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# Abstract Title: Supplier Facilitated Transshipments in a Multi-Retailer Distribution System

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## Abstract

We study the role of inventory transshipment for achieving supply chain coordination in a multi-retailer distribution system. The previous transshipment literature focuses on coordination via retailer-negotiated transshipments, which requires a certain degree of cooperation among the independent retailers. We find that to achieve coordination in a system with non-cooperative retailers, including the supplier in the transshipment system is critical. We thus propose and examine a new coordination scheme: supplier-facilitated transshipments. We study a two-period production and inventory distribution problem in which transshipments among the retailers occur at the start of second period. These transshipments are implemented through a bi-directional adjustment contract, offered by the supplier to each individual retailer. Under this contract, in the second period, each retailer can either buy additional inventory from the supplier or sell back excess inventory to the supplier. We show that a properly designed adjustment contract, along with a wholesale contract with appropriately chosen wholesale price, can be used to achieve coordination, with guaranteed participation from all parties in the system. In developing the coordination scheme, we observe two types of incentive conflict: one associated with vertical inventory flow and another associated with horizontal inventory flow. While the double marginalization associated with vertical inventory flow is known to exist in decentralized supply chains with multiple stages, the horizontal incentive conflict exists only in supply chains with multiple retailers, and has not been previously studied in the literature. We show that this horizontal incentive conflict cannot be removed through the use of any coordination mechanism from the existing literature. Instead, a unique state-dependent adjustment contract (or transshipment) price, where the state is the system inventory level, is required to coordinate the single supplier, multiple retailer system.

*Keywords: Inventory transshipment, supply chain coordination*

# 1  Introduction and Motivation

Manufacturers of specialized products often distribute their goods through independent retailer networks. For example, apparel manufacturers, such as Benetton and Mango, and large equipment manufacturers, such as Caterpillar, generally rely on exclusive retailer networks to distribute their products and, in the latter case, to provide after-sales service. In these networks, the retailers are independently-owned and operated, under a contractual arrangement with the manufacturer, and often have exclusive rights to serve a given geographical region. The benefits of such exclusive distribution to the manufacturer include stronger selling effort on the part of the retailer, as well as greater control over pricing, image and customer service (Kotler and Armstrong 1997).

For a supplier who distributes his products through such a retailer network, the success of the manufacturer becomes intrinsically linked to the success of the retailers. The retailers serve as the "face" of the manufacturer, interacting with customers and providing technical support and customer service. Thus, it is not uncommon for suppliers to introduce programs and technology that assist the retailers in performing key tasks, such as inventory control and after-sales service. As Fites (1996) notes, manufacturers understand the importance of their partnerships with the retailers in their distribution networks and are continually searching for ways to assist their retailers, with the belief that such initiatives will have long-term benefits for the manufacturer. For example, as discussed in Zhao et al. (2005), manufacturers such as Caterpillar have provided their dealers with inventory-sharing information systems that enable an individual dealer to search the dealer network to find a needed part or product. Some manufacturers also provide their dealers with monetary incentives to encourage the sharing of inventory in the network. Such supplier-facilitated inventory sharing programs serve as a form of risk pooling and thus allow the retailers to reduce the amount of safety stock on-hand, while maintaining high levels of service. These programs also benefit the manufacturer by providing the end-customer with improved service. However, the best way to manage these programs, i.e., the

best way to encourage transshipments in the retailer network, remains an open question.

There is a vast amount of previous research on inventory transshipment, most of which assumes that the retailers belong to a centralized system. The literature on inventory transshipment in decentralized systems is more limited and has focused on coordination via *retailer-negotiated* transshipments. Here, coordination usually means profit maximization across the retailers, treated as whole; the supplier is generally excluded from consideration. Two types of schemes, coordinating transshipment prices and coordinating profit allocation rules, have been proposed and studied using single-period models where transshipments occur only at the end of the period. However, in both schemes, a certain degree of cooperation from the retailers is required to achieve coordination.

In contrast, inspired by the transshipment practice at Comicfans Culture, Ltd., the largest Chinese publisher for animation and caricature journals, as described below, we study coordination in systems with *supplier-facilitated* transshipments, using a more realistic two-period model in which transshipments occur at the end of the first period (or, equivalently, in the middle of the selling season). Unlike transshipments in single-period models, which are purely reactive, transshipments in our two-period model are both proactive and reactive, occurring both before and after demand is observed. In addition, coordination in our model has an additional source of complexity due to the inclusion of the supplier (and his production decisions for both periods) in the model. We find that coordination for non-cooperative independent retailers can be attained through appropriately designed contracts.

## 1.1 Motivating Example

Comicfans Culture, Ltd. (http://www.comicfans.net/mywh/companyen.htm) was founded in 1997 and has annual sales of more than 40 million animation books and journals through about 350 wholesalers, who have about 30,000 newsvendors. Private interviews with the VP for sales, marketing and advertisement, reveal that Comicfans has been facilitating transshipments among their wholesalers

for at least 8 years. The selling season for a comic book title is usually two to three weeks and less than a week for weekly magazines. Before the selling season starts, Comicfans determines the retail price and the total production quantity to be produced prior to the start of the season at a given production cost. This production is followed by the wholesalers' orders, at prices ranging from 55-65% of the retail price. The selling season then begins. In the middle of the selling season, i.e., after a week to ten days, some wholesalers will typically approach Comicfans with additional orders, to be procured at a price equal to the wholesale price they paid for orders placed prior to the start of the selling season. At the same time, Comicfans contacts the other wholesalers and determines whether any of them would be interested in returning products. These wholesalers would be paid a price equal to the wholesale price they paid for orders placed prior to the start of the selling season. Any returns are used to satisfy the midseason orders. If the returns are insufficient, Comicfans will produce to fill the gap. At the end of the selling season, the wholesalers can return leftover inventory to Comicfans, up to a certain limit, at full price. However, if such end-of-season returns are made, the contract between Comicfans and their wholesalers dictates that the price for that wholesaler's next purchase may increase. In this way, Comicfans discourages returns at the end of the selling season.

The wholesalers are located in different provinces and belong to different systems, e.g., government-owned bookstores and private book markets of different scales. Due to distance, a lack of trust, and the complexities associated with contacting many different wholesalers, they generally do not contact each other directly to arrange for transshipments, instead relying on Comicfans to facilitate the exchange of goods between them. Such supplier-facilitated transshipment has a number of practical advantages over retailer-negotiated transshipment. In retailer-negotiated transshipment, the retailers must arrange for, and negotiate, transshipments among themselves. However, such a retailer-initiated process is often difficult, or even infeasible, to manage. It thus becomes beneficial for the manufacturer to facilitate a higher level of transshipment than what would occur "naturally" through intra-retailer

negotiation. This is particularly true when the retailers are geographically-distributed.

## 1.2  Problem Statement

We study the role played by the manufacturer in encouraging and facilitating inventory sharing in an exclusive retailer network, consisting of multiple independently-owned and operated retailers. As Anupindi et al. (2001) note, there are two necessary components for the implementation of transshipments in such a system. First, information systems are required to track excess stock/demand across the retailers. Second, the independent retailers must be willing to participate in the transshipment system, which "depends on the existence of an associated process for allocating the relevant costs and revenues in a way that is consistent with the self-interests of the relevant parties." Under the assumption that the necessary information system is already in place, we consider the second issue, i.e., the problem of designing incentive schemes to encourage retailer participation in the transshipment system. We do so under the assumption that the supplier has made a conscious decision to implement such a system, with the goal of increasing his own profit by improving retailer performance, and that the supplier will be an active participant in the system. The mechanism we propose is a *bi-directional adjustment contract*, in which the retailers are able to either purchase extra stock from or sell excess stock back to the manufacturer, as needed. Any product returned to the manufacturer can be reallocated to retailers in need of additional stock. Thus, with appropriately chosen buying and selling prices, an adjustment contract is one way in which a manufacturer can implement inventory transshipment, while allowing for independent decision-making by the retailers. Finally, we note that although the adjustment contracts are signed between the supplier and each retailer, when a transshipment is needed, the physical movement of the inventory takes place between the retailers.

