Abstract number: 025-0227

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POMS 23rd Annual Conference

Chicago, Illinois, U.S.A.

April 20 to April 23, 2012

Optimal decisions for a decentralized assembly system with dual supply modes

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Abstract

We consider a decentralized assembly system with one assembler and two suppliers, where one supplier is perfectly reliable and another generates yield uncertainty. With customer's random demand, we derive each component's optimal production (input) quantity through a static Nash game model. Finally, we propose a contract to achieve supply chain coordination.

Keywords: Ordering mode; VMI mode; Nash equilibrium; supply chain coordination

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1 Introduction

Assembly systems have been widely applied in automobile, electronics and many other manufacturing industries. Taking full advantages of every participant's high quality resources is one of the biggest strengths of assembly systems. It is well-known that there are a large number of companies, such as Hewlett-Packard and Toyota, outsourcing the production of components to external suppliers around the world in order to reduce costs and increase flexibility. However, the accompanying challenge is how to ensure every node enterprise in the system move harmonized and synchronized. Difficulties mainly exist in: (1) Uncertain factors. Uncertain conditions lie in the fields of delivery time, customer demand and production yield, and they may cause the system suffer production interruption and sales loss; (2) Disunity of objectives. Each company within a decentralized assembly system makes decisions only in the interest of maximizing its own profit, and it is always contradictory to optimizing the performance of the entire system. Both of these reasons can lead to an undesirable result in the final output of a decentralized assembly system. Therefore, in order to improve the system's performance, many scholars have completed extensive researches on making and optimizing decisions for assembly systems (Yano, 1987; Tang & Grubbstrom, 2003; Gurnani & Gerchak, 2007; Granot & Yin, 2008).

Among the available researches on assembly systems, most of them consider that all kinds of components are supplied to the assembler in a same mode. In one case, the assembler needs to order components from all his suppliers before the assembling starts (referred as "ordering mode"). Yano (1987) studied the optimal order time of a assembler with two different suppliers, when both suppliers have a stochastic supply lead time. Song et al. (2000) investigated an assembler's optimal order decision in the situation of both random demand and stochastic supply lead time. In another case, the customer's demand information is shared between the assembler and his suppliers. Then, all the suppliers are required to provide the assembler with components under VMI arrangement (referred as "VMI mode"). Gerchack & Wang (2004) analyzed how to achieve coordination in an assembly system under VMI mode with both revenue-sharing contract and buy-back contract when the market demand is uncertain.

However, in practice, due to the differences among the suppliers' bargain power and the components' importance, ordering mode and VMI mode always coexist in one assembly system. For example, in China's automobile industry, the assembler needs to order key components, such as engine, from overseas suppliers. Meanwhile, the matching components needed to assemble an entire car, such as windshield wipers, are provided by the domestic suppliers under VMI mode. There also exists a similar phenomenon that in electronics industry, the assembler always place orders in advance to key components' providers, such as Intel, while the matching components are provided by local suppliers with VMI arrangement. Whereas, to date, the decision making and optimizing in assembly systems with these two supply modes are seldom considered. As such, in this paper, we expect to derive the optimal decisions for a decentralized assembly system with both ordering mode and VMI mode. Specifically, we firstly study the decentralized system's optimal decisions with random yield and demand. Then, we introduce a centralized system as the benchmark and propose a proper contract to achieve supply chain coordination in the decentralized system.

Besides the literature reviewed above, another stream of the related research is random yield and demand. Yield and demand uncertainty in assembly systems have been extensively studied by many scholars (Gerchak et al., 1994; Gurnani et al., 2000; Güler & Bilgiç, 2009; Xiao et al., 2010). In which, the majority of the yield uncertainty models have been developed are in the situation of all the suppliers enrolled in an assembly system generate production yield uncertainty. But, actually, there coexist two types of supply in a supply chain (Chopra et al., 2007; Giri, 2010): (1) one kind of suppliers are subject to yield uncertainty, which is mainly due to production of poor quality, machine breakdown, etc; (2) the others are perfectly reliable, which relies on their stable production capacity and flexible management capability. Approximately, in assembly systems, the key component providers, such as Intel, are stable in production capacity and can operate quiet well. Therefore, they are nearly perfectly reliable. Meanwhile, the matching component providers may suffer yield uncertainty, because they are usually in small size and unreliable in managing production. Whereas, to date, only Pan & So (2010) analyzed an assemble-to-order system with these two types of supply. In their paper, one decision maker determines both two kinds of suppliers' production input quantity in the interest of maximizing system's profit when facing price-dependent demand. Different from them, in our research, we have multiple decision makers: the assembler decides the production quantity of the key components, while the suppliers of the matching components determine their own production quantity. Under this setting, our research goal is to derive the optimal decisions for each player and achieve the supply chain coordination.

The remainder of this paper is organized as follows. In section 2, we briefly describe the model. The optimal decisions of a decentralized system and its coordination are derived in Section 3. In section 4, we investigate the reason of incoordination in the decentralized system and the contract's effectiveness on supply chain coordination via numerical analysis. Section 5 concludes the paper and shows the future research directions.

