Impact of Cournot competition and commitment contract on strategic inventory in a one manufacturer,

two-retailer supply chain

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ABSTRACT

Strategic Inventory (SI) has been studied in a supply chain with no downstream competition. This research shows Cournot competition between downstream retailers decreases SI between periods. It also shows a commitment contract increases SI in the Cournot-competing retailer supply chain while it completely eliminates SI in the monopoly downstream case.

INTRODUCTION

Inventory in a supply chain helps us cope with demand uncertainty, transportation delays and price fluctuations and such other exigencies and hence is inevitable. On the other hand, inventory incurs holding costs. Due to these conflicting effects, determining the optimal amount of inventory in a supply chain has been an important research issue in the supply chain literature. Economic competition has also been an important research area in a supply chain. Seldom does one firm dominate a market, and competition is inevitable. Research on economic competition usually models a real world case either as one of the standard competition models such as Bertrand, Cournot and Stackelberg models, or hybrid models. Research in this area has typically focused on the interaction of control and incentives in scenarios where one of the competing entities is in a stronger position than the other(s) and the design of appropriate incentive schemes to remedy this inequity.

This research considers these two issues together: inventory policy and economic competition. We aim to understand the effect of competition on Strategic Inventory (SI) carried by competing downstream retailers in a supply chain. SI are inventories carried by supply chain entities for purely strategic reasons, even in the absence of its "traditional" reasons to hold inventory (Anand et al., 2008). Traditional reasons for holding inventory at a supply chain entity (e.g. manufacturer, retailer, and

distributor) have been economies of scale in production, hedging against production or distribution delays, safety stock, hedging against price fluctuations, smooth production and lower production costs.

LITERATURE REVIEW

Prior work in economics such as Rotemberg and Saloner (1989), Mollgaard et al. (2000) and Matsumura (2002) have analyzed SI in a single-echelon inventory game among market competitors. In most economics literature, the producer himself has been the seller in the market. Anand et al. (2008) present a two-period as well as an n-period supply chain model consisting of a manufacturer supplying a single downstream retailer at a different wholesale price in each period. They consider a price-dependent linear demand function as well as arbitrary end-customer demand function and present a detailed analysis on the presence and effect of SI held by downstream retailers.

   Krishnan and Winter (2007) propose a scheme of joint price and inventory control in a one-manufacturer, two-retailer supply chain where the downstream retailers compete as differentiated duopolists under uncertain demand. They find that a combination of a buy-back contract with a resale price ceiling maximizes joint profit. Keskinocak et al. (2008) extend the two-period model from Anand et al. (2008) to the case where the manufacturer's first period capacity is limited. Zhang et al. (2008) extend the same two-period model of Anand et al. (2008) to the case of asymmetric information n-period model.   Desai et al. (2010) focus on the effects of forward buying by retailers and prove that the motivations for forward buying are more complex than just manufacturer trade promotions. They find that retailer stock piling helps the retailer but hurts the manufacturer, a view shared by Anand et al. (2008) who subsequently develop coordinating contracts to remedy this inequality. This work adds to the model considered by Anand et al. (2008) the possibility of manufacturer trade promotions as well as uncertainty of demand and find that, regardless of whether the manufacturer offers a trade promotion,

forward buying can be beneficial to both the retailer and the manufacturer. Also, they find that, in the case of uncertain demand, strategic forward buying is encouraged and that the retailer orders more than they expect to sell even in the most optimistic demand scenario. This work further bolsters evidence that inventory can play a strategic role in the supply chain in interesting ways that merit more careful research and analysis.

Viswanathan and Jang (2009, 2010) extend the work of Anand et al. (2008) to the supply chain with a competitive downstream retailers. Viswanathan and Jang (2009) presents a basic two-period model of a manufacturer supplying to a Cournot duopoly competing retailer at its downstream. Viswanathan and Jang (2010) presents a similar model but with the downstream retailers competing as part of a leader-follower (Stackelberg) duopoly arrangement. Table 1 summarises the literature discussed here on SI in a supply chain.

In this paper, we present a further refinement of the two-period model presented in Viswanathan and Jang (2009) that considers more real world constraints while deriving the equilibrium decisions as well as provides a more rigorous analysis of the two-period Cournot duopoly supply chain model. We also analyze the impact of a commitment contract on SI.

Table 1. Literature on SI in a supply chain

Author (year)	Vertical control Present?	Competititve downstream present?	Type of demand functions considered	Number of periods	Coordinating contracts developed?
Krishnan and Winter (2007)	Yes	Yes	Uncertain demand	Single period	Yes
Anand et al. (2008)	Yes	No	Price-dependent linear, general	2 period, n-period	Yes
Keskinocak (2008)	Yes	No	Price-dependent linear	2-period	Yes
Desai et al. (2010)	Yes	Yes, two manufacturer one retailer and one manufacturer two retailer configurations	Uncertain, price-dependent linear, depending on only the particular retailer's prior retail price	2-period	No
Viswanathan and Jang (2009)	Yes	Yes, Cournot competing downstream duopoly	Price-dependent linear, depending on the total quantity on sale in the market in the given period	2-period	No
Viswanathan and Jang (2010)	Yes	Yes, Stackelberg Competing downstream duopoly	Price-dependent linear, depending on the total quantity on sale in the market in the given period	2-period	No

The MODEL

The model consists of one manufacturer supplying a product to two competing downstream retailers. The two downstream retailers form a Cournot dupoly, i.e, they compete on quantity. The demand is assumed to be price-dependent linear: $P(Q) = a-bQ$, where Q is the total quantity on the market (the quantity that both retailers together put out for sale in a given period) and a and b are exogenous model parameters. The retailers are allowed to carry from the 1st period to the 2nd period at a holding cost of h per unit. We model this case as a two period dynamic game. The players' decisions are made as follows:

PERIOD 1

- Manufacturer sets the 1st period wholesale price (w_1).

- Retailers decide their 1st period order quantity, which is the sum of the quantities they want to sell in the 1st period (q_{11} and q_{12} respectively) and the inventory to be carried to the next period, I. (Each retailer carries the same inventory, and the total inventory of the supply chain is $2I$.)

- The first selling season begins, and the product is sold to the end customer at the unit price of $P(Q) = \text{a-b}Q$, where $Q = q_{11} + q_{12}$.

- Each retailer carries inventory to the 2nd period.

PERIOD 2

- Manufacturer sets the 2nd period wholesale price (w_2).

- Retailers decide their 2nd period order quantity, q_{21} and q_{22} respectively.

- The second selling season begins. The retailers sell $q_{21}+I$ and $q_{22}+I$ units in the 2nd period, respectively.

- The products are sold at the price of $P(Q) = \text{a-b}Q$ where $Q = q_{21} + q_{22} + 2I$.

ASSUMPTIONS

This model assume the following:

- Both competing downstream retailers are identical.

- The holding costs of both retailers are the same ($h_1=h_2=h$), and hence they carry the same inventory, I.

- All players are rational and act to maximize their own profits

We use the backward induction procedure to determine the equilbrium quantities and prices, i.e., start with the final decision in the last period and roll back to the 1st period manufacturer decision.

PROFIT FUNCTION FORMULATION

In this section, we present the formulation of the problem for obtaining manufacturer's 2nd period decision (2nd period wholesale price - w_2) and retailer's 1st period decisions (q_{11} and I). These are intended to give the reader an overview of how manufacturers' and retailers' decisions are modeled. The detailed calculations and sequence of decisions are available from the authors.

Manufacturer's 2nd period decision

Given (assumptions) : *a, b, h >0, I ≥ 0, a -2 bI ≥ 0*

Decision variable: w_2

Requirement (constraints) for the decision variables: *0≤w₂≤a*

Set w_2 to maximize the manufacturer's 2nd period profit $\Pi M_2 = w_2(q_{21} + q_{22})$.

Retailer 1's 1st period decisions

Given: (assumptions): *a, b, h >0, 0≤ w₁+h< a, q₁₂≥0*

Decision variables: *q₁₁, I*

Requirement (constraints) : $q_{11} \geq 0$, $a-b\,(q_{11}+q_{12}) \geq 0$, $I \geq 0$, $a-2bI \geq 0$.

The 1st period profit function for retailer 1 is

$$\Pi R_{11} = (a-b(q_{11}+q_{12}))(q_{11})-w_1(q_{11}+I)-hI$$

Retailer 1's 1st period problem is to maximize the sum of 1st and 2nd period profits, where the 1st period profit is given in the above profit function and the 2nd period optimal profit is rolled back from the optimal 2nd period decisions. The equilibrium quantities are:

1st period decisions

$$w^*_1 = \frac{105\,a-96\,h}{176}, \quad q^*_{11}=q^*_{12}=\frac{a-w_1}{3b}=\frac{71\,a+96\,h}{528\,b}, \quad I^* = \frac{4a}{15b}-\frac{2}{5b}(w_1+h)=\frac{37\,a-240\,h}{1320\,b}$$

2nd period decisions

$$w^*_2 = \frac{a}{10}+\frac{3}{5}(w_1+h)=\frac{403\,a+240\,h}{880}, \quad q^*_{21}=q^*_{22}=\frac{a}{5b}-\frac{1}{5b}(w_1+h)=\frac{71\,a+80\,h}{880\,b}$$

EFFECT OF COURNOT COMPETITION ON STRATEGIC INVENTORY

Table 2 compares the results of this model to the two-period one manufacturer, one-retailer model of Anand et al. (2008). In the table, Q_1 and Q_2 are the total purchase quantities $Q_1 = q_{11} + I$ and $Q_2 = q_{12} + I$. Also, p_1 and p_2 are the retail prices in period 1 and 2: $p_1 = a-bQ_1$ and $p_2 = a-bQ_2$.

Table 2. Comparison of the dynamic two-period Cournot and the two-period dynamic model without competition

Equilibrium quantity	Two-period Cournot Model results from Section 3.4	Two-period dynamic model without competition (Anand et al., 2008),
Wholesale prices (w_1, w_2)	$\left\{\dfrac{105\,a-96\,h}{176}\,,\,\dfrac{403\,a+240\,h}{880}\right\}$	$\left\{\dfrac{9\,a-2\,h}{17}\,,\,\dfrac{6\,a+10\,h}{17}\right\}$
Purchase Quantities (Q_1, Q_2)	$\left\{\dfrac{503\,a-480\,h}{2640b}\,,\,\dfrac{71\,a+80\,h}{440b}\right\}$	$\left\{\dfrac{13\,a-18\,h}{34\,b}\,,\,\dfrac{3\,a+5\,h}{17\,b}\right\}$
Total Channel Inventory (I)	$\dfrac{37\,a-240\,h}{660\,b}$	$\dfrac{5(a-4\,h)}{34\,b}$
Total Manufacturer Profit ($\Pi M_1 + \Pi M_2$)	$\dfrac{0.1719\,a^2-0.09256\,a\,h+0.1487\,h^2}{b}$	$\dfrac{9\,a^2-4\,a\,h+8\,h^2}{34\,b}$
Sales Quantities (q_1, q_2)	$\left\{\dfrac{211\,a+48\,h}{264\,b}\,,\,\dfrac{287\,a-240\,h}{1320\,b}\right\}$	$\left\{\dfrac{4a+h}{17b}\,,\,\dfrac{11\,a-10\,h}{34\,b}\right\}$
Retail Prices (p_1,p_2)	$\left\{\dfrac{193\,a-96\,h}{264}\,,\,\dfrac{1033\,a+240\,h}{1320}\right\}$	$\left\{\dfrac{13a-h}{17}\,,\,\dfrac{23\,a+10\,h}{34}\right\}$
Total Retailer Profit	$\dfrac{1.2475\,a^2-0.6508\,a\,h-0.8262\,h^2}{b}$	$\dfrac{155\,a^2-118\,a\,h-304\,h^2}{1156\,b}$
Total Supply Chain Profit	$\dfrac{1.4194\,a^2-0.7433\,a\,h-0.6775\,h^2}{b}$	$\dfrac{0.398\,a^2-0.2197\,a\,h-0027\,h^2}{b}$

From Column 2 and Column 3 of Table 2, we make the following observations about the effect of Cournot competition on SI. It should be noted here that the reservation price (model parameter "a"), which is the maximum price that an end customer is willing to pay for the product is usually much greater than the holding cost per unit (a >> b) and this holds for most goods.

COMPARISON OF WHOLESALE AND RETAIL PRICES

- 1st period wholesale price is mostly greater in Cournot competition than without (when $a > 6.1h$, which mostly holds)

- 2nd period wholesale price is mostly greater in Cournot competition than without. (when $a > 3.1h$, which mostly holds)

- The 1st period retail price is always lower in Cournot competition than without.

- 2nd period retail price is always higher in Cournot competition than without.

COMPARISON OF PURCHASE AND SALE QUANTITIES

- 1st period total purchase quantity is mostly lesser in Cournot competition than without (when $a > 2.61h$, which mostly holds).

- 2nd period total purchase quantity is always lesser in Cournot competition than without

- The 1st period sales quantity is always higher in Cournot competition than without

- The 2nd period sales quantity is mostly lower in Cournot competition than without (when $a > 1.04h$, which mostly holds)

COMPARISON OF TOTAL CHANNEL INVENTORY

- Total channel inventory is always lesser in Cournot competition than without.(when $a > -8.66h$, which always holds)

COMPARISON OF TOTAL PROFITS

- Manufacturer's total profit is lower under Cournot competition (when $a >> h$, which usually

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holds).

- Total retailer profit is higher under Cournot competition (when $a \gg h$, which usually holds).

EFFECT OF COMMITMENT CONTRACT ON STRATEGIC INVENTORY:

Anand et al. (2008) find that a commitment contract completely eliminates SI in a two-period model with one manufacturer supplying to a single downstream retailer. Under a commitment contract, the manufacturer announces the wholesale price for the 1st and 2nd period at the beginning of the 1st period. In this case, the uncertainty over the 2nd period wholesale price is absent (for the retailers) and hence there is intuitively less incentive for SI . This is also an example of a contractual structure that completely eliminates SI in the case without downstream retailer competition. In this section, we study the effect of this contract on SI in the case with downstream retailer competition. We consider the same model as in the previous section except for that w_1 and w_2 are announced at the beginning of the first period. The results are as follows.

$$w^*_1 = w^*_2 = \frac{a}{2} , q^*_{11} = q^*_{12} = \frac{a-w_1}{3b} = \frac{a}{6b} , q^*_{21} = q^*_{22} = 0, I^* = \frac{a-w_2}{3b} = \frac{a}{6b}$$

In Table 3, we present a comparison of the equilibrium decision variable values of the Cournot duopoly downstream model under a commitment contract with those of the one-manufacturer, one-retailer model under a commitment contract.

Table 3. Comparison of Cournot duopoly commitment contract and monopoly commitment contract

Equilibrium quantity	Two-period commitment contract without competition	Two-period commitment contract model under a Cournot duopoly
Wholesale prices (w_1, w_2)	$\{\dfrac{a}{2}, \dfrac{a}{2}\}$	$\{\dfrac{a}{2}, \dfrac{a}{2}\}$
Purchase Quantities (Q_1, Q_2)	$\{\dfrac{a}{4b}, \dfrac{a}{4b}\}$	$\{\dfrac{2a}{3b}, 0\}$
Total Channel Inventory (I)	0	$\dfrac{a}{3b}$
Retail Prices $P(Q) = a - bQ$	$\{\dfrac{3a}{4}, \dfrac{3a}{4}\}$	$\{\dfrac{2a}{3}, \dfrac{2a}{3}\}$
Sale Quantities	$\{\dfrac{a}{4b}, \dfrac{a}{4b}\}$	$\{\dfrac{a}{3b}, \dfrac{a}{3b}\}$

From the above table and the results from our commitment contract model with Cournot duopoly downstream competition, we observe the following:

COMPARISON OF WHOLESALE AND RETAIL PRICES

– The manufacturer wholesale prices are the same.

– Retail prices in either period are lower under a commitment contract with competitionthan without.

COMPARISON OF PURCHASE AND SALE QUANTITIES

– The first period total purchase quantity including inventory to be carried is higher under Cournot competition compared to a monopoly. No second period ordering occurs in the Cournot duopoly case under commitment contract.

– First and second period total sale quantities are higher under a commitment contract in a cournot duopoly than under a commitment contract supplying to a single retailer downstream.

COMPARISON OF TOTAL CHANNEL INVENTORY

SI is higher under a commitment contract in the presence of a Cournot duopoly downstream than when

a manufacturer supplies to a single retailer downstream.

In summary, we can say that competition has the effect of increasing SI , and the commitment contract is not sufficient to deter this.We can also say that competition encourages higher first period ordering, as well as a higher level of SI into the 2nd period. The strategic inventory carried is high enough so that no second period ordering is needed.

In Table 4, we present a comparison of the equilibrium quantities in the model without (previous section) and with a commitment contract (this section). This gives us an insight into the effect a commitment contract has in the presence of downstream retailer competition.

Table 4. Comparison of Cournot duopoly model with and without commitment contract

Equilibrium Quantity	Cournot duopoly downstream without commitment contract	Cournot duopoly downstream with commitment contract
Wholesale prices (w_1, w_2)	$\dfrac{105\,a-96\,h}{176}$, $\dfrac{403\,a+240\,h}{880}$	$\{\dfrac{a}{2}, \dfrac{a}{2}\}$
Purchase Quantities (Q_1, Q_2)	$\{\dfrac{503\,a-480\,h}{2640b}, \dfrac{71\,a+80\,h}{440b}\}$	$\{\dfrac{2a}{3b}, 0\}$
Total Channel Inventory (I)		
Retail Prices P(Q) = a-bQ	$\{\dfrac{193\,a-96\,h}{264}, \dfrac{1033\,a+240\,h}{1320}\}$	$\{\dfrac{2a}{3}, \dfrac{2a}{3}\}$
Total manufacturer profit		
Total retailer profit		
Total supply chain profit		

From Table 4, we make the following observations on the effect of a commitment contract on strategic inventory in a cournot duopoly downstream supply chain.

COMPARISON OF WHOLESALE AND RETAIL PRICES

- Manufacturer's 1st period wholesale price (w_1) is lesser in commitment contract than without

(when $a > 5.6h$, which mostly holds).

- Manufacturer's 2nd period wholesale price (w_2) is greater in commitment contract than without (when $a > 6.477\ h$, which mostly holds).

- 1st period retail price is lesser in a commitment contract (when $a > 5.71h$ which mostly holds) 2nd period retail price is always lesser in commitment contract.

COMPARISON OF PURCHASE AND SALE QUANTITIES

- Retailers' second period selling quantities are always lesser in commitment contract than without.

- Retailers' first period selling quantities are greater in a commitment contract than without.

COMPARISON OF TOTAL CHANNEL INVENTORY

- Total channel inventory is always greater in commitment contract than without (when $a > -1.317\ h$, which always holds).

COMPARISON OF MANUFACTURER AND RETAILER PROFITS

- Total manufacturer profit is greater under a commitment contract than without (when $a \gg h$, which usually holds).

- Total retailer profit is lesser under a commitment contract than without (when $a \gg h$, which usually holds).

    In summary, in a commitment contract case with

Cournot duopoly down stream, the manufacturer sets a lower first period wholesale price and higher second period wholesale price in equilibrium than in the same case without a commitment contract. This in turn leads to the retailers ordering more in the first period and less in the 2nd period. This also leads to a higher first period sale quantity as well as higher SI carried. Manufacturer profit is higher

but retailers total profit (sum of profit of both downstream retailers over the two selling seasons) is lesser under commitment contract.

CONCLUSIONS AND FUTURE WORK

Total channel SI is lower when a manufacturer supplies to a Cournot duopoly downstream over two periods compared to a manufacturer supplying to a single retailer downstream. A commitment contract does not have the same effect under competition as it does in the absence of competition with respect to SI. More SI is carried under a commitment contract with a Cournot duopoly downstream, both compared to a monopoly downstream, as well as a Cournot duopoly downstream without a commitment contract. It shows that the commitment contract does not have the effect of completely eliminating SI , as in the monopoly downstream case, but rather only serves to increase SI . The supply chain with the Cournot duopoly downstream is actually better off, in terms of SI without a commitment contract. Manufacturer's total profit is lower while supplying to a Cournot duopoly downstream than a monopoly downstream over the two selling seasons. However, under a commitment contract, the manufacturer's total profit is higher supplying to the same Cournot duopoly downstream than without the contract. We can hence say that the imposition of a commitment contract benefits the manufacturer in the presence of competing downstream retailers.

Retailers total profits are higher in a Cournot duopoly downstream (total profit of both retailers over the two selling seasons) than compared to the profit of a single monopoly retailer (Anand et al. (2008)), implying that Cournot competition benefits the retailers. However the total profit under this same arrangement is lower in the presence of a commitment contract by the manufacturer than without one. We can hence infer that though retailers are benefited in a competitive environment, That benefit is diminished with the introduction of a commitment contract by the manufacturer. So, we see that a commitment contract results in both higher SI and reduced retailer profits. Total supply chain profit is

higher in the supply chain consisting of a Cournot duopoly downstream compared to a monopoly downstream, implying that competition leads to increased total supply chain profits. Total supply chain profit of a Cournot duopoly downstream supply chain under commitment contract is lesser than in the case without the contract, implying that a commitment contract makes the Cournot duopoly downstream supply chain worse off.

The development of an effective contractual structure for a Cournot duopoly downstream supply chain that mitigates SI is left for future research.

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