Specifically, we study a two-period problem for a distribution system consisting of a single supplier and multiple symmetric retailers. The supplier can produce at the start of each period. In the first

period, the retailers receive inventory from the supplier, which is used to fill independent customer demands. In the second period, inventory is redistributed between the various locations, i.e., between the retailers and the supplier, as needed. We study centralized and decentralized versions of the system, where the optimal production and inventory distribution policy for the centralized system serves as a benchmark for the design of a coordination mechanism in the decentralized system.

In the decentralized system, we model the relationship between the supplier and retailers using a Stackelberg game, in which the supplier is the leader, and the relationship among the retailers using a Nash game. Prior to the start of the planning horizon, the supplier offers two contracts to the retailers, a wholesale price contract for period 1 and an adjustment contract for period 2. Under the adjustment contract, at the start of the period 2, each retailer can procure inventory from the supplier at a specified buying price or can sell inventory back to the supplier at a specified selling price. We allow the contract prices to be state-dependent and assume forced compliance for the supplier, i.e., the supplier is required to satisfy the retailers' orders and adjustments. Given the contract prices, the retailers simultaneously determine their optimal ordering and adjustment quantities, arriving at a Nash equilibrium (NE), if one exists. Given this NE, the supplier, as the leader of the game, determines the optimal contract parameters to maximize his own profit. We assume information symmetry, i.e., all parties have complete information about the supply chain as a whole.

The Comicfans example fits our problem setting quite well. A manufacturer (Comicfans) of a specialized product (animation journals) distributes that product through a network of independent, geographically-dispersed firms (the wholesalers). The product is sold for just a single selling season, with the wholesalers making independent ordering decisions prior to the start of the selling season. In addition, the manufacturer seeks to encourage and facilitate inventory transshipment by accepting additional orders and product returns halfway through the selling season. Production is possible both prior to the selling season and at the midpoint of the season. Production capacity is generally

not a concern. However, while Comicfans allows mid-season purchases and returns to be placed at the same wholesale price charged at the start of the selling season, in this paper we investigate how the manufacturer can *optimally* determine the adjustment (purchasing and return) prices, as well as the pre-season wholesale price, with the goal of maximizing his own profit and achieving supply chain coordination. To demonstrate the value, to the manufacturer, of our proposed coordination method, in our numerical study we compare the performance of the Comicfans' approach (with a single wholesale price used for all purchases and returns) with the optimal adjustment and wholesale prices. We demonstrate that the Comicfans' approach can perform quite poorly, while using the coordinating contract prices can substantially improve the system profit.

## 1.3 Literature Review

Most previous research on transshipment in a single supplier, multi-retailer setting assumes that the locations belong to a centralized system. However, in practice the locations are often independent. In discussing the recent work on inventory transshipment in such decentralized systems, we follow the framework used in Shao et al. (2010), who distinguish between systems which are horizontally decentralized (the retailers are independent of each other), those that are vertically decentralized (the retailers are independent of the supplier), and those that are both vertically and horizontally decentralized. Horizontally decentralized systems can be further categorized according to the extent of cooperation between the retailers, and according to the method of profit allocation assumed: transshipment prices (TP) or profit allocation (PA). TP must be calculated prior to observing demand while PA is calculated after observing demand, and thus depends on the state of the system.

The work on transshipments in horizontally decentralized systems includes Rudi et al. (2001), who were among the first to consider inventory transshipment in a decentralized system. They consider a single-period model consisting of two retailers, with transshipment of excess stock at the end of the

period (after demand is realized, but before demand is filled). The transshipment stage is assumed to be completely cooperative. The authors demonstrate that there exists a unique Nash equilibrium for the order quantities and characterize the coordinating transshipment prices. Hu et al. (2007), consider a similar model and demonstrate that coordinating transshipment prices do not always exist.

Several other researchers have considered trannsshipment in horizontally decentralized systems with partial cooperation between the retailers. In this case, PA is generally used to allocate the profits generated by transshipment. For example, Anupindi et al. (2001) consider a single-period decentralized system with $N$ retailers and a central inventory location. The authors characterize a fractional allocation rule for the profit earned from transshipment, under which the allocations are in the core. Granot and Sosic (2003) and Sosic (2006) extend this work to allow the retailers to decide (non-cooperatively) how much of their excess supply or demand to share with other retailers. Huang and Sosic (2010) compare the two methods of profit allocation: TP (with complete cooperation) and PA (with partial cooperation). The authors propose a heuristic, which combines the properties of TP and PA, in which transshipments are coordinated by a neutral central depot.

Yan and Zhao (2008) and Hanany et al. (2010) consider a horizontally decentralized system with non-cooperative the retailers. However, both papers require the assumption of a neutral third party to achieve coordination. While these papers, like Huang and Sosic (2010), consider a third party who facilitates the transshipment between the retailers, they do not seek to model the third party's profit, or to coordinate the system as a whole, including the third party. In particular, they do not consider the question of whether the third party would be willing to act as the coordination mechanism. In contrast, we consider a single supplier, multi-retailer system in which the supplier, as Stackelberg leader, facilitates transshipments between the retailers and thus is an integral part of the system.

Dong and Rudi (2004) study transshipment in a vertically decentralized newsvendor setting. Unlike most previous work, the authors explicitly model the supplier's wholesale pricing decision. Shao

et al. (2010) also consider a newsvendor model in which the wholesale price charged by the supplier to the retailers is set endogenously. They compare a fully decentralized system, in which the supplier and all retailers are independently-owned, and the transshipment price is set by the retailers, with a system in which the retailers are jointly-owned, and thus the transshipment price plays no role. Thus, Shao et al. (2010) were among the first to consider transshipments in a system which is both vertically and horizontally decentralized. In this paper, we also consider a vertically and horizontally decentralized system. However, unlike Shao et al. (2010), we seek to coordinate the system.

## 1.4 Contributions

We conclude by discussing our contributions to the literature and to practice. First, we study supply chain coordination in a system that is both vertically and horizontally decentralized, with retailers who are completely non-cooperative. To date, only Shao et al. (2010) have considered transshipments in such a completely decentralized system. However, they do not consider coordination. Second, we study a multi-period model. In a single-period model, transshipments are reactive and thus a retailer will always be willing to share all of her excess stock. In a multi-period setting, however, transshipments can be used both to fill shortages that occurred during the first period (reactive action) and to position inventory in anticipation of second period demand (proactive action). Thus, retailers have an incentive to ration inventory to satisfy their own future demands. Third, we consider supplier-facilitated transshipments, rather than retailer-negotiated transshipments, which are insufficient to coordinate a system with non-cooperative retailers. We obtain a flexible coordination scheme, consisting of bi-directional adjustment and wholesale price contracts, which allows for the free allocation of the maximum system profit among all parties in the system. Finally, we identify two factors which may cause incentive misalignment in a completely decentralized system: vertical double marginalization and horizontal incentive conflict. The first is well-studied (Cachon 2003) in decentralized supply

chains consisting of a single supplier and a single retailer. The second is directly related to the existence of multiple retailers and will occur in any decentralized supply chain with multiple retailers served by a single supplier.