2 Model descriptions

A decentralized assembly system consisting of two suppliers and one assembler is considered. In which, all participants are risk-neutral and aim to maximize its own profit. We assume that the final product only consists of one key component and one matching component. The key component's unit production cost is c_1 and unit wholesale price is w_1 ($w_1 \geq c_1$), and it is provided by supplier 1 under ordering mode with perfect reliability. Whereas, supplier 2 provides the matching component under VMI mode with unit production cost c_2 and unit wholesale price w_2 ($w_2 \geq c_2$), but he is unreliable in the sense that it generates supply yield uncertainty. Specifically, if the input quantity of production is q, then the output quantity of components that meet the quality standard is θq, where θ is a random variable in $[0, 1]$ with probability distribution function $g(\cdot)$ and cumulative distribution function $G(\cdot)$ (Yano & Lee, 1995; Xu, 2010; Pan & So, 2010). As to the assembler, he assembles the components into final products and sales them to the customers with random demand D, which follows probability distribution function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ in $[0, \infty)$. The final product's unit price is p ($p \geq w_1 + w_2$).

Without loss of generality, we assume that the unit assembly cost equals 0 (Actually, if the cost is $c_a > 0$, the final product's unit price can be modified to $p' = p + c_a$).

In a single-period setting, the assembler will receive an order with a fixed demand time T from customer in advance. But, the one-time demand quantity D will not be realized until time T. To further simplify, we assume that assembly time of final products is negligible, which equals zero for the assembler. As a result, the assembler just needs to tell every supplier that the component's delivery time is T. Besides, the assembler should make component purchasing contracts respectively with two suppliers. In the contract with supplier 1, the assembler determines the order quantity (i.e., the production input quantity) Q_1 of the key component and must pay for supplier 1 immediately after he finished delivery. While in the contract with supplier 2, supplier 2 should decide the input quantity Q_2 of the matching component's production and will not receive the payment until the component is consumed. Supplier i starts his production at time B_i, which is ahead of T for a period of a fixed lead time L_i, i.e., $B_i = T - L_i$ $(i = 1, 2)$. After both components' production and delivery are finished, the assembler will assemble them together and sale the final product to the customer. With yield and demand uncertainty, the assembler will be penalized by the customer if he suffers stockout ($\min(Q_1, \theta Q_2) < D$), unit penalty cost is β. In turn, supplier 2 will receive a penalty from the assembler if his supply quantity θQ_2 is insufficient ($\theta Q_2 < D$), unit penalty cost is β_2 ($\beta_2 \leq \beta$). Besides, we assume that the salvage value of the mismatched components, $Q_1 - \min(Q_1, \theta Q_2, D)$ and $\theta Q_2 - \min(Q_1, \theta Q_2, D)$, is zero.

In the system described above, all the information except the purchasing contracts are freely shared among the participants. In summary, there are two sequential decisions need to be made. The system's decision sequence is that the assembler and supplier 2 simultaneously decide the production input quantity of the key component and the matching component. Therefore, we can develop a static Nash game model between the assembler and supplier 2 to investigate the optimal decisions. In the following section, we make an in-depth analysis of optimal decisions for the decentralized system and its coordination. As a matter of convenience, we will say that X^d represents the condition in decentralized system and X^c represents the condition in centralized system.

3 System's optimal decisions

In this section, we firstly prove that there exists a unique static Nash equilibrium between the assembler and supplier 2. Upon this, the decentralized system's optimal decisions are derived. Then, we import the centralized system as a benchmark and propose an advance payment contract to achieve coordination in the decentralized system.

3.1 Decentralized System

In the decentralized system, the assembler (denoted as A) will pay supplier 1 (denoted as S_1) immediately after he finishes the key component's delivery. Therefore, his profit function is $\Pi^d_{S_1} = (w_1 - c_1)Q_1$, which is nonnegative and meets the participant constraint.

As to supplier 2 (denoted as S_2), he will not receive the payment until the matching component is consumed. Besides, supplier 2 will be penalized by the assembler if his output is less than the customer's demand. Then, supplier 2's expected profit function can be formulated as:

$$\Pi_{S_2}^d = w_2 E\{\min(Q_1, \theta Q_2, D)\} - c_2 Q_2 - \beta_2 E[D - \theta Q_2]^+ \tag{1}$$

Here, the first term is the expected revenue. The second term is the production cost, which is based on the input quantity. The third term is the expected penalty when the stockout of the matching component occurs. Similarly, we have the assembler's expected profit function as follows.

$$\Pi_A^d = (p - w_2) E\{\min(Q_1, \theta Q_2, D)\} - w_1 Q_1 - \beta E[D - \min(Q_1, \theta Q_2)]^+ + \beta_2 E[D - \theta Q_2]^+ \tag{2}$$

In which, the first two terms are the final product's expected sales revenue minus the procurement cost of the key and the matching components. The third term is the penalty penalized by the customer when the stockout of the final product occurs. The fourth one is the compensation from supplier 2 when his yield can not meet the customer's demand.

The following lemma states that the objective functions, $\Pi_{S_2}^d$ and Π_A^d, are concave in supplier 2's and supplier 1's production (input) quantity respectively. In this paper, the proofs of all lemmas and theorems are given in the Appendix.

Lemma 1. *Let $Q_i^{d*}(Q_j)$ denote the optimal production input quantity of the key / matching component for a given Q_j $(i, j = 1, 2;\ i \neq j)$. Then, we have:*

i) Supplier 2's profit function, $\Pi_{S_2}^d$, is concave in $Q_2 \in [0, \infty)$. And, the optimal input quantity $Q_2^{d*}(Q_1)$ meets the first-order condition:

$$\frac{\partial \Pi_{S_2}^d(Q_1, Q_2)}{\partial Q_2} = w_2 \int_0^{\frac{Q_1}{Q_2}} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) dD d\theta + \beta_2 \int_0^1 \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) dD d\theta - c_2 = 0 \tag{3}$$

ii) The assembler's profit function, Π_A^d, is concave in $Q_1 \in [0, \infty)$. And, the optimal input quantity $Q_1^{d*}(Q_2)$ meets the first-order condition:

$$\frac{\partial \Pi_A^d(Q_1, Q_2)}{\partial Q_1} = (p + \beta - w_2) \int_{\frac{Q_1}{Q_2}}^{1} \int_{Q_1}^{\infty} f(D) g(\theta) dD d\theta - w_1 = 0 \tag{4}$$

With Lemma 1, we can characterize the optimal production quantity of both the key and the matching components through their F.O.Cs and the static Nash game equilibrium. In particular, we have the following theorem:

Theorem 1. *There is a unique static Nash equilibrium solution $D(Q_1^{d*}, Q_2^{d*})$ to the assembler's and supplier 2's decisions of the key and the matching components' production quantity. The solution (Q_1^{d*}, Q_2^{d*}) meets the combination of equation (3) and (4).*

With Theorem 1, the unique optimal production quantity of the key component (Q_1^{d*}) and the matching component (Q_2^{d*}) can be solved when the operational parameters, e.g., the wholesale price w_1 and w_2, are given. Then, they can support the assembler and suppliers to achieve the maximization of their own profit.

3.2 Benchmark

An alternative scenario is the case where the assembler and two suppliers cooperate to achieve system optimum for the entire supply chain. To this end, we formulate a centralized system's (denoted as B) expected profit function.

Note that, in the centralized case, the business between the assembler and two suppliers can be eliminated with each other. Therefore, we have:

$$\Pi_B^c(Q_1, Q_2) = pE\{\min(Q_1, \theta Q_2, D)\} - (c_1 Q_1 + c_2 Q_2) - \beta E[D - \min(Q_1, \theta Q_2)]^+ \tag{5}$$

The first term is the final product's expected sales revenue. The second term is the production cost of both the key and the matching components. The third term is the expected penalty when stockout occurs.

Fix either Q_i $(i = 1, 2)$, the maximization of $\Pi_B^c(Q_1, Q_2)$ becomes a newsvendor problem. As such, we have the following lemma for characterizing the property of $\Pi_B^c(Q_1, Q_2)$.

Lemma 2. *The centralized system's expected profit function, $\Pi_B^c(Q_1, Q_2)$, is jointly concave in $Q_1 \in [0, \infty)$ and $Q_2 \in [0, \infty)$. And, the unique optimal input quantity of both the key and the matching components, Q_1^{c*} and Q_2^{c*}, meet the F.O.Cs:*

$$\frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_2} = (p+\beta) \int_0^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) dD d\theta - c_2 = 0 \tag{6}$$

$$\frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_1} = (p+\beta) \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} f(D) g(\theta) dD d\theta - c_1 = 0 \tag{7}$$

From Lemma 2, we can always find the unique pair of (Q_1^{c*}, Q_2^{c*}) that maximizes the $\Pi_B^c(Q_1, Q_2)$. Furthermore, the equation (4) and (7) are similar and actually independent of θ. As such, we can derive the relationship of Q_1^{x*} and Q_2^{x*} (x represents d and c). The following theorem states that $Q_2^{x*} > Q_1^{x*}$ always holds.

Theorem 2. *No matter in decentralized system or in centralized system, the optimal input quantity of the matching component is always strictly larger than the optimal input quantity of the key component, i.e, $Q_2^{d*} > Q_1^{d*}$ and $Q_2^{c*} > Q_1^{c*}$.*

For Theorem 2, it is easy to understand that with random yield, if the input quantity of the matching component is less than the key component, the output θQ_2 must be less than Q_1. As such, the penalty of the shortage of the matching component can be reduced by enlarging the input quantity, which increases the profit of supplier 2 and the system. When Q_2 exceeds Q_1, they will reach an equilibrium.

Besides, to find the inefficiency of the decentralized system, we regard the centralized system as the benchmark and make a comparison between them. Then, we have the following theorem.

Theorem 3. *In the decentralized system, the summation of all the participants' optimal profit is strictly less than the centralized system's optimal total profit, i.e.,*

$$\left(\Pi_{S_1}^d + \Pi_{S_2}^d + \Pi_A^d\right)\big|_{Q_1=Q_1^{d*},Q_2=Q_2^{d*}} < \Pi_B^c\left(Q_1^{c*}, Q_2^{c*}\right)$$

From theorem 3, we can see that although there exists a unique static Nash equilibrium in the decentralized system, it is motivated to introduce a proper contract to coordinate the supply chain move as in a centralized system. In the following subsection, we propose an advance payment contract to achieve the supply chain coordination.

3.3 Supply chain coordination

Among the previous literature, a number of different contract types aiming at coordinating the supply chain are identified and their benefits and drawbacks are illustrated, e.g., the revenue sharing contract, the buy-back contract, etc. For a detailed review, please refer to Cachon (2003). Here, we propose an *advance payment contract*. Specifically, at the beginning of a production period, the assembler pays $(\lambda_1\Pi_B^{c*} + c_1Q_1^{c*})$ to supplier 1 and $(\lambda_2\Pi_B^{c*}+c_2Q_2^{c*})$ to supplier 2. After receiving the customer's order, the assembler announces the production input quantity of both the key and the matching components, Q_i^{c*} $(i = 1, 2)$.

Then, the suppliers carry out the production according to them. Finally, the assembler assembles the components into products and sales them to the customer. After that, he gains the profit of $(1-\sum_{i=1}^{2}\lambda_i)\Pi_B^{c*}$, where $\lambda_i \in [0,1]$ and $\sum_{i=1}^{2}\lambda_i \leq 1$.

The major feature of the above contract is that the assembler should pay the suppliers in advance. Thus, if supplier i's profit $(\lambda_i \Pi_B^{c*})$ is less than what he can gain in the decentralized system, the contract will fail in practice. Therefore, we have the following theorem to state that there always exist a proper pair of (λ_1, λ_2) which can successfully implement the contract.

Theorem 4. *The decentralized assembly system with dual supply modes always can be coordinated through the advance payment contract. In which, the contract parameter λ_i meets the following conditions:*

$$\frac{\Pi_{S_i}^{d*}}{\Pi_B^{c*}} \leq \lambda_i \leq 1 - \frac{\Pi_A^{d*} + \Pi_{S_j}^{d*}}{\Pi_B^{c*}} \qquad \text{and} \qquad \sum_{i=1}^{2}\lambda_i \leq 1 - \frac{\Pi_A^{d*}}{\Pi_B^{c*}} \tag{8}$$

Wherein, $\Pi_y^{d*} = \Pi_y^d(Q_1^{d*}, Q_2^{d*}),\ y \in \{S_1, S_2, A\},$ *and* $i, j = 1, 2,\ i \neq j.$

With theorem 4, we can draw the insight that the advance payment makes the suppliers operate as the assembler's subsidiaries, therefore can achieve the supply chain coordination. Besides, under this contract, the assembler actually changes the business scenario with supplier 2 by deciding the input quantity of the matching component himself, which indicates that when facing two kinds of supply, the VMI arrangement with the supplier who generates yield uncertainty is not beneficial for the entire supply chain. To this point, we further study the comparison of the decentralized and centralized systems, and investigate the effectiveness of the advance payment contract in the following section.

4 Numerical analysis

In this section, we conduct two numerical examples. In the first example, we make a comparison of the components' optimal input quantity and each party's expected optimal profit under three alternative scenarios: the decentralized system with dual supply modes, the decentralized system only with ordering mode (Yano, 1987; Song et al., 2000) and the centralized system. Based on this, we can investigate the reason why the decentralized system with dual supply modes generates incoordination and study the unprofitability of the VMI arrangement with the supplier who generates yield uncertainty. The second example is to show the effectiveness of the advance payment contract on the supply chain coordination by making sensitivity analysis on λ_i ($i = 1, 2$).

4.1 Comparison of the alternative scenarios

In Section 3.3, we discussed that the VMI arrangement is not beneficial to the entire supply chain. However, in the decentralized system only with ordering mode (denoted as d'), both the key and the matching components' input quantity are decided by the assembler, which is similar with the case under the advance payment contract. Therefore, the supply chain's performance should be better than the one with dual supply modes. To this point, it is also introduced as an alternative scenario.

In such a decentralized system, the assembler decides both the input quantity of the key and the matching components. Then, each party's expected profit functions change to:

$$\Pi_{S_1}^{d'} = (w_1 - c_1)Q_1, \quad \Pi_{S_2}^{d'} = w_2 E\{\theta Q_2\} - c_2 Q_2 \tag{9}$$

$$\Pi_A^{d'} = pE\{\min(Q_1, \theta Q_2, D)\} - w_1 Q_1 - w_2 E\{\theta Q_2\} - \beta E[D - \min(Q_1, \theta Q_2)]^+ \tag{10}$$

Through a similar proof of Lemma 2, we can easily find that equation (10) is jointly concave in Q_1 and Q_2, and the optimal $(Q_1^{d'*}, Q_2^{d'*})$ meets the following F.O.Cs:

$$\frac{\partial \Pi_A^{d'}(Q_1, Q_2)}{\partial Q_2} = (p+\beta) \int_0^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) dD d\theta - w_2 \int_0^1 \theta g(\theta) d\theta = 0 \tag{11}$$

$$\frac{\partial \Pi_A^{d'}(Q_1, Q_2)}{\partial Q_1} = (p+\beta) \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} f(D) g(\theta) dD d\theta - w_1 = 0 \tag{12}$$

Due to the F.O.Cs and the profit functions' complexity, we now make the comparison of the three alternative scenarios via numerical approaches. In the following numerical example, we conduct several comparisons under different groups of parameters.

Example 1 Assume that the customer's demand follows the normal distribution and the production random variable of supplier 2 follows the uniform distribution. Then, suppose we have the following data:

$$\text{Case 1}: c_1 = 4,\ w_1 = 6,\ c_2 = 3,\ w_2 = 8,\ \beta_2 = 4,\ p = 25,\ \beta = 15$$

$$\text{Case 2}: c_1 = 4,\ w_1 = 5,\ c_2 = 3,\ w_2 = 9,\ \beta_2 = 5,\ p = 23,\ \beta = 13$$

Table 1 contains the key and the matching components' optimal input quantity, the expected profits of two suppliers and the assembler, and the expected total profit of the supply chain for three alternative scenarios.

Table 1: Performance with different penalty, price, and demand

Demand	x	(Q_1^{x*}, Q_2^{x*})		$\Pi_{S_1}^{x*}$		$\Pi_{S_2}^{x*}$		Π_A^{x*}		$\sum_y \Pi_y^{x*}$	
		Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
N(40,1.5)	d	(39.5,56.1)	(39.8,60.1)	79.00	39.80	32.46	54.15	184.05	162.60	295.51	256.54
	d'	(40.6,70.9)	(40.5,64.0)	81.20	40.50	70.90	96.00	175.54	128.07	327.64	264.57
	c	(41.3,82.0)	(41.0,78.8)	/						333.63	274.89
N(35,2)	d	(34.3,48.6)	(34.8,52.9)	68.60	34.80	24.44	42.23	147.40	134.52	240.44	211.55
	d'	(35.8,62.8)	(35.6,56.5)	71.60	35.60	62.80	84.75	139.21	98.96	273.61	219.31
	c	(36.7,73.4)	(36.5,70.1)	/						279.39	229.05
N(45,1)	d	(44.6,62.6)	(44.9,68.4)	89.20	44.90	40.10	63.72	213.29	191.80	342.59	300.42
	d'	(45.4,79.4)	(45.3,71.7)	90.80	45.30	79.40	107.56	208.46	153.89	378.66	306.75
	c	(45.8,92.0)	(45.7,88.2)	/						384.89	318.10

As observed from the above table, we have $\sum_y \Pi_y^{d*} < \sum_y \Pi_y^{d'*} < \Pi_B^{c*}$, which demonstrates that the decentralized systems generate incoordination, and the decentralized system only with ordering mode performs better than the one with dual supply modes. Specifically, both the key and the matching components' optimal input quantity in the decentralized systems are less than those in the centralized system, i.e., $Q_1^{d*} < Q_1^{d'*} < Q_1^{c*}$ and $Q_2^{d*} < Q_2^{d'*} < Q_2^{c*}$, which directly makes the performance of the decentralized systems be worse than the centralized system. Besides, there exist $\Pi_{S_1}^{d'*} > \Pi_{S_1}^{d'*}$, $\Pi_{S_2}^{d'*} > \Pi_{S_2}^{d'*}$, and $\Pi_{S_2}^{d'*} < \Pi_{S_2}^{d*}$. As such, for the suppliers, they prefer to providing components with the ordering mode. Oppositely, under the decentralized situation, to maximize his own profit, the assembler will certainly choose the VMI mode with the supplier who generates yield uncertainty. It illustrates that VMI is widely applied in practice.

4.2 Sensitivity analysis on λ_i

In the previous subsection, we studied that the decentralized systems perform worse than the centralized system. Now we show that the advance payment contract can always coordinate the decentralized supply chain move as in the centralized system.

Example 2 Suppose we have the following data: $c_1 = 4, \quad w_1 = 6, \quad c_2 = 3, \quad w_2 = 8, \quad \beta_2 = 4, \quad p = 25, \quad \beta = 15; \quad D \backsim N(40, 1.5), \quad \theta \backsim U(0, 1)$.

Let λ_1 take the value uniformly distributed over $\left[\frac{\Pi_{S_1}^{d*}}{\Pi_B^{c*}}, 1 - \frac{\Pi_A^{d*}+\Pi_{S_2}^{d*}}{\Pi_B^{c*}}\right]$ and λ_2 take the value of $\left[\left(1 - \lambda_1 - \frac{\Pi_A^{d*}}{\pi_B^{c*}}\right) + \left(\frac{\Pi_{S_2}^{d*}}{\Pi_B^{c*}}\right)\right]/2$, which is in $\left[\frac{\Pi_{S_2}^{d*}}{\Pi_B^{c*}}, 1 - \frac{\Pi_A^{d*}+\Pi_{S_1}^{d*}}{\Pi_B^{c*}}\right]$. Figure 1 shows the effectiveness of the advance payment contract.

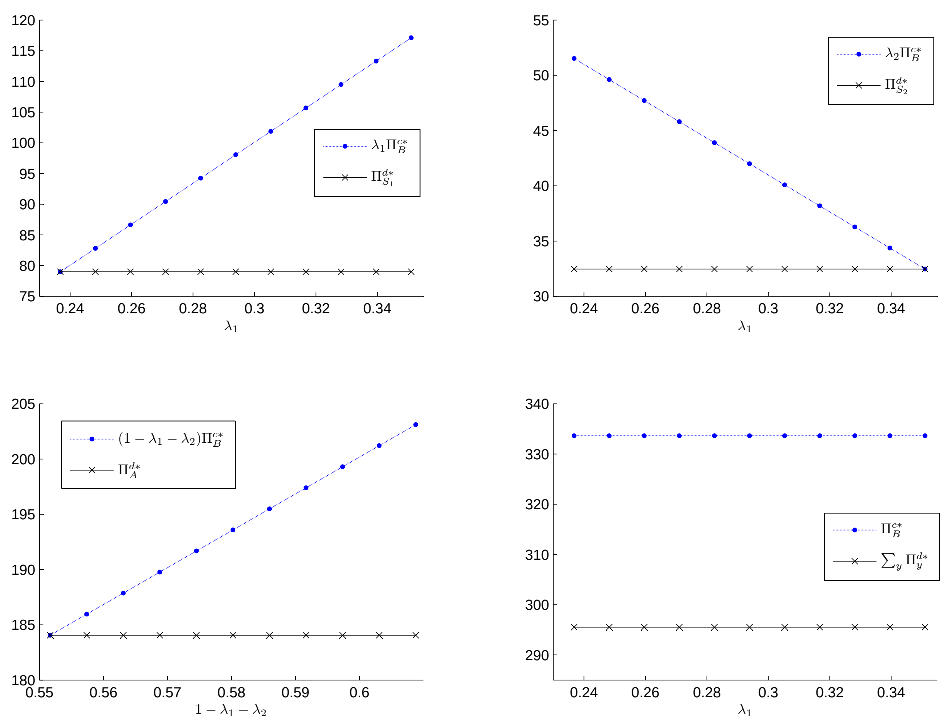

Figure 1: Effectiveness of the advance payment contract under different λ_i

In figure 1, we can see that as long as the λ_i is in its valid interval, all the suppliers and the assembler can be better off and the decentralized system can be perfectly coordinated with the advance payment contract. Therefore, it is effective and practicable.

5 Conclusion and future works

In this paper, we focus on the decision making and optimizing of the components' production input quantity in a decentralized assembly system consisting of a single assembler and two suppliers. In which, the assembler orders the key component from supplier 1 who is perfectly reliable, while supplier 2 provides the matching component under VMI mode with yield uncertainty. We analyze the key and the matching components' optimal production input quantity in a static Nash game model. Besides, a centralized system is introduced as the benchmark and an advance payment contract is proposed to achieve supply chain coordination. Furthermore, via numerical analysis, we find that (1) the less of the components' optimal production input quantity makes the decentralized systems perform worse than the centralized system, (2) the assembler prefers VMI mode with supplier 2 in the interest of maximizing his own expected profit, and (3) the proposed advance payment contract is effective and practical.

One extension of our model is to consider a more general assembly system with one assembler and N (≥ 3) suppliers, where the decisions of those suppliers with yield uncertainty can affect each other. Besides, it can be extended to other random yield model. For example, in semiconductor industry, the output of chips $Y(q)$ is a nonlinear function of the input of silicon wafers q, and other related factors. These extensions will be conducted in the future work.

Acknowledge

This work was supported by the National Natural Science Foundation of China, No.71072035.

Appendix

Proof of Lemma 1: Equation (1) can be re-written as $\Pi^d_{S_2} = w_2 A - \beta_2 B - c_2 Q_2$, wherein,

$$
\begin{aligned}
A &= \int_{\frac{Q_1}{Q_2}}^{1}\int_{Q_1}^{\infty} Q_1 f(D)g(\theta)dDd\theta + \int_{0}^{\frac{Q_1}{Q_2}}\int_{\theta Q_2}^{\infty} \theta Q_2 f(D)g(\theta)dDd\theta + \int_{0}^{Q_1}\int_{\frac{D}{Q_2}}^{1} Dg(\theta)f(D)d\theta dD \\
B &= \int_{0}^{1}\int_{\theta Q_2}^{\infty} (D-\theta Q_2)f(D)g(\theta)dDd\theta
\end{aligned}
$$

Fix Q_1, taking first- and second-order derivatives with respect to Q_2 in equation (1), we get:

$$
\begin{aligned}
\frac{\partial \Pi^d_{S_2}}{\partial Q_2} &= w_2 \int_0^{\frac{Q_1}{Q_2}} \int_{\theta Q_2}^{\infty} \theta f(D)g(\theta)dDd\theta + \beta_2 \int_0^1 \int_{\theta Q_2}^{\infty} \theta f(D)g(\theta)dDd\theta - c_2 \\
\frac{\partial^2 \Pi^d_{S_2}}{\partial Q_2^2} &= -w_2 \left\{ \int_0^{\frac{Q_1}{Q_2}} \theta^2 f(\theta Q_2)g(\theta)d\theta + \int_{Q_1}^{\infty} \frac{Q_1^2}{Q_2^3} f(D)g(\frac{Q_1}{Q_2})dD \right\} - \beta_2 \int_0^1 \theta^2 f(\theta Q_2)g(\theta)d\theta
\end{aligned}
$$

It is easy to find that $\frac{\partial^2 \Pi^d_{S_2}}{\partial Q_2^2} < 0$. Therefore, $\Pi^d_{S_2}$ is concave in Q_2. Besides, we observe that $\left.\frac{\partial \Pi^d_{S_2}}{\partial Q_2}\right|_{Q_2=0} = w_2 + \beta_2\bar{\theta} - c_2 > 0$ and $\left.\frac{\partial \Pi^d_{S_2}}{\partial Q_2}\right|_{Q_2\to\infty} = -c_2 < 0$. As such, there must exist a $Q_2 \in [0, \infty)$ meets the first-order condition $\frac{\partial \Pi^d_{S_2}}{\partial Q_2} = 0$.

Similarly, fix Q_2, taking first- and second-order derivatives with respect to Q_1 in equation (2), we get:

$$\begin{aligned}
\frac{\partial \Pi_A^d}{\partial Q_1} &= (p+\beta-w_2)\int\limits_{Q_1/Q_2}^{1}\int\limits_{Q_1}^{\infty} f(D)g(\theta)dDd\theta - w_1 \\
\frac{\partial^2 \Pi_A^d}{\partial Q_1^2} &= -(p+\beta-w_2)\left\{\int\limits_{Q_1/Q_2}^{1} f(Q_1)g(\theta)d\theta + \frac{1}{Q_2}\int\limits_{Q_1}^{\infty} f(D)g(\frac{Q_1}{Q_2})dD\right\}
\end{aligned}$$

We can easily derive $\frac{\partial^2 \Pi_A^d}{\partial Q_1^2} < 0$, $\left.\frac{\partial \Pi_A^d}{\partial Q_1}\right|_{Q_1=0} = p+\beta-w_2-w_1 > 0$ and $\left.\frac{\partial \Pi_A^d}{\partial Q_1}\right|_{Q_1\to\infty} = -w_1 < 0$. Therefore, we can claim that Π_A^d is concave in Q_1 and the optimal $Q_1^{d*}(Q_2)$ meets the first-order condition $\frac{\partial \Pi_A^d}{\partial Q_1} = 0$. □

Proof of Theorem 1: Considering the profit curves of supplier 2 and the assembler in equation (1) and (2), we let

$$N_1(Q_1,Q_2) = \frac{\partial \Pi_A^d(Q_1,Q_2)}{\partial Q_1} \qquad \text{and} \qquad N_2(Q_1,Q_2) = \frac{\partial \Pi_{S_2}^d(Q_1,Q_2)}{\partial Q_2}$$

Taking first-order derivative with respect to Q_1 in $N_1(Q_1,Q_2)$ and $N_2(Q_1,Q_2)$, we have

$$\begin{aligned}
\frac{\partial N_1(Q_1,Q_2)}{\partial Q_1} &= \frac{\partial^2 \Pi_A^d(Q_1,Q_2)}{\partial Q_1^2} \\
&= -(p+\beta-w_2)\left\{\int\limits_{Q_1/Q_2}^{1} f(Q_1)g(\theta)d\theta + \frac{1}{Q_2}\int\limits_{Q_1}^{\infty} f(D)g(\frac{Q_1}{Q_2})dD\right\} < 0 \\
\frac{\partial N_2(Q_1,Q_2)}{\partial Q_1} &= \frac{\partial^2 \Pi_{S_2}^d(Q_1,Q_2)}{\partial Q_2 \partial Q_1} = w_2\int\limits_{Q_1}^{\infty} \frac{Q_1}{Q_2^2} f(D)g(\frac{Q_1}{Q_2})dD > 0
\end{aligned}$$

Then, we get $\frac{\partial N_1(Q_1,Q_2)}{Q_1} - \frac{\partial N_2(Q_1,Q_2)}{Q_1} < 0$, for all (Q_1,Q_2). As such, the players' profit functions, $\Pi_{S_2}(Q_1,Q_2)$ and $\Pi_A(Q_1,Q_2)$, will meet once at most (Gurnani & Gerchak, 2007, Proposition 2.1). Also, referring to the Theorem 2.4 in Friedman (1986), there is always a Nash equilibrium existing for a concave-payoff-functions game like ours. Therefore, the static Nash equilibrium of both component's production input quantity is unique and the solution, (Q_1^{d*}, Q_2^{d*}), meets the combination of equation (3) and (4). □

Proof of Lemma 2: Equation (5) can be re-written as: $\Pi_B^c(Q_1, Q_2) = pC - \beta D - (c_1 Q_1 + c_2 Q_2)$, wherein,

$$C = \int_{\frac{Q_1}{Q_2}}^{1} \int_{Q_1}^{\infty} Q_1 f(D) g(\theta) dD d\theta + \int_{0}^{\frac{Q_1}{Q_2}} \int_{\theta Q_2}^{\infty} \theta Q_2 f(D) g(\theta) dD d\theta + \int_{0}^{Q_1} \int_{\frac{D}{Q_2}}^{1} D g(\theta) f(D) d\theta dD$$

$$D = \int_{\frac{Q_1}{Q_2}}^{1} \int_{Q_1}^{\infty} (D - Q_1) f(D) g(\theta) dD d\theta + \int_{0}^{\frac{Q_1}{Q_2}} \int_{\theta Q_2}^{\infty} (D - \theta Q_2) f(D) g(\theta) dD d\theta$$

Taking first- and second-order derivatives with respect to Q_1 and Q_2 in equation (5) separately, we get:

$$\frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_2} = (p + \beta) \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) dD d\theta - c_2$$

$$\frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_1} = (p + \beta) \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} f(D) g(\theta) dD d\theta - c_1$$

and

$$\frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_2^2} = -(p + \beta)(E + \frac{Q_1^2}{Q_2^3} F)$$

$$\frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1^2} = -(p + \beta)(G + \frac{1}{Q_2} F)$$

$$\frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1 \partial Q_2} = (p + \beta) \frac{Q_1}{Q_2^2} F$$

Here, $E = \int_{0}^{Q_1/Q_2} \theta^2 f(\theta Q_2) g(\theta) d\theta > 0$, $F = \int_{Q_1}^{\infty} f(D) g(\frac{Q_1}{Q_2}) dD > 0$, $G = \int_{Q_1/Q_2}^{1} f(Q_1) g(\theta) d\theta >$

0. Then, we derive the Hessen Matrix as follows:

$$H = \begin{vmatrix} \frac{\partial^2 \Pi_B^c(Q_1,Q_2)}{\partial Q_1^2} & \frac{\partial^2 \Pi_B^c(Q_1,Q_2)}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 \Pi_B^c(Q_1,Q_2)}{\partial Q_2^2} & \frac{\partial^2 \Pi_B^c(Q_1,Q_2)}{\partial Q_2 \partial Q_1} \end{vmatrix} = (p+\beta) \begin{vmatrix} -(G+\frac{1}{Q_2}F) & \frac{Q_1}{Q_2^2}F \\ \frac{Q_1}{Q_2^2}F & -(E+\frac{Q_1^2}{Q_2^3}F) \end{vmatrix} \tag{13}$$

In equation (13), the value of the first-order determinant is $-(p+\beta)(G+\frac{1}{Q_2}F) < 0$ and the value of the second-order determinant is $(p+\beta)(EG+\frac{Q_1^2}{Q_2^3}FG+\frac{1}{Q_2}EF) > 0$. As such, the Hessen Matrix is negative definite. Therefore, $\Pi_B^c(Q_1,Q_2)$ is joint concave in $Q_1 \in [0,\infty)$ and $Q_2 \in [0,\infty)$. And, the optimal production input quantity (Q_1^{c*}, Q_2^{c*}) meet the F.O.Cs, which is consisting of $\frac{\partial \Pi_B^c(Q_1,Q_2)}{\partial Q_2} = 0$ and $\frac{\partial \Pi_B^c(Q_1,Q_2)}{\partial Q_1} = 0$. □

Proof of Theorem 2: Simplify equation (4) and (7), we have:

$$\left[1-F(Q_1^{d*})\right] \times \left[1-G\left(\frac{Q_1^{d*}}{Q_2^{d*}}\right)\right] = \frac{w_1}{p+\beta-w_2} \tag{14}$$

$$[1-F(Q_1^{c*})] \times \left[1-G\left(\frac{Q_1^{c*}}{Q_2^{c*}}\right)\right] = \frac{c_1}{p+\beta} \tag{15}$$

Consider the value of $G\left(\frac{Q_1^{x*}}{Q_2^{x*}}\right)$. If $Q_1^{d*} \geq Q_2^{d*}$ or $Q_1^{c*} \geq Q_2^{c*}$, $G\left(\frac{Q_1^{x*}}{Q_2^{x*}}\right) = 1$. As such, the left parts of equation (14) and (15) both equal to 0. Meanwhile, the right part of equation (14) and (15) both are larger than 0. Thus, both equation (14) and (15) can not be balanced unless $Q_1^{x*} < Q_2^{x*}$. □

Proof of Theorem 3: Sum $\Pi_{S_1}^d$, equation (1) and (2), we get the decentralized system's total profit function:

$$\Pi_{S_1}^d + \Pi_{S_2}^d + \Pi_A^d = pE\{\min(Q_1, \theta Q_2, D)\} - (c_1 Q_1 + c_2 Q_2) - \beta E[D - \min(Q_1, \theta Q_2)]^+ \tag{16}$$

Notice that the formulation is the same as the one of the centralized system. As such, we can compare their optimal performance through the components' optimal production input quantity. Considering equation (14) and (15), we have:

$$\frac{w_1}{p+\beta-w_2} - \frac{c_1}{p+\beta} = \frac{(p+\beta)(w_1-c_1)+c_1w_2}{(p+\beta)(p+\beta-w_2)} > 0$$

As a result, we can derive that $(Q_1^{d*}, Q_2^{d*}) \neq (Q_1^{c*}, Q_2^{c*})$. Otherwise, if $Q_1^{d*} = Q_1^{c*}$ and $Q_2^{d*} = Q_2^{c*}$, equation (14) and (15) can not hold simultaneously. Then, plug (Q_1^{d*}, Q_2^{d*}) and (Q_1^{c*}, Q_2^{c*}) into the system's profit function separately, we can easily get:

$$\left(\Pi_{S_1}^d + \Pi_{S_2}^d + \Pi_A^d\right)\big|_{Q_1=Q_1^{d*}, Q_2=Q_2^{d*}} < \Pi_B^c\left(Q_1^{c*}, Q_2^{c*}\right),$$

since (Q_1^{c*}, Q_2^{c*}) is the unique optimal decisions. □

Proof of Theorem 4: Under the advance payment contract, each party's expected profit (denoted as Π_y^{co}) can be formulated as follows.

$$\Pi_{S_1}^{co} = \lambda_1 \Pi_B^{c*}, \quad \Pi_{S_2}^{co} = \lambda_2 \Pi_B^{c*}, \quad \text{and} \quad \Pi_A^{co} = \left(1 - \sum_{i=1}^{2} \lambda_i\right) \Pi_B^{c*}$$

To ensure the contract's success, there must satisfy the following constraints:

$$\Pi_y^{co} \geq \Pi_y^{d*}, \quad \forall y \in \{S_1, S_2, A\} \tag{17}$$

Substituting λ_i $(i = 1, 2)$ into equation (17) and combining with $\Pi_B^{c*} > \sum_x \Pi_x^{d*}$, we can easily derive that λ_i meets equation (8). □

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