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Energy Portfolio Management with Abandonment Option Over An Infinite Horizon

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Abstract

We study the optimal time to abandon a plant of a firm with a portfolio of plants to maximize the expected profit over an infinite horizon. Under geometric Brownian motion, we formulate the problems as mixed optimal stopping/control problems, and characterize the optimal strategies in closed-form.

1 Introduction

We study the optimal abandonment decision to shut down a power plant in an energy portfolio, with the objective to maximize expected long-term profit over an infinite horizon. The electricity price is modeled as a stochastic process, and auxiliary problems are introduced to solve the original problem. We model the problem as stochastic control and optimal stopping problem.

Evaluations of power plants have been widely studied. Tseng and Barz [6] evaluated a power plant in short-term with unit commitment constraints by real-options approach. Deng et al. [2] valued electricity derivatives by futures-based replication due to the non-storable nature of electricity. By the same approach, Tseng and Lin [7] evaluated a power plant involving processes of electricity and fuel prices. Deng et. al. [1] studied the optimal entry time to construct an alternative plant to form an energy generating portfolio.

This paper studies the timing that a firm shuts down a conventional power plant on an infinite horizon. We assume the firm owns a power plant portfolio, which includes an alternative plant and a conventional plant, and considers shutting down the conventional power plant while maximizing the expected cumulative profit. Under the geometric Brownian motion of electricity prices, we formulate the decision problem as a mixed stochastic control problem. Due to the intractability of the mixed problem, we decompose it into two auxiliary problems: one is a regular stochastic control problem, the other one is an optimal stopping problem. We construct the closed-form solution of the problems, and obtained the optimal solution with the help of value-matching and smooth-pasting conditions.

The rest of the paper is organized as follows. We formulate the decision problem as a non-standard stochastic control problem in Section 2. In Section 3, we write the equivalent form of the value function to the control problem. By standard arguments, we obtain the closed-form of the value function. Finally, conclusions and future research directions are presented in Section 4.

2 Problem Formulation

We introduce the following notion to formulate the problem.

- X_t : Electricity price [$/MWh]
- ρ : the risk-adjusted discount rate
- α_{max}: maximum proportion of total wealth invested in alternative method (AL)
- $\alpha \in [0, \alpha_{max}]$: proportion of total wealth invested in AL [Decision variable]
- c_1: Production rate of the conventional method (CON)
- c_2: Production rate of AL
- D_1: Total cost of generating c_1 units of electricity from CON
- D_2: Total cost of generating c_2 units of electricity from AL

- τ: time to abandon CON [Decision variable]
- T: planning horizon
- K : liquidation value for abandoning CON[$]

We assume the long-term electricity price follows the standard geometric Brownian motion [5]:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \tag{1}$$

where μ and σ are expected growth rate and volatility of the electricity price, respectively, and B_t is a Wiener processes. Assume x is the initial position of electricity price. That is, $X_0 = x$.

The objective of the firm is to choose an optimal stopping time τ to abandon CON due to its life time, and an optimal proportion α before τ in order to maximize the expected profit. If we define the expected discounted profit functional J given initial electricity price x, proportional investment in AL α, and the time to abandon CON τ as

$$\begin{aligned} J(x;\alpha,\tau) \equiv \; & \mathbf{E}_x[\textstyle\int_0^\tau [\alpha_t(c_1 X_t - D_1) + (1-\alpha_t)(c_2 X_s - D_2)]e^{-\rho t}dt \\ & +Ke^{-\rho\tau} + \textstyle\int_\tau^{+\infty}(c_2 X_s - D_2)e^{-\rho t}dt] \end{aligned} \tag{2}$$

subject to (1), where $\mathbf{E}_x$ is the expectation with respect to x, and α_t is obviously a function of time t, then the value function u is defined as

$$u(x) \equiv \sup_{\alpha \in [0, \alpha_{max}], \tau \in \Gamma} J(x; \alpha, \tau), \tag{3}$$

where Γ is the set of stopping times.

3 Solution Methods for the Problem

In order to solve the non-standard stochastic control problem (3), we proceed as follows: By introducing an auxiliary function v, we first obtain the equivalent function w to the value function u, which solves an optimal stopping problem. We then get the closed-form solution of w by value-matching and smooth-pasting conditions.

3.1 Equivalent Problem to (3)

We define the auxiliary function $v(x)$ as the expected cumulative discounted profit from AL:

$$v(x) \equiv E_x[\int_0^\infty (c_2 X_s - D_2) e^{-\rho t} dt]. \tag{4}$$

It is easy to get

$$v(x) = \frac{c_2 x}{\rho - \mu} - \frac{D_2}{\mu} \tag{5}$$

by the fact that $E[X_t] = x e^{\mu t}$.

If we define the value function w as

$$w(x) = \sup_{\alpha,\tau} \mathbf{E}_x[\int^{\tau} [\alpha(c_1 X_s - D_1) + (1-\alpha)(c_2 X_s - D_2)] e^{-\rho t} dt + (v(X_\tau) + K) e^{-\rho\tau}], \quad (6)$$

we can prove that w is equivalent to u in (3) ([4]).