Specifically, *horizontal incentive conflict* refers to the retailers' incentive misalignment associated with horizontal inventory flow, i.e., the transshipments among the retailers that are *required* to obtain the system optimal inventory reallocation. In the centralized system, transshipments are optimally determined based on the system marginal value of inventory (i.e., the value to the system). In the decentralized system, however, the system inventory is carried and owned by the individual retailers and thus each retailer, when considering whether or not to transship, will compare her own marginal value of inventory (i.e., the value to herself) to the offered transshipment price. The main driver of this conflict is the difference between the retailers' individual marginal values and the system's marginal value. For coordination, transshipment prices must be set to induce *every* retailer to use the *system* marginal value for her inventory. Such a coordinating transshipment price can be offered by the supplier to the retailers, but would be unlikely to result from retailer negotiations. Thus, understanding horizontal incentive conflict helps to explain (1) why coordination for non-cooperative retailers cannot be achieved through retailer-negotiated transshipments, as seen in the literature, and (2) why engaging the supplier (by using supplier-facilitated transshipments) is a must for achieving coordination. Finally, horizontal incentive conflict cannot be removed through constant transfer prices, revenue sharing, or any other coordination mechanism with constant parameters.

## 2 Centralized Distribution System with Inventory Redistribution

We next describe the centralized distribution system that we study. We then develop a dynamic programming model of the system and prove the structure of the optimal production and inventory distribution and redistribution policy.

## 2.1 Model Notation and Assumptions

A single supplier (he) provides goods to $n$ symmetric retailers (she). We use subscript $k$, $k = 1, \ldots, n$, to denote the different retailers. We consider two periods and use subscript $i$, $i = 1, 2$, to denote the different periods. We will use $F_X(\cdot)$ to denote the cumulative distribution function (cdf) of any random variable $X$ defined on probability space $(\Omega, \mathcal{F}, P)$. Let $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$, for any $a, b \in \mathbb{R}$. The additional key notation and assumptions are as follows:

- $D_{ki}$ is the demand seen by retailer $k$ in period $i$, with probability density function (pdf) $f_i(s)$ and cdf $F_i(s)$, $s \in \mathbb{R}$.

- $h_i$ is the unit holding cost for period $i$ for the supplier and for each retailer. While, for simplicity, we assume equal holding costs at the supplier and retailers, throughout the paper we will discuss whether and how our results need to be adjusted to cope with the case in which the holding cost at the supplier is less than the holding cost at the retailers.

- $c_i$ is the unit cost of production for period $i$. We assume $c_2 - c_1 < h_1$. This is applicable in the Comicfans example, where the holding cost, including the opportunity cost, warehousing cost, and logistics cost, is quite significant compared to the production cost difference. This assumption may also apply to any business with tight working capital. Intuitively, if $c_2 - c_1 < h_1$, it is cheaper to fill a demand in period 2 with second period production at cost $c_2$ than with first period production at cost $c_1 + h_1$. Thus, the supplier has no incentive to over-produce and hold inventory in period 1.

- $p_i$ and $r_i$ are the unit penalty cost and unit revenue for each retailer in period $i$. It is reasonable to assume $r_i + p_i \geq c_2$; otherwise the period 2 production is never used. Any excess demand at retailer $k$ in period 1 is backlogged, with penalty cost $p_1$, and filled in period 2. Any excess demand at retailer $k$ in period 2 is lost, with penalty cost $p_2$.

- $v$ is the unit salvage value for any party in the supply chain at the end of period 2.

In the centralized system, there is a single decision-maker who maximizes the system's expected profits. Following the literature (see Robinson 1990) and Comicfans' practice, we assume inventory redistribution (or transshipment) occurs only at the start of period 2. For analytical tractability, we assume the physical shipping cost is negligible. However, we also note that the shipping cost is not the main driver of the incentive misalignment identified in this paper and thus this assumption will not affect our main insights, as discussed in Section 1.4 and Section 4.1. See Karmarkar (1987) for a discussion of the analytical difficulties associated with positive shipping costs in multi-period distribution models. Finally, the assumption of zero shipping costs fits the Comicfans example, where the shipping cost of comic books is negligible relative to the production cost.

The sequence of events is as follows. At the start of period 1 (or at some time prior to the start of period 1, due to the production lead time), the supplier decides how much to produce at unit cost $c_1$ and how to distribute this amount among the retailers (as explained above, the supplier should not hold inventory in period 1). At the start of period 2, customer demand for period 1 is filled at each retailer. At this point, the system inventory may be unbalanced, i.e., some retailers may have more than the optimal inventory level for the second period, while others may have less than the optimal inventory level for the second period. Given the inventory levels at the retailers, the supplier decides how much to produce at unit cost $c_2$ and how to redistribute the system inventory among the retailers. Once the redistribution is complete, the second period demand occurs at the retailers. At the end of the horizon, the remaining system inventory is salvaged for $v$ per unit.

## 2.2  Model Formulation and Analysis

We develop a dynamic programming formulation for the centralized system. We first derive an expression for the maximum expected system profit for period 2 and solve for the optimal production

and re-distribution decisions. We then work backwards to find the system optimal decisions for period 1, which include how much to produce and how to allocate the inventory among the retailers.

### 2.2.1 Period 2

Let $\Pi_2^c(x)$ denote the maximum profit in period 2 for the centralized system, given the system on-hand inventory level is $x = \sum_{k=1}^{n} x_k$, where $x_k$ is retailer $k$'s on-hand inventory level. This profit function depends only on a single state variable, the total inventory level $x$, rather than on the inventory level at each individual location, $x_1, \ldots, x_n$. This is because the shipping cost is assumed to be negligible.

The supplier must determine whether and how much to produce in period 2 and how to re-distribute the system inventory. In other words, the supplier should determine $z_k$, which denotes the stock level at retailer $k$ after inventory re-distribution at the start of period 2. Note that $z_k$ may be larger (less) than $x_k$, if retailer $k$ receives (returns) stock at the start of period 2. Since the salvage value is assumed to be less than the production cost, the supplier has no incentive to carry inventory in period 2, i.e., the supplier should redistribute all the system inventory among the retailers. Thus, the total stock level at the retailers, i.e., the system's stock level, must satisfy $\sum_{k=1}^{n} z_k \geq x(= \sum_{k=1}^{n} x_k)$. We can now write the optimal system profit for the period 2 as:

$$\Pi_2^c(x) = \max_{\sum_{k=1}^{n} z_k \geq x} \left\{ -c_2 \left( \sum_{k=1}^{n} z_k - x \right) + \sum_{k=1}^{n} \pi_{k2}(z_k) \right\} = \max_{z \geq x} \left\{ -c_2 \left( z - x \right) + \pi_{c2}(z) \right\}, \qquad (1)$$

where $z_k \geq 0$, $k = 1, \ldots, n$, $z = \sum_{k=1}^{n} z_k$ is the total stock at the retailers,

$$\pi_{k2}(\xi) = r_2 E[\xi \wedge D_{k2}] + (v - h_2)E[\xi - D_{k2}]^+ - p_2 E[\xi - D_{k2}]^-, \quad \forall \xi \geq 0, \qquad (2)$$

and $\pi_{c2}(z) = \max_{\sum_{k=1}^{n} z_k = z} \sum_{k=1}^{n} \pi_{k2}(z_k)$ represents the maximum expected revenue received in period 2, minus the holding and penalty costs, for the centralized system with on-hand inventory $z$. Equation

(1) indicates that setting the stock level for each retailer, $(z_1, \ldots, z_n)$, is equivalent to setting the total stock at the retailers, $z$, and then determining how to split that total stock among the retailers.

We next characterize the optimal production and re-distribution decisions, i.e., the optimal $z_k$, $k = 1, \ldots, n$. We first define Case P as the case in which the supplier finds it optimal produce in period 2. We then note that in Case P, the optimal system inventory level can be written as $Z_P^0 = nZ_{kP}^0$, where $Z_{kP}^0 = F_2^{-1}\left(\frac{p_2+r_2-c_2}{p_2+r_2+h_2-v}\right)$ denotes the optimal inventory level for period 2 for retailer $k$, i.e., $Z_{kP}^0$ is the value of $z_k$ that satisfies $\pi'_{k2}(z_k) = c_2$. We are now ready to state the following proposition. Note that proofs of the propositions, lemmas and corollaries are contained in Appendix B.

**Proposition 2.1** *The optimal production and redistribution policy for the system at the start of period 2, given the system on-hand inventory level $x \in \mathbb{R}$, is:*

(i) *If $x < Z_P^0$, the supplier should produce to raise the system stock level up to $Z_P^0$ and redistribute this system stock equally across the retailers, so that each retailer's stock level is $Z_k^0(x) = \frac{Z_P^0}{n}$.*

(ii) *If $x \geq Z_P^0$, the supplier should not produce, but redistribute the system stock equally across the retailers, so that each retailer's stock level is $Z_k^0(x) = \frac{x}{n}$.*

The optimal policy for the system is a produce-up-to $Z_P^0$ policy, with equal redistribution of inventory among the retailers. Note that case (ii) above involves transshipments only. Finally, if the retailers' holding cost is greater than the supplier's holding cost, Proposition 2.1 would need to be modified to include a take-back-to level, $Z_T^0$, so that if the system inventory level is greater than $Z_T^0$, the supplier should take back inventory to reduce the total stock level at the retailers down to $Z_T^0$.

### 2.2.2 Period 1

We next derive the optimal production and inventory distribution policy for period 1. We assume the system starts with zero inventory. The supplier will not hold inventory during period 1 since it

is cheaper to fill any second period demand using second period production than using first period production. Thus, the supplier only needs to decide $y_k$, the inventory allocated to retailer $k$ in period 1, and $y = \sum_{k=1}^{n} y_k$, the supplier's total production quantity for period 1. We now derive an expression for the maximum expected profit-to-go for the two-period centralized system, denoted by $\Pi_1^c(0)$.

$$\Pi_1^c(0) = \max_{y_1,\ldots,y_n} \pi_c(y_1,\ldots,y_n) = \max_y \pi_c(y), \tag{3}$$

where $y = \sum_{k=1}^{n} y_k$,

$$\pi_c(y_1,\ldots,y_n) = -c_1 \sum_{k=1}^{n} y_k + \sum_{k=1}^{n} \pi_{k1}(y_k) + \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \Pi_2^c(x) \prod_{k=1}^{n} dF_1(s_k),$$

$$\pi_c(y) = -c_1 y + \pi_{c1}(y) + \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \Pi_2^c(x) \prod_{k=1}^{n} dF_1(s_k), \tag{4}$$

where $x = y - \sum_{k=1}^{n} s_k$ is the on-hand system inventory at the start of period 2, given the period 1 demand realization at the retailers $(s_1,\ldots,s_n)$; retailer $k$'s expected revenue in period 1, minus the holding and penalty cost, given that period 1 demands are backlogged, is

$$\pi_{k1}(\xi) = r_1 E[D_{k1}] - h_1 E[\xi - D_{k1}]^+ - p_1 E[\xi - D_{k1}]^-, \quad \forall \xi \geq 0; \tag{5}$$

and $\pi_{c1}(y) = \max_{\sum_{k=1}^{n} y_k = y} \sum_{k=1}^{n} \pi_{k1}(y_k)$ is the expected revenue received in period 1 for the system.

We next characterize the optimal production and distribution decisions for period 1, i.e., the optimal values of $y_k$, $k = 1,\ldots,n$. We can show that, for any period 1 production quantity $y$, it is optimal for the supplier to distribute that quantity equally across the $n$ retailers. Using this property, we can simply maximize $\pi_c(y)$ over $y$, rather than solving for optimal values of $y_1,\ldots,y_n$ simultaneously. We define the optimal production quantity $Y^0$ as the value of $y$ that solves

$$-c_1 + \pi'_{c1}(y) + \frac{d}{dy} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \Pi_2^c \left( y - \sum_{k=1}^{n} s_k \right) \prod_{k=1}^{n} dF_1(s_k) = 0. \tag{6}$$

**Proposition 2.2** *It is optimal for the supplier to produce to raise the system stock level to $Y^0$ and distribute it equally across the retailers.*

# 3 Decentralized Distribution System with Inventory Redistribution

We next consider a decentralized distribution system in which each party makes decisions to maximize its own profit. The supplier has chosen to encourage inventory transshipment by offering the retailers an adjustment contract for the start of period 2, in addition to a standard wholesale price contract for period 1. This adjustment contract allows the retailers to either purchase additional inventory from, or return excess inventory to, the supplier. We assume these transactions are performed virtually, so that the actual exchange of goods takes place directly between the retailers. We study the relationship between the supplier and the retailers using a Stackelberg game with the supplier acting as the leader. We model the relationship among the retailers using a Nash game.

## 3.1 Model Formulation and Analysis

The notation and assumptions outlined in Section 2 for the centralized system will also apply for the decentralized system. Previously, we used the superscript 0 to denote the optimal policy for the centralized system. In this section, we use the superscript $*$ to denote the optimal policy for the decentralized system. Next, we define the contracts offered by the supplier to the retailers:

- $w_k$ is the unit wholesale price for retailer $k$'s procurement at the start of period 1.

- $P_{kB}$ and $P_{kS}$ are the upward and downward adjustment prices for retailer $k$'s adjustments at the start of period 2, respectively. We assume $P_{kB} \geq P_{kS}$ to avoid pathological cases.

- $\gamma_k$ is the side payment for retailer $k$. If $\gamma_k > 0$, retailer $k$ must pay a lump sum $\gamma_k$ to participate in the wholesale and adjustment contracts. If $\gamma_k < 0$, retailer $k$ receives a lump sum $|\gamma_k|$ for participating in the contracts. We assume that retailer $k$ will participate in these contracts only if her resulting expected profit is no less than her reservation profit, which is her expected profit earned under a wholesale price contract, in a system which may or may not have transshipments negotiated among the retailers.

We assume that both wholesale orders and adjustments of any amount are permitted and are 100% satisfied (i.e., forced compliance). The contract prices, $w_k, P_{kB}, P_{kS}$, may be constants or functions of some real-time information that is observable to all parties in the system. Later we will show that the coordinating adjustment prices must depend on the system on-hand inventory level.

At the start of the planning horizon, the supplier offers the contract terms to the retailers. If acceptable (i.e., if each retailer's expected profit under the contract is no less than her reservation profit), they will enter into an agreement with the supplier; otherwise, they will walk away. We model this setting using a Stackelberg game. Since the supplier is the leader, if the proposed contracts can help maximize the system profit, the supplier should facilitate coordination and use side payments to guarantee that each retailer earns exactly her reservation profit. The supplier then extracts all of the additional system profit achieved under this scheme. Thus, for the remainder of this section, we consider the case in which all retailers will accept the supplier's contract terms. The retailers will play a Nash game to simultaneously determine their wholesale order quantities and adjustment policies. If the contract prices are constants, the Nash game can be reduced to independent decision-making by the individual retailers, i.e., each retailer can decide her wholesale order quantity and adjustments solely based on her corresponding prices. However, if the contract prices are not constant, e.g., if the adjustment prices depend on the system on-hand inventory level, the Nash game may not be reducible. Finally, we assume information symmetry among the parties in the supply chain.

## 3.2 The Retailer's Problem

Let $y_k$ denote retailer $k$'s order quantity for period 1. Let $z_{kB}$ and $z_{kS}$ denote retailer $k$'s buy-up-to stock level and sell-down-to stock level, respectively, if she decides to adjust her inventory upwards or downwards at the start of period 2. We first derive retailer $k$'s maximum expected profit in period 2, denoted by $\Pi_2^{kr}(x_k|x_{-k})$, where $x_k$ and $x_{-k}$ represent the on-hand inventory levels of retailer $k$ and of all the other retailers, respectively, at the start of period 2. In other words, $x_{-k} = [x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n]$ and subscript $-k$ is associated with arrays for the retailers other than retailer $k$. Then $\Pi_2^{kr}(x_k|x_{-k})$ can be written as

$$\Pi_2^{kr}(x_k|x_{-k}) = \max_{z_{kB} \geq x_k} \left\{-P_{kB}(z_{kB} - x_k) + \pi_{k2}(z_{kB})\right\} \vee \max_{z_{kS} \leq x_k} \left\{-P_{kS}(z_{kS} - x_k) + \pi_{k2}(z_{kS})\right\}. \tag{7}$$

Next, retailer $k$'s maximum expected profit-to-go for period 1, denoted $\Pi_1^{kr}(0|y_{-k})$, can be written as

$$\Pi_1^{kr}(0|y_{-k}) = \max_{y_k \geq 0} \pi_{kr}(y_k|y_{-k}) = \max_{y_k \geq 0} \left\{-w_k y_k + \pi_{k1}(y_k) + \int_{-\infty}^{\infty} \Pi_2^{kr}(y_k - s_k|y_{-k} - s_{-k}) dF_1(s_k)\right\}, \tag{8}$$

where $y_k$ and $y_{-k}$ represent the wholesale order quantities of retailer $k$ and all the other retailers.

It is known that, for any adjustment contract prices $(P_{kB}, P_{kS})$, a buy-up-to or sell-down-to policy[1] is optimal for each retailer in period 2 (see, for example, Taylor 2001). The optimal buy-up-to level for retailer $k$, $Z_{kB}^*$, and the optimal sell-down-to level for retailer $k$, $Z_{kS}^*$, for $k = 1, \ldots, n$, are

$$Z_{kB}^* = F_2^{-1}\left(\frac{p_2 + r_2 - P_{kB}}{p_2 + r_2 + h_2 - v}\right) \quad \text{and} \quad Z_{kS}^* = F_2^{-1}\left(\frac{p_2 + r_2 - P_{kS}}{p_2 + r_2 + h_2 - v}\right).$$

Finally, the retailers' optimal order quantities for period 1, i.e., the Nash equilibrium (if one exists)

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[1]Under a buy-up-to $Z_B$ or sell-down-to $Z_S$ policy, the retailer should raise her inventory level up to $Z_B$ if her on-hand inventory level is lower than $Z_B$; the retailer should sell to reduce her inventory level down to $Z_S$ if her on-hand inventory is higher than $Z_S$, where $Z_S \geq Z_B$; if the retailer's inventory level is between $Z_B$ and $Z_S$, she should do nothing.

$Y_1^*, \ldots, Y_n^*$, can be found by simultaneously solving the equations, $\pi'_{kr}(y_k|y_{-k}) = 0$, $k = 1, \ldots, n$.

## 3.3 The Supplier's Problem

The supplier must determine the optimal production quantity in each period, given the retailers' optimal policies. Under the contract agreement, at the start of period 2, the supplier must comply with the retailers' adjustment requests. Specifically, the supplier first consolidates the retailers' adjustment orders by using the returns, i.e., the downward adjustment orders, to fill the purchases, i.e., the upward adjustment orders. We refer to this task as the supplier-facilitated transshipments. Note that although the information and cash flow is through the supplier, the physical shipment of goods in this case takes place directly between the corresponding retailers. In addition to supplier-facilitated transshipments, if more inventory is needed the supplier must produce the necessary amount at unit cost $c_2$. Thus, the supplier has only one real production decision, which occurs at the start of period 1. To characterize the supplier's period 1 production decision, recall that it is cheaper for the supplier to fill his second period demand with second period production than first period production. Thus, the supplier's optimal period 1 production decision is simply to produce exactly the retailers' order quantity in period 1, i.e., the supplier should produce $Q^* = \sum_{k=1}^n Y_k^*$. Then, in period 2, the supplier meets all of the retailers' adjustment requests through a combination of supplier-facilitated inventory transshipments and second period production.

## 4 Coordination of the Decentralized Distribution System

We next explore how the decentralized system can be coordinated via appropriately set contract prices, which we refer to as the coordinating contract prices. Recall the decisions of each party in the decentralized system at the start of the horizon. Given the contract prices, the retailers, the followers in the Stackelberg game, choose their optimal order quantities for period 1 and their optimal inventory

policies for period 2 simultaneously. The supplier, the leader of the game, determines and announces the optimal contract terms, using this NE, with the goal of maximizing his own expected profit. Note that since the supplier is the leader of the game, he can use the side payments to the retailers to freely allocate the system profit among the parties in the system, as long as the retailers are willing to participate. Therefore, maximizing his own expected profit is equivalent to maximizing the system profit, i.e., coordinating the supply chain.

If we compare the centralized system to a decentralized system with transshipments, we find that incentive misalignment arises from *horizontal incentive conflict*, in addition to the usual (vertical) double marginalization. Horizontal incentive conflict refers to the retailers' incentive misalignment associated with the horizontal flow of inventory (transshipments) that is *required* to achieve a system optimal inventory redistribution. For the vertical inventory flow in periods 1 and 2, vertical double marginalization results from the fact that each retailer's profit margin differs from the system's profit margin. Thus, coordination can be obtained by using a wholesale price that equates these two profit margins. For the horizontal inventory flow in period 2, the system incurs no cost (or, if the shipping cost is assumed to be non-negligible, the system only incurs the shipping cost) in *both* the centralized and decentralized systems. Thus, the shipping cost is not the main driver of the incentive misalignment. Instead, the main driver of horizontal incentive conflict is the difference between the individual retailer's marginal value of inventory and the system's marginal value of inventory, as discussed in detail in the next section.

### 4.1  Coordinating Period 2

We first consider how to coordinate the second period decisions using coordinating adjustment prices. The optimal inventory redistribution for period 2 in the centralized system, $Z_1^0(x), \ldots, Z_n^0(x)$, as defined in Proposition 2.1, depends on the system on-hand inventory level, $x$. Specifically, if $x < Z_P^0$,

$Z_k^0(x) = \frac{Z_P^0}{n}$; if $x \geq Z_P^0$, $Z_k^0(x) = \frac{x}{n}$. In the decentralized system, the retailers make independent decisions, choosing $Z_{kB}^*$ and $Z_{kS}^*$, which depend only on the adjustment prices, $P_{kB}$ and $P_{kS}$. Thus, we see that the supplier must use state-dependent adjustment prices, i.e., $P_{kB}(x)$ and $P_{kS}(x)$, to coordinate the second period decisions, i.e., to induce each independent retailer to make adjustment decisions that optimize the system.

Intuitively, when only transshipments are needed, i.e., when the supplier does not produce, the centralized system redistributes the system inventory so that all retailers have the same inventory level, and thus have the same marginal value of inventory, which is just equal to the *system* marginal value. Notice that this marginal value depends on the system on-hand inventory level, i.e., the higher the system on-hand inventory level, the lower the value of an additional unit of inventory to the system. The retailers in the decentralized system, however, determine their optimal inventory levels by equating their *own* marginal value of inventory to the adjustment prices. Therefore, to induce the retailers to use the *system* marginal value for their own inventories, i.e., to coordinate the decentralized system, the supplier must use the system marginal value of inventory as the state-dependent adjustment price for all the retailers. This coordinating adjustment price can be offered by the supplier to the retailers, but would be unlikely to result from retailer negotiations. Thus, identifying and understanding horizontal incentive conflict helps to explain (1) why coordination for non-cooperative retailers cannot be achieved through retailer-negotiated transshipments and (2) why engaging the supplier (by using supplier-facilitated transshipments) is a must for achieving coordination. We next formally state how to design $P_{kB}(x)$ and $P_{kS}(x)$ to achieve coordination in the second period.

**Proposition 4.1** *Coordination (or coordinated transshipment) is achieved in period 2 if and only if conditions (i) and (ii) hold, where*

(i) *if $x < Z_P^0$, the supplier should set $P_{kB}(x) = P_{kS}(x) = c_2 = \pi_{k2}'\left(\frac{Z_P^0}{n}\right) = \pi_{c2}'(Z_P^0)$, $k = 1, \ldots, n$*

(ii) *if $x \geq Z_P^0$, the supplier should set $P_{kB}(x) = P_{kS}(x) = \pi_{k2}'\left(\frac{x}{n}\right) = \pi_{c2}'(x)$, $k = 1, \ldots, n$.*

If the system stock level is low (so that the retailers, considered as a whole, want an upward adjustment), the coordinating adjustment price is just the second period unit production cost, $c_2$, and is the same for all retailers. In this case, we observe two types of flows to the retailers: vertical inventory flow between the supplier and some retailers and horizontal inventory flow (transshipment) between other retailers. For the vertical flows, the usual double marginalization effect exists. A simple and direct way to remove this double marginalization is to use $c_2$ as the adjustment price. For the horizontal flows, we observe the horizontal incentive conflict. As explained above, removing this conflict requires the use of the system marginal value of inventory as the adjustment (transshipment) price for all retailers. In this case, the system marginal value is just equal to the marginal net revenue at the optimal produce-up-to level, $Z_P^0$, i.e., $\pi'_{c2}(Z_P^0) = c_2$. Thus, only by charging this precisely defined adjustment price can the supplier induce the retailers to share the system's perspective.

If the system stock level is high, so that no additional production is needed and there is only horizontal inventory flow, the coordinating adjustment price is also equal for buying and for selling, and for all retailers. However, this price is not a constant, but depends on the system on-hand inventory level, $x$. In this case, we only observe horizontal incentive conflict and the system marginal value of inventory is just equal to the marginal net revenue for the system, $\pi'_{c2}(x)$, which is a non-linear function of $x$. As discussed above, the supplier can induce the retailers to share the system's perspective only by charging this marginal value as the adjustment price for all the retailers. Figure 2, in Section 5, shows the typical behavior of this coordinating adjustment price. Finally, recall that if the retailers' holding cost is greater than the supplier's holding cost, Proposition 2.1 would be modified to include a take-back-to level, $Z_T^0$. In this case, we would also need to modify Proposition 4.1 to note that when the system inventory level is greater than $Z_T^0$, the coordinating adjustment price is $P_{kB} = P_{kS} = v - h_{s2}$, where $h_{s2}$ denotes the supplier's holding cost in the second period.

The horizontal incentive conflict identified here cannot be removed by revenue sharing or any

other coordination mechanism in the existing literature with constant parameters. To understand this, consider a single-period supply chain with a single retailer. Cachon (2003) shows that two coordinating schemes, revenue sharing and buyback contracts, are equivalent in this setting. Both schemes promote coordination, i.e., induce the retailer to use a constant system optimal order quantity, with a free allocation of the system profit. In contrast, constant transfer prices can be used to promote coordination, but with the result that the retailer extracts all of the system profit. The free allocation of profits in the revenue sharing and buyback contracts is achieved through the use of an additional constant control parameter, in addition to the wholesale price, i.e., the portion of revenue shared in the revenue sharing contract and the buyback price in the buyback contract. Mathematically, inducing the retailer to use the constant system optimal inventory level is achieved through the introduction of an additional degree of freedom for the control parameters. In our two-period supply chain with *multiple retailers*, however, coordination requires the retailers to use state-dependent, rather than constant, system optimal inventory levels. In addition, the state, i.e., the system on-hand inventory level, is continuous. Thus, the retailers cannot be induced to use the state-dependent system optimal adjustment quantity through a finite number of additional degrees of freedom for the control parameters. Therefore, the horizontal incentive conflict cannot be removed by revenue sharing or any other coordination mechanism with constant parameters.

To achieve coordination, each retailer should be charged the *same* adjustment price for upward and downward adjustments. For the remainder of the paper, we will denote this coordinating adjustment price by $P(x)$, which represents the system marginal value of inventory. This equal and unique adjustment price induces each retailer to value any unit of system inventory equally, regardless of its ownership. We next show that this price, $P(x)$, is non-increasing in $x$. This implies that, when the system is short of inventory (case (i) of Proposition 4.1), the supplier should set the upwards and downwards adjustment prices equal to the production cost in order to encourage more buying than

selling. Otherwise, when the system has enough inventory (case (ii) of Proposition 4.1), the supplier should charge a price lower than the production cost in order to ensure that the net adjustment quantity across all retailers is equal to 0 and thus no additional production is needed.

**Corollary 4.2** $P(x)$, $k = 1, \ldots, n$, is non-increasing function of $x$ and $P(x) \leq c_2$.

Finally, although the adjustment contracts are signed between the supplier and each retailer, when transshipment is needed inventory should be physically transshipped between the retailers. This process would be facilitated by the supplier, e.g., the supplier would ask the selling retailers to deliver the sold inventory to an appropriate buying retailer. In practice, given the optimal final inventory allocation, a linear program can be formulated and solved to minimize the total transportation cost and determine an optimal transshipment plan.

## 4.2 Coordinating Period 1

We now consider how the supplier can coordinate the period 1 decisions, under the assumption that the coordinating adjustment prices characterized in the previous section are used. In other words, we characterize a coordinating wholesale price contract, which together with the coordinating adjustment contract makes the decentralized system act like a centralized system.

The supplier must design the wholesale prices, $w_1, \ldots, w_k$, to encourage each independent retailer, say retailer $k$, to order the system optimal amount, $Y_k^0$, under the Nash equilibrium. The supplier will then produce the quantity $Y^0 = \sum_{k=1}^{n} Y_k^0$. We start by rewriting the retailer's expected profit-to-go, $\pi_{kr}(y_k)$, as in (8), given that the coordinating state-dependent adjustment price $P(x)$ is used.

$$\pi_{kr}(y_k|y_{-k}) = -w_k y_k + \pi_{k1}(y_k) + \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left\{ P(x)(y_k - s_k - Z_k^0(x)) + \pi_{k2}(Z_k^0(x)) \right\} \prod_{k=1}^{n} dF_1(s_k). \quad (9)$$

Recall that $x = \sum_{k=1}^{n} y_k - \sum_{k=1}^{n} s_k$ is the total system inventory at the start of period 2, i.e., the total inventory at the start of period 1 ($\sum_{k=1}^{n} y_k$) minus the total demand in period 1 ($\sum_{k=1}^{n} s_k$).

Differentiating this profit function with respect to $y_k$, we obtain the marginal profit function:

$$\pi'_{kr}(y_k|y_{-k}) = -\frac{d}{dy_k}(w_k y_k) + \pi'_{k1}(y_k) + c_2 \bar{F}_{\sum_{k=1}^{n} D_{k1}} \left( \sum_{k=1}^{n} y_k - Z_P^0 \right) + \int \cdots \int_{x \geq Z_P^0} \pi'_{k2}(Z_k^0(x)) \prod_{k=1}^{n} dF_1(s_k). \quad (10)$$

Note that $w_k$ may be set as a function of $y_k$. By jointly solving the equations $\pi'_{kr}(y_k) = 0$, $k = 1, \ldots, n$, we can obtain a unique Nash equilibrium.

To guarantee that this Nash equilibrium is system optimal, we compare the above marginal profit function to retailer $k$'s marginal profit function in the centralized system, $\frac{d\pi_c}{dy_k} = \frac{d\pi_c(y_1,\ldots,y_n)}{dy_k}$, where $\pi_c(y_1, \ldots, y_n)$ is defined by (4). This comparison is valid because the optimal period 1 solution for the centralized system, $(Y_1^0, \ldots, Y_n^0)$, is the unique solution to $\frac{d\pi_c}{dy_l} = 0$, $l = 1, \ldots, n$, where

$$\frac{d\pi_c}{dy_k} = -c_1 + \pi'_{k1}(y_k) + c_2 \bar{F}_{\sum_{k=1}^{n} D_{k1}} \left( \sum_{k=1}^{n} y_k - Z_P^0 \right) + \int \cdots \int_{x \geq Z_P^0} \pi'_{k2}(Z_k^0(x)) \prod_{k=1}^{n} dF_1(s_k). \quad (11)$$

It is not difficult to see that the only difference between $\pi'_{kr}(y_k|y_{-k})$ and $\frac{d\pi_c}{dy_k}$ is the first term, which corresponds to the marginal cost. Thus, setting $\frac{d}{dy_k}(w_k y_k) = c_1$ will completely eliminate the vertical double marginalization in the first period, which is the only source of incentive conflict, and thus coordinates the system, given that the coordinating adjustment price, $P(x)$, is used in the second period. Note that $P(x)$ is defined in Proposition 4.1. We thus obtain the following coordination results for the two period problem.

**Proposition 4.3** *Coordination is achieved in the decentralized supply chain at the unique equilibrium of the Stackelberg and Nash games if and only if the supplier offers the retailers with the wholesale price contract[2] with $w_k = c_1$, the state-dependent adjustment price, $P(x)$, and a side payment, $\gamma_k$, such that each retailer earns exactly her reservation profit.*

Finally, we summarize the coordination results. The wholesale and adjustment contracts are an-

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[2]In general, this wholesale price can be a menu defined as $w_k = c_1 + \beta_k(y_{-k})$, where $\beta(\cdot)$ can be any function.

nounced at the start of the horizon. The retailers then participate in a Nash game and simultaneously determine their wholesale order quantities for period 1, as well as their adjustment policies for period 2. The actual adjustment prices and quantities will be calculated at the start of period 2, when the system stock level, $x$, is observable. The unique Nash equilibrium will achieve the system optimal allocation for both periods, i.e., $(Y_1^0, \ldots, Y_n^0)$ for period 1 and $(Z_1^0(x), \ldots, Z_n^0(x))$ for period 2. As long as the coordinating contract prices are offered to the retailers, the retailers' decisions will be in line with the system optimal decisions and thus the system profit will be maximized. As the leader of the Stackelberg game, the supplier can theoretically extract this entire maximum system profit. However, to ensure the participation of the retailers, the supplier should charge side payments (denoted $\gamma_k$) such that each retailer earns exactly her reservation profit, with the supplier extracting the rest of the maximum system profit.

# 5 Computational Study

We next present a computational study to supplement the analytical results provided in this paper and to demonstrate the value of our proposed coordination scheme.

## 5.1 Experiment Setup

We study a distribution system consisting of a single supplier and five retailers. Our base case model parameters, which were motivated by the Comicfans example provided in Section 1.1, are shown in Table 1. In the base case, the wholesale price is set at 60% of the retail price, while the unit production cost in the first period is set at 35% of the retail price. For our base case, we set the unit production cost for the second period equal to the unit production cost for the first period. Finally, for the base case, the upward and downward adjustment prices are set equal to the wholesale price in period 1, as in the policy used at Comicfans.

Table 1: Base case parameters for the computational study for period $i$, $i = 1, 2$

| Retail price | $r_i$ | 15 |
|---|---|---|
| Production cost | $c_i$ | $0.35 \times r_i$ |
| Wholesale price | $w$ | $0.60 \times r_i$ |
| Holding cost | $h_i$ | $0.05 \times r_i$ |
| Salvage value | $v$ | $0.10 \times r_i$ |
| Upward adjustment price | $P_{kB}$ | $w$ |
| Downward adjustment price | $P_{kS}$ | $w$ |

While the penalty cost is generally difficult to accurately assess, it does have a significant impact on the inventory levels and on the value of coordination. Therefore, we consider three difference cases. As shown in Table 2, in case P1, the unit penalty cost, as a percentage of the retail price, is the same in both periods. In case P2, the penalty cost as a percentage of the retail price is higher in period 1 than in period 2. This assumption causes the retailers to hold more inventory in period 1 and to order less inventory at the start of period 2. In case P3, the penalty cost as a percentage of the retail price is higher in period 2 than in period 1. This causes the retailers to hold less inventory in period 1 than in period 2. Intuitively, in case P2 the retailers are more likely to want to sell inventory back to the supplier at the end of the first period, which makes coordination more important. Thus, we expect to see coordination have the most value in case P2.

Table 2: Penalty cost for the retailers

| Case | Period 1 | Period 2 |
|---|---|---|
| P1 | $0.50 \times r_1$ | $0.5 \times r_2$ |
| P2 | $r_1$ | $0.25 \times r_2$ |
| P3 | $0.25 \times r_1$ | $r_2$ |

Finally, we consider the distribution of demand at each of the retailers. We assume that all five retailers have the same distribution of demand in each period. In addition, we assume that demand follows a truncated Normal distribution (to ensure non-negative demands), with different means and standard deviations (std) in each period. The parameter values for the three cases are shown in Table 3. In cases D2 and D3 we have lower mean demand in the second period than in the first period. In

these cases, we would expect the retailers to have a greater need to return inventory to the supplier at the end of the first period, which should make coordination more important than in case D1.

Table 3: Parameters of the demand distributions

| Case | Period 1 | | Period 2 | |
|------|----------|-----|----------|-----|
|      | Mean | Std | Mean | Std |
| D1 | 10000 | 5000 | 10000 | 5000 |
| D2 | 10000 | 5000 | 5000 | 2500 |
| D3 | 10000 | 5000 | 3000 | 1500 |

## 5.2 Value of the Coordination

We first compare three systems: the centralized system; the decentralized system with no adjustment contract, in which the retailers can order additional quantities in the beginning of period 2, but cannot sell inventory back to the supplier; and the decentralized system with a fixed-price adjustment contract, where the adjustment prices are as shown in Table 1. Our goal in making this comparison is to demonstrate the importance of coordination for improving supply chain performance. Table 4 shows the total system profit for these three systems for the nine combinations of the demand and penalty cost settings, with all other parameters set to their base values.

There are two ways to implement the adjustment contract in the decentralized system. The simple method is to use constant adjustment prices. This is the approach used by Comicfans, who have chosen to set the adjustment prices just equal to the wholesale price. The other method uses a state-dependent adjustment price and carefully-selected wholesale price to coordinate the system. In Table 4, the percentage gains for the centralized system (G1) and decentralized system with constant adjustment prices (G2), as shown in parentheses, are calculated with respect to the system profit for the decentralized system with no adjustment contract. From the table, we see that Comicfans' current practice can result in a lower system profit than having no adjustment contract, i.e., having no transshipments. In contrast, the coordinated decentralized system achieves the same profit as the

Table 4: Comparison of centralized system with decentralized systems (in $100,000)

| Demand Distribution | Penalty Cost | Decentralized without adjustment contract | Decentralized with adjustment contract (G2) | Centralized (G1) | Decentralized with $w = c_1$ (G3) |
|---|---|---|---|---|---|
| D1 | P1 | 7.7149 | 7.6850 (-0.39%) | 8.0392 (+4.20%) | 7.9690 (+3.33%) |
| D1 | P2 | 7.7624 | 7.6817 (-1.04%) | 8.1081 (+4.45%) | 8.0253 (+3.39%) |
| D1 | P3 | 7.6077 | 7.5984 (-0.12%) | 7.9154 (+4.04%) | 7.8474 (+3.19%) |
| D2 | P1 | 5.9427 | 5.7400 (-3.41%) | 6.2512 (+5.19%) | 6.0754 (+2.22%) |
| D2 | P2 | 5.8283 | 5.3395 (-8.39%) | 6.1854 (+6.13%) | 5.9397 (+1.92%) |
| D2 | P3 | 5.9992 | 5.9785 (-0.34%) | 6.2504 (+4.19%) | 6.1246 (+2.04%) |
| D3 | P1 | 4.9224 | 4.6737 (-5.05%) | 5.4126 (+9.96%) | 4.9992 (+1.53%) |
| D3 | P2 | 4.5596 | 4.2104 (-7.66%) | 5.1963 (+13.96%) | 4.6498 (+1.91%) |
| D3 | P3 | 5.1847 | 5.0940 (-1.75%) | 5.5197 (+6.46%) | 5.2651 (+1.51%) |

centralized system, and thus outperforms the decentralized system with no adjustment contact in all cases. We find that the performance gap between the coordinated (or centralized) system and the decentralized system with no adjustment contract varies from about 4% to close to 14%.

We explain these results by considering the objective of the supplier when implementing the adjustment contract. The supplier offers the adjustment contract to encourage the retailers to increase their order quantities in the first period. However, if the downward adjustment price ($P_{kS}$) is set too high (equal to the wholesale price, $w$), the retailers will order large quantities in the first period, causing the supplier to incur a loss in the second period. In contrast, using a state-dependent adjustment price can encourage the retailers to order larger quantities without the associated loss at the supplier, since the upward and downward adjustment quantities are balanced when the adjustment price is less than the supplier's production cost in the second period ($c_2$).

We also observe that coordination has the most value when the demand in period 2 is low relative to the demand in period 1 (cases D2 and D3). This is due to the fact that, with lower demand in period 2, the retailers are more likely to sell products back to the supplier, in which case the system benefits more from coordination. Also, when the penalty cost in period 2 is lower than the penalty

cost in period 1 (case P2), the profit gain from coordination is the highest. As noted above, in case P2 the system has a higher inventory level in period 1 and a lower desired inventory level in period 2, which makes it more likely that the retailers will want to sell inventory back to the supplier at the start of period 2, making coordination more important.

One question that remains is whether the profit gain from coordination (G1) is obtained largely by removing double marginalization in period 1, rather than by implementing the state-dependent adjustment price in period 2. We study this question by examining the system profit for the decentralized system with no adjustment contract and $w = c_1$ (G3). In such a system, double marginalization in period 1 is removed. The profit gain for this decentralized system relative to the system with no adjustment contract (G3) is shown in the last column of Table 4. In case D1, i.e., when the mean demand is the same in both periods, G3 is close to G1, which implies that the coordinated system benefits mainly from the removal of double marginalization in period 1. In case D3, i.e., when the mean demand in period 2 is much less than in period 1, G3 is much smaller than G1, and thus the removal of double marginalization is only a portion of the value of coordination. This demonstrates the value of the state-dependent adjustment price for the second period: coordination cannot be achieved by simply removing double marginalization or applying a fixed-price adjustment contract.

Finally, we consider the impact of the number of retailers on the performance gap between the decentralized and coordinated system. As shown in Figure 1, as the number of retailers increases, the gain from coordination also increases. (All the figures are contained in Appendix A.) However, in cases D1 and P1, i.e., when the demand distributions and penalty costs are the same in both periods, the number of retailers has a minimal impact. The intuition is as described above: when the demand distribution is the same in both periods, the increased profit under coordination is mainly achieved by removing double marginalization in period 1. In this case, having more retailers will not help.

## 5.3 Coordination in the Decentralized System

We next discuss the proposed coordination strategy, including the adjustment contract price at the start of period 2, the wholesale price at the start of period 1, and the profit split under coordination. Figure 2 shows the coordinating adjustment price $P(z)$ for the system under different retail prices. We see that when the system inventory is less than $Z_P^0$, the adjustment price equals $c_2$. The contract price then gradually decreases as the system inventory increases. When the system inventory approaches infinity, the contract price converges to $v - h_2$, which equals the profit the retailers can earn from holding their excess inventory and salvaging it at the end of the second period.

In Section 4.2, we showed the supplier should choose $\gamma_k$, together with the coordinating contract parameters, so that each retailer earns exactly her reservation profit. The supplier then extracts all of the additional system profit achieved under this coordination scheme. To provide the retailers with more incentive to participate, the supplier can consider reducing the side payments. Here we study the range of $\gamma_k$ that will lead to Pareto-improvement, i.e., that will make all parties in the system better off. The upper limit of $\gamma_k$, denoted by $\gamma_k^U$, can be determined by making the profit earned by each retailer the same as that earned in the decentralized system with no adjustment contract. Similarly, the lower limit, denoted by $\gamma_k^L$, can be set so that the supplier's profit equals that earned by the supplier in the decentralized system. However, $\gamma_k^L$ is not uniquely determined for each retailer $k$, since the supplier has numerous ways to allocate the remaining profit. Hence, we add the constraint that, when determining $\gamma_k^L$, all retailers should see the same percentage gain in their profits compared to the decentralized system. With this constraint, the $\gamma_k^L$ can be uniquely determined. In Figure 3, for retailer 1 (in a system with five retailers), we illustrate how we can determine the range of $\gamma_1$ that will achieve Pareto-improvement. The dashed line shows the profit of retailer 1 with respect to $\gamma_1$, and the two solid horizontal lines show the profits earned by the retailer in two cases: (a) when the supplier earns the same profit as in the decentralized system ($\pi_a$), and (b) the retailer earns

the same profit as in the decentralized system ($\pi_b$). Hence, the intersection of the solid lines with the dashed line determines the two limits on the side payment $\gamma_1$, within which the system achieves Pareto-improvement.

# 6    Conclusions

We find that coordination in a distribution system with non-cooperative independent retailers can *only* be attained through appropriately designed contracts which serve to implement supplier-facilitated transshipments. Thus, we believe we are the first to successfully coordinate a vertically and horizontally decentralized supply chain with non-cooperative retailers. We have characterized the optimal policies for the two-period centralized and decentralized systems. We have identified two causes of incentive misalignment: vertical double marginalization and horizontal incentive conflict. While the first factor is well-studied in decentralized supply chains consisting of a single supplier and a single retailer, the second factor is directly related to the existence of multiple retailers in our supply chain setting. We show that constant wholesale prices and state-dependent adjustment prices can be used to address the incentive misalignment and to coordinate the system. Under coordination, the supplier can extract all of the additional system profit achieved from coordination. To demonstrate the value, to the manufacturer, of our proposed coordination method, in our numerical study we compared the performance of the Comicfans' approach with the coordinating adjustment and wholesale prices. We demonstrated that the Comicfans' approach to setting the adjustment contract prices can perform quite poorly, while using the coordinating contract prices can substantially improve the system profit.

# 7    Reference

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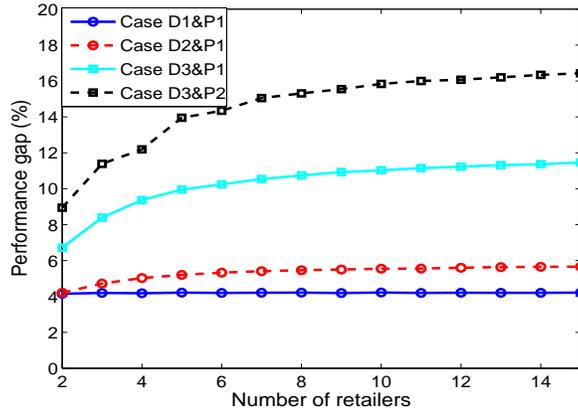
Figure 1: