table 6. Run times for different number of items

Number of items	Run time (s)	hh:mm:ss
9	41	00:00:41
10	133	00:02:13
12	300	00:05:00
14	880	00:14:40
16	5947	01:39:07
18	8212	02:16:52
20	25639	07:07:19

Therefore, to obtain a promptly results and to be able to run as many instances we need to do the analysis, we used ten food items, two items per food category k.

For the numerical study, using PRSMP scenario characteristics, we defined all the parameters for the food items that belong to the same food category, $i \in k,$ with an equal value, being this scenario known as the base model scenario (see Table 7). In order to describe the structure of the optimal assortment and illustrate how the firm should take the assortment planning decisions taking into consideration the food items characteristics, we made several runs, using Lingo© software, isolating the effects of one or more parameters to identify when a food item becomes attractive to be carried in the assortment compared to the base items[3]. The next section outlines the results and observations that were obtained in order to explain how the difference in value of one or more parameters can determine which items are attractive to belong to an assortment while the value of all others parameters of the food items of the same category remained equal.

Table 7. Base model parameters values

Item	Food Category	c_{p_i}	c_{c_i}	C_i[4]	μ_{γ_i}	σ_{γ_i}	β_i[5]	U_i
1	Meats	0.38	0.13	0.50	0.95	0.04	9	2.01
2	Cereals	0.11	0.04	0.15	0.93	0.05	19	3.02
3	Grains	0.04	0.01	0.05	0.70	0.20	13	2.16
4	Vegetables	0.08	0.03	0.10	0.85	0.10	10	2.40

[3] The base item is the other item of the same food category that has all parameters values equally set as in the base model.
[4] The units' purchasing and cooking costs (C_i) were obtained using 2003-04 CNPP Food Prices Database (http://www.cnpp.usda.gov/usdafoodplanscostoffood.htm). On the other hand, the unit purchasing costs can be calculated dividing the total cost of the food batch between the total quantities (i.e. ounces, units). In like manner, unit cooking costs can be estimated by calculating labor costs to prepare the items and dividing such cost per unit produced.
[5] In a real scenario, we could calculate β_i values by performing a regression analysis given that sufficient observations are used. For the purposes of this work (and due to insufficient data) β_i were selected so that we could observe similar assortment demand values as those observed at the school selected for the study.

5	Fruits	0.15	0.05	0.20	0.90	0.12	12	3.36
6	Cereals	0.11	0.04	0.15	0.93	0.05	19	3.02
7	Grains	0.04	0.01	0.05	0.70	0.20	13	2.16
8	Vegetables	0.08	0.03	0.10	0.85	0.10	10	2.40
9	Fruits	0.15	0.05	0.20	0.90	0.12	12	3.36
10	Meats	0.38	0.13	0.50	0.95	0.04	9	2.01

5.2.1 Purchasing and cooking costs parameter's characteristic

In preparation for the next results, we considered the PRSMP scenario characteristics with the assumption that that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level, $I_i = Q_i U_i \quad \forall\, i.$ For all observations, the parameters values were changed to higher values and lower values using as reference the base model value to identify if this characteristic has significance when a firm has to select an item to be part of the assortment

As first observation, the model was ran changing the purchasing and cooking costs for one item i of each food category k. The results obtained for a food item i, that has a base value $C_i = 0.50$, are show in the next Figure 5. Figure 5 shows the costs incurred by the firm if item i is offered, total costs incurred by the firm if base item is offered in the assortment (turning point) and the optimal region (minimum total costs in each run). The interpretation is defined in Observation 1.

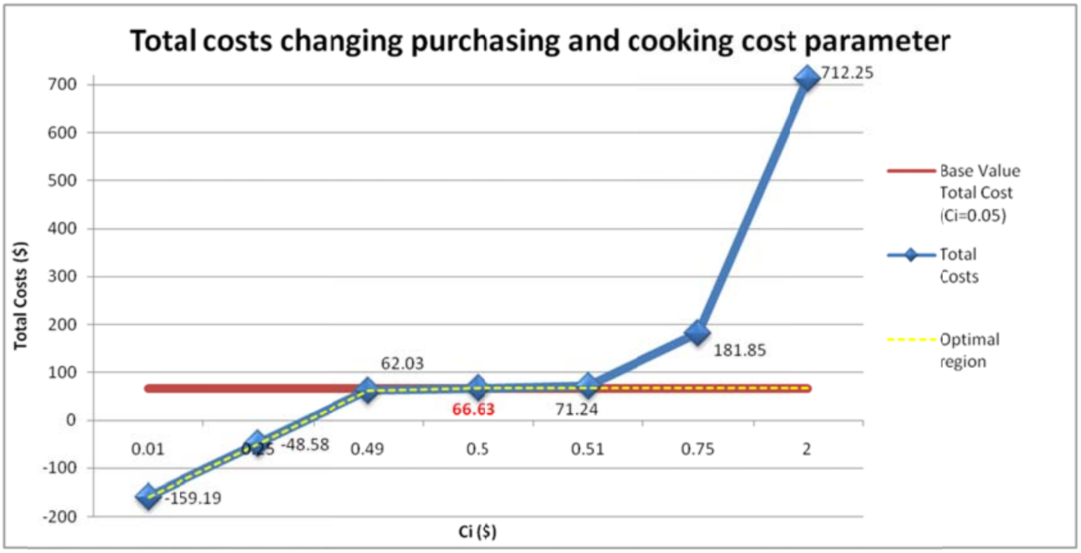

Figure 5. Total costs changing purchasing and cooking costs parameter

Observation 1. *If two items i of the same category k have the same parameters' values except for the value of the purchasing and cooking cost* C_i *, then the item with the lower purchasing and cooking cost* C_i *is more attractive to belongs to the assortment.*

The previous observation indicates that the purchasing and cooking costs parameter has significance when a firm has to decide between two or more items that are identical except in this value and which can also be equally combined with the other food category items. For this instance, to obtain a minimal total cost, the item with lower purchasing and cooking cost must be selected.

5.2.2 Number of consumer's participation parameter's characteristic

Using the same scenario that we used with the purchasing and cooking cost parameter, we next changed the number of consumer's participation parameter for one item of each food category *k,* maintaining the other parameters with its base values. This parameter was studied for two different cases, when the firm receives lower funding revenues and when they receive higher funding revenues. Figure 6 shows when an item i is attractive to belong to the assortment (when the firm incur in lower total costs or are inside the optimal region) in the two cases. Two observations were formulated, which are demonstrated in next example whose item i presented has a base value of $\beta_i = 9$.

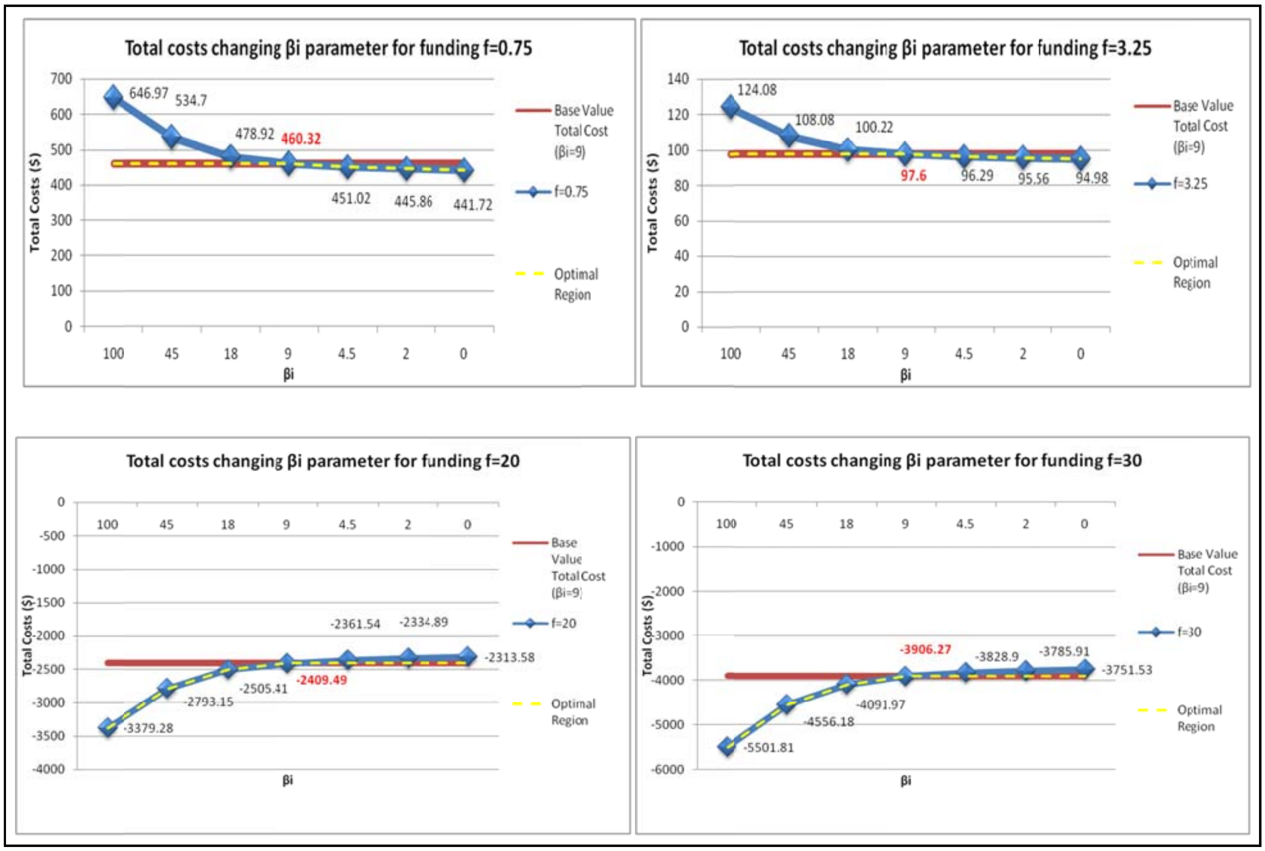

Figure 6. Total costs changing β_i parameter for lower and higher refunds

Observation 2a. *For lower refund values, if two items i of the same category k have the same parameters' values except for the value of the number of consumer's participation* β_i *, then the item with the lower number of consumer's participation* β_i *is more attractive to belongs to the assortment.*

Observation 2b. *For higher refund values, if two items i of the same category k have the same parameters' values except for the value of the number of consumer's participation* β_i *, then the item with the higher number of consumer's participation* β_i *is more attractive to belongs to the assortment.*

From the results, can be observed that for lower revenues values it is more attractive to offer an item that has lower values in the number of consumer's participation β_i. This result is based in the fact that the lower the revenue the higher the likelihood for negative profits (losses) and then is more attractive to offer an item that has lower consumer's participation to incur in less total costs. On the other hand, it can be noted that for higher revenues values it is more attractive to offer an item that has higher values in the number of consumer's participation β_i. Contrary as the case of lower revenues, in this instance the likelihood for obtain profits are higher and then is more attractive to offer an item that has higher consumer's participation to receive more revenues.

5.2.3 Variability parameter's characteristic

We next explore how a change of value in the item *i* variability can affect the assortment decision. As was formulated in section 3.1, the variability of food item i is calculated as the squared product of the expected assortment demand Y_s and the standard deviation of the rate that food item i is requested by the consumers, σ_{γ_i}. Then, for this observation, the standard deviation σ_{γ_i} was changed to a higher and lower value for one item of each food category k to identify if the variability of a food item has significance in the decision variable. In this case, the results obtained per each item didn't have the same behavior; for some items the results obtained were opposite to the ones obtained with other items of other categories. The assortment demand standard deviation, σ_i, affects two terms of the objective function in our model, the total purchasing and cooking costs and the total expected salvage value. Therefore, this parameter affects cost's terms and revenue's terms, affecting the net profit value. Using PRSMP example and assuming that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level, $I_i = Q_i U_i \quad \forall\, i$, the total inventory cost term is equal to zero and the total funding revenue remain equal for any value of σ_i. Then, to understand these results, we changed the purchasing and cooking cost parameter, C_i, together with the variability parameter, σ_{γ_i}, for different items *i*.

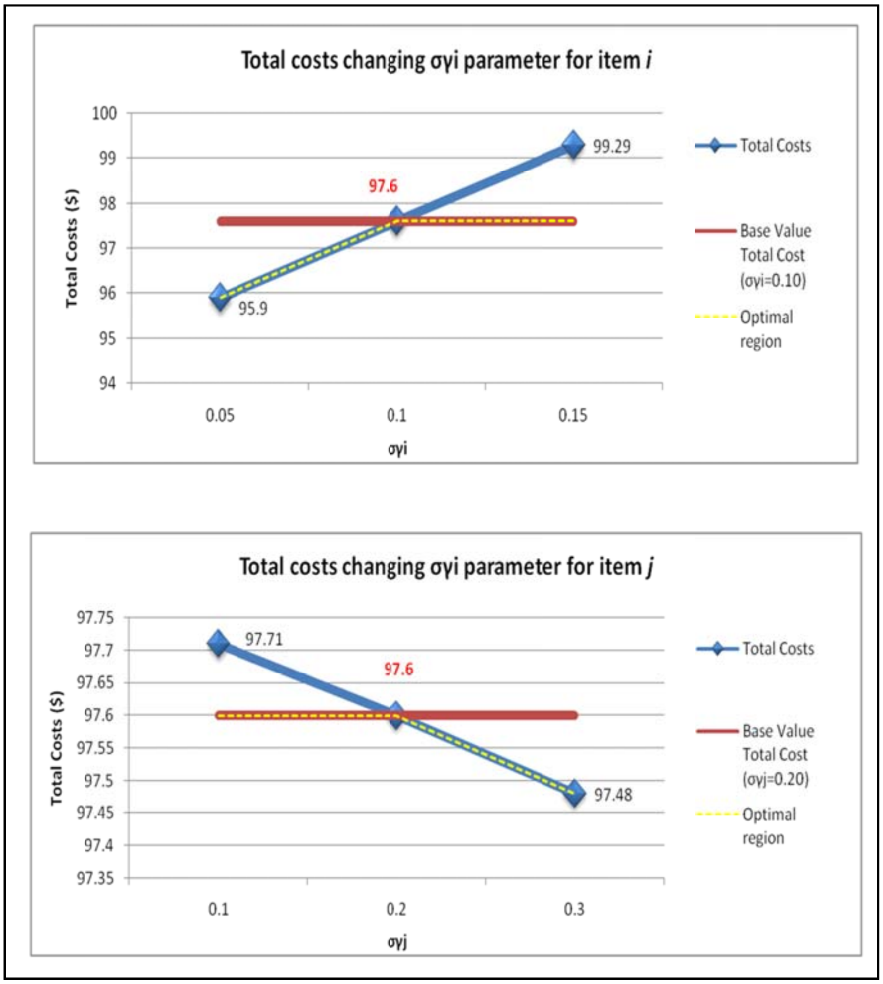

Figure 7. Total costs changing variability parameter for two items of different categories *k*

In the previous example (see Figure 7) item i has a standard deviation base value of $\sigma_{\gamma_i} = 0.10$ and the item j has a base value of $\sigma_{\gamma_j} = 0.20$. Note from above that when item i increasing the coefficient of variability (σ_{γ_i}) from 0.05 to 0.15 leaves the item out of the assortment (total cost is out the optimal region). In this case, the firm should favor the item with less variability that is fairly intuitive. For item j the opposite occurs item j increasing the coefficient of variability (σ_{γ_j}) from 0.10 to 0.30 the model will include the item in the assortment (total cost is inside the optimal region). The last result suggests that it is possible that in optimality the firm will favor items with higher variability in the assortment. The last two examples trigger the next observation.

Observation 3. *Given two items that are the same in all respects except for the value of the standard deviation* σ_{γ_i}*, the choice on which item to add to carry in the assortment is not trivial.*

The explanation for the results is related to the inventory decision. Recall that the model assumes that the inventory decision is exogenously fixed once a service level ("in-stock rate") is picked. Therefore, the inventory decision will not necessarily be the one that minimizes cost (i.e. Q_i may not be optimal).

5.2.4 Mean parameter's characteristic

Consider that the firm offers its products for the whole assortment expected demand, i.e. $\mu_{\gamma_i} = 1$. Hence, the expected demand for food item *i* will be equal to the assortment expected demand, i.e. $\mu_i = Y_{s.}$ To determine how μ_{γ_i} affects the decision variable with the previous PRSMP assumptions, we ran several instances where one of the food items had the parameter μ_{γ_i} set to equal to one. For the next example (see Figure 8), the expected rate base value for item *i* is equal to $\mu_{\gamma_i} = 0.95$.

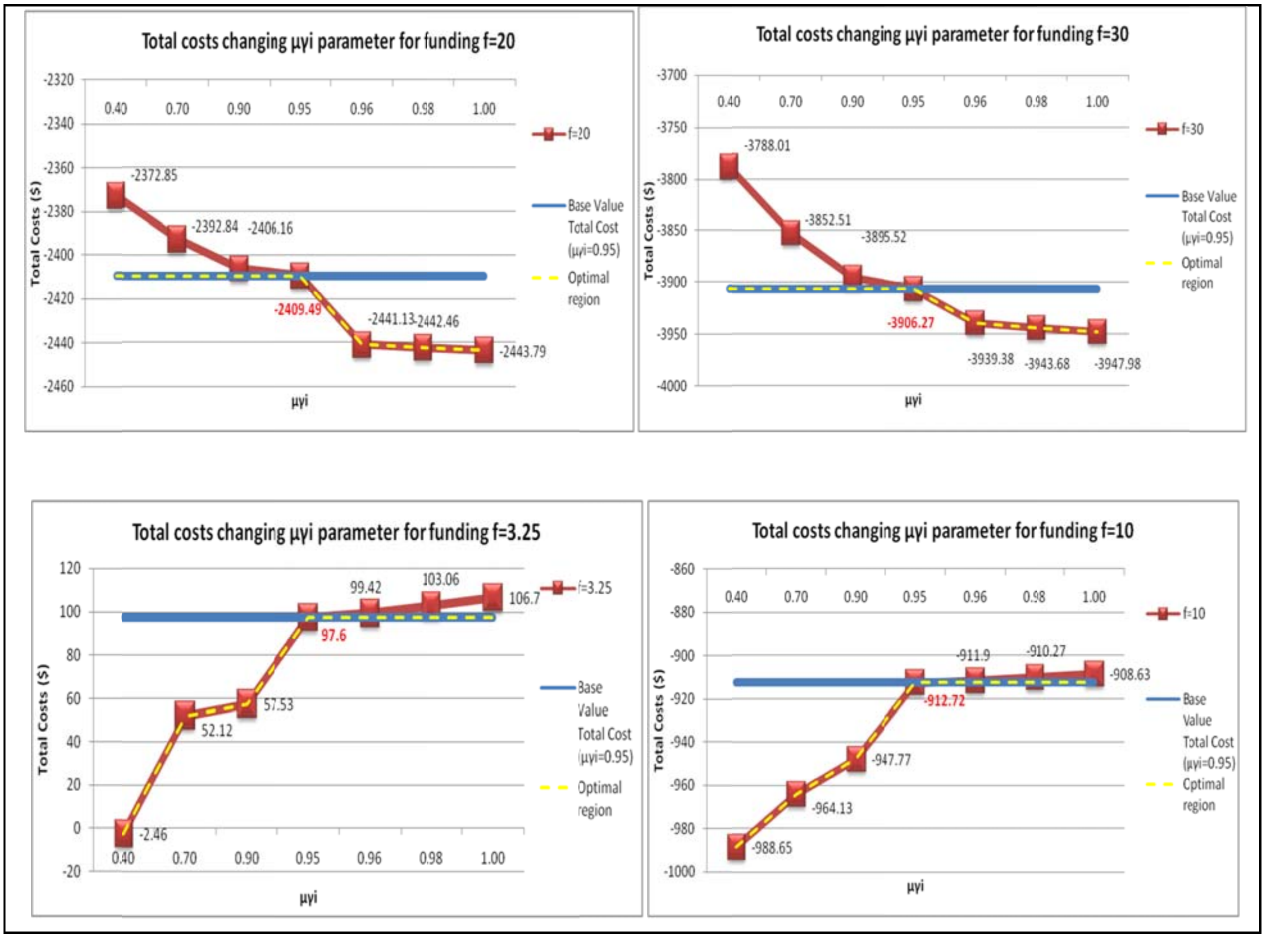

Figure 8. Total costs changing μ_{γ_i} parameter for lower and higher refunds

Observation 4a. The *higher the refund values, the more attractive are the items with higher* μ_{γ_i}.

Observation 4b. The *lower the refund values, the more attractive are the items with lower* μ_{γ_i}.

Similar to the number of consumer's participation parameter (Observations 2a and 2b), note from above that for higher revenue values it is more attractive to offer an item that has a greater expected demand having a greater net profit. This result is grounded in the fact that the higher the refund the higher the likelihood for profiting on offering the item. But, if the revenue is lower, it is more attractive to offer an item that is less attractive. This last result is grounded in the fact that the lower the revenue the higher the likelihood for negative profits (losses) and hence it is desirable to meet a lower consumer demand. For example, if the firm receives higher revenues per food tray served, it will like to offer an item that is more attractive (i.e. steak) because more consumers will choose it and hence a refund (revenue) will be granted making some profit. But, if the revenues are lower, the firm has to offer an item less attractive (i.e. sausage) because less consumers will choose it and by doing this, the firm will incur in less associates costs (i.e. purchasing and cooking costs).

5.2.5 Holding cost parameter's characteristic

In preparation for the next results, we know considered the PRSMP scenario characteristics with the assumption that the initial inventory level is greater than the total quantity per serving needed to satisfy the type 1-service level. To consider how the assortment composition can be affected when inventory costs are greater than zero, we explored this observation changing the value of the holding cost parameter, considering an interest per period of 0.068% (annual interest of 25%[6]). Following the same procedure as the previous observations the next interpretation was done based on the results obtained for item *i*, which are shown in Figure 9. The item *i* presented in next example has a base value of $h_i = 8\ x\ 10^{-5}$.

[6] Using as reference the capital cost used by the United States Department of Transportation Federal Transit Administration (FTA) in November 1999 during the San Juan, Puerto Rico Minillas Extension. http://www.fta.dot.gov/publications/reports/reports_to_congress/planning_environment_2947.html

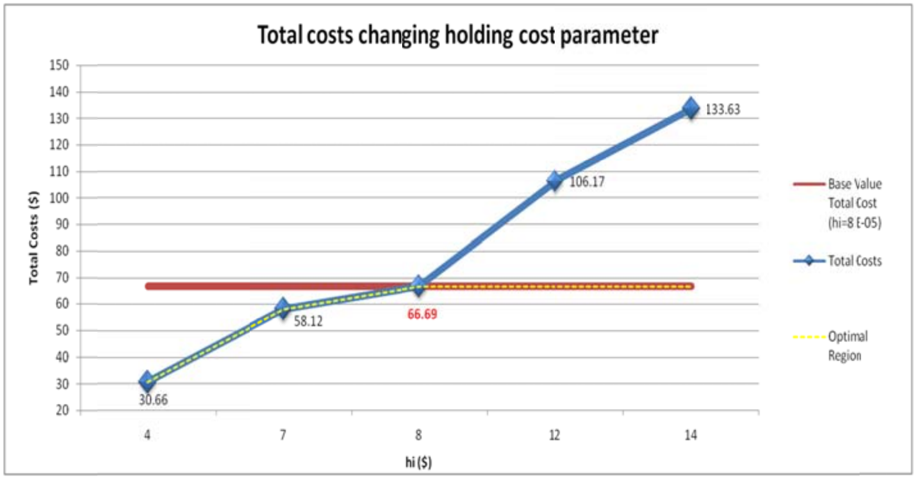

Figure 9. Total costs changing holding cost parameter

Observation 5. *If two items i of the same category k have the same parameters' values except for the value of the holding cost,* h_i*, then the item with the lower holding cost* h_i *is more attractive to belongs to the assortment.*

The previous observation indicates that the holding cost parameter has significance on the food optimal assortment in the sense that items with lower holding cost are favored. The above result is not surprising once given Observation 1 and the fact that the holding cost is linear with respect to c_{p_i} and c_{c_i} is maintained constant. However, it illustrates that the result holds even when now it is assumed that the holding cost does in fact play a role in the cost function.

6. Conclusions and Future Work

In conclusion, we present a cost minimization model adapted for the case of PRSMP. The model is subject to assortment composition constraints where demand influence between items is considered. The demand of the items that we considered is influenced by the presence of other items in the assortment, because the consumer preference of a particular item is influenced by the combination of items. On the other hand, this model minimizes costs for a firm (e.g. non-profit firms, like Publics School Meals Programs) contrary to some works in food management area that has an objective of minimize costs for the consumers. The minimization model has constraints to guarantee nutritional requirements, assortment composition and food items availability, which can be adjusted to different real scenarios, e.g. PRSMP. The work that we presented considers some operational costs, like purchasing and cooking costs and inventory holding costs. Furthermore, this model considers two types of revenues, like funding and items' salvage value. In addition to the case study presented, this work can be applied to other non-profits firms that offer nutritional food assortment incurring in costs and receiving external revenues, like hospitals, Department of Corrections and Rehabilitation[7], Food Banks and other organizations like Meal on Wheels Program[8] and Child & Adult Care Food Program[9].

To verify the model, it was introduced into optimization software and several instances were solved. It was found that as the number of items increased the software running time also increased but always obtaining an optimal assortment. To describe the structure of the optimal assortment different instances were modeled illustrating how the firm should take the assortment planning decisions based solely on food items' characteristics. We performed a numerical analysis using the Puerto Rico School Meal Program (PRSMP) as motivational example and several observations were obtained. The most influential parameter on the assortment composition found was the purchasing and cooking cost C_i, which between two items from the same category k and all other parameter been equal, the item with the lower purchasing and cooking costs is the most attractive to be part of the assortment. This observation can be obtained

[7] http://www.cdcr.ca.gov/Regulations/Adult_Operations/docs/DOM/NCDOM/2010NCDOM/10-15/DOM%20Chp5%20Art51%20Food%20Service.pdf
[8] http://www.mowaa.org
[9] http://www.fns.usda.gov/cnd/care/

regardless of the inventory level I_i is equal or not to the quantity needed to satisfy a type-1 service level, $Q_i U_i$. As for the items demand variability, it was observed that it is not a trivial decision of what item to carry based solely on this parameter, which highlights the importance of the inventory decision for the problem.

Furthermore, we observed that if the firm receives lower funding revenues the lower the value of the number of consumer participation β_i the more attractive is item i, and vice versa. Similarly, if the firm has two items from the same category k with equal parameters values, except for the expected rate value at which an item is requested by the consumers, μ_{γ_i}, if the firm receives higher revenues is more attractive to offer the item that is more attractive to the consumer, therefore the item with higher μ_{γ_i}; whereas, if the firm receives lower revenues is more attractive to offer the item with lower μ_{γ_i}, incurring in less total costs. Finally, the model was verified and the parameters studied, and we can conclude that our model complies with its function and an optimal menu can be found for a firm.

With this model, we are providing to non-profit firms a tool to plan their nutritional food assortment at lower costs. On the other hand, if these firms do not have the resources to obtain all the necessary data or to run a programming like this, we are providing some guides that describe an optimal menu and how they should plan their assortment considering some food items characteristics in order to offer a nutritional assortment at the minimum total costs.

An immediate extension for this work is to consider several periods. The scope of this work was concentrated in one period plan horizon. As next step, this model can be extended to consider more than one period. Considering more than one period, some features can be included, as is that the items that we are considering are perishables, then contrary as we considered in this work, exists the possibility that these items are damaged and can't be offered to the consumers. This event will affect the inventory holding costs calculation and the assortment planning for future periods. Also, the inventory holding cost formulation can be modified adding the cost that the firm will incurred in holding inventory of items that has not be offered in a single period and are expected to be offered in next periods. On the other hand, the consumers' preferences also can be affected when more than one period is considered. A consumer may like some items but not necessarily consecutively repeating the same. As a further extension, can be incorporated in the analysis more echelons of this Supply Chain (for example: distribution centers and the purchasing process at the firm main offices), including how the operational costs are affected in the whole supply chain and considering how this echelons are connected.

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