The value function w satisfies the combination of variational inequality of optimal stopping and the Hamilton-Jacobi-Bellman equation of stochastic as follows

$$\min\{\sup_{\alpha}[Lw + [\alpha(c_1 x - D_1) + (1-\alpha)(c_2 x - D_2)] - \rho w], w - v - K\} = 0. \quad (7)$$

where the generator L is defined as

$$Lw \equiv \mu x \frac{\partial w}{\partial x} + \frac{1}{2}\sigma_x^2 x^2 \frac{\partial^2 w}{\partial x^2}. \quad (8)$$

It is obvious that the optimal proportion α^* is

$$\alpha^*(x) = \begin{cases} \alpha_{max}, & \text{if } x > \frac{D_1 - D_2}{c_1 - c_2} \equiv x^* \\ 0, & \text{if } x \leq \frac{D_1 - D_2}{c_1 - c_2} \end{cases} \quad (9)$$

Equation (7) is transformed into

$$\min\{Lw + [\alpha^*(c_1 x - D_1) + (1-\alpha^*)(c_2 x - D_2)] - \rho w, w - v - K\} = 0. \quad (10)$$

3.2 Closed-Form Solution to w

First we consider the solution to the following ordinary differential equation (ODE):

$$Lw + [\alpha^*(c_1 x - D_1) + (1-\alpha^*)(c_2 x - D_2)] - \rho w = 0. \tag{11}$$

A special solution w_0 to (11) can be easily identified as

$$w_0(x) = \alpha^*(x)(\frac{c_1 x}{\rho - \mu} - \frac{D_1}{\mu}) + (1-\alpha^*(x))(\frac{c_2 x}{\rho - \mu} - \frac{D_2}{\mu}) \tag{12}$$

If we try a function w of the form

$$w(x) = Cx^\beta, \text{ for some constant } \beta, \tag{13}$$

we get

$$Lw - \rho w = x^\beta[\mu\beta + \frac{1}{2}\sigma^2\beta(\beta-1) - \rho] \equiv x^\beta h(\beta), \tag{14}$$

where

$$h(\beta) \equiv \mu\beta + \frac{1}{2}\sigma^2\beta(\beta-1) - \rho. \tag{15}$$

Note that

$$h(1) = \mu - \rho \text{ and } \lim_{\beta\to\infty} = \infty. \tag{16}$$

Therefore if we assume that $\mu < \rho$, then we can get there exists $\beta_1 > 1$ such that $h(\beta_1) = 0$.

Next we solve for the explicit form of solution to Problem (10). With the value of β_1, we put

$$w(x) = \begin{cases} Cx^{\beta_1} + w_0(x), & \text{if } x \le \overline{x} \\ v(x) + K, & \text{if } x > \overline{x} \end{cases} \tag{17}$$

for constants C and $\overline{x}$ to be determined.

By "high contact" conditions, we have

$$C\overline{x}^{\beta_1} + w_0(\overline{x}) = v(\overline{x}) + K \qquad \text{(value matching condition at } x = \overline{x}) \tag{18}$$

and

$$C\beta_1\overline{x}^{\beta_1 - 1} + w_0'(\overline{x}) = v'(\overline{x}) \qquad \text{(smooth pasting condition at } x = \overline{x}) \tag{19}$$

It is easy to see that

$$C\overline{x}^{\beta_1} = \alpha^*(\overline{x})(\frac{(c_2 - c_1)\overline{x}}{\rho - \mu} - \frac{D_2 - D_1}{\mu}) + K. \tag{20}$$

and

$$C\beta_1\overline{x}^{\beta_1 - 1} = \alpha^*(\overline{x})\frac{c_2 - c_1}{\rho - \mu}. \tag{21}$$

(21) requires that

$$\alpha^*(\overline{x}) = \alpha_{max}, \text{ i.e., } \overline{x} \geq x^*. \tag{22}$$

Therefore, we obtain

$$\overline{x} = \frac{\beta(\rho - \mu)[(D_2 - D_1)\alpha_{max} - K\rho]}{(\beta - 1)\rho\alpha_{max}(c_2 - c_1)} \tag{23}$$

(21) yields

$$D_2 - D_1 \geq \frac{K\beta_1}{\rho}(\rho - \mu)\alpha_{max}(\rho - \mu\beta), \text{ and } \beta_1 < \frac{\rho}{\mu}. \tag{24}$$

Plugging (23) into (21) yields

$$C = \frac{\alpha_{max}(c_2 - c_1)}{(\rho - \mu)\beta_1\overline{x}^{\beta_1 - 1}}. \tag{25}$$

In summary, the value function u is equivalent to w, which has the closed-form (21). The unknown constants are determined by (23) and (25) with constraints (24). The optimal policy is characterized by (9) and the free boundary $\overline{x}$.

4 Conclusion

In this paper we study the optimal abandonment decision for the conventional power plant given fixed liquidation value with and the optimal dispatch decision between the two power plants. The objective function is to maximize the expected cumulative profit under geometric Brownian Motion for electricity prices. Optimal control method is employed to solve the abandonment decision problem in energy portfolio.

The results provide some valuable guildlines for the administration of the generators to make decisions. The free boundary can be employed to decide when the generator will shut down the conventional power plant - The generator should shut down the old power plant if the electricity price is above the free boundary at a particular time in order to gain the most profit.

This paper can be generalized in the following ways. First, the analysis in this paper is just based on one stochastic process (the electricity price); we admit that there are other stochastic processes that can affect the decisions, such as the cost of the carbon dioxide emission. We would have more stochastic processes in addition to electricity prices process, and we need to solve the multidimensional optimal control problem. Second, switching costs will be incurred when we abandon the conventional method, and singular control technique would be employed to study this problem.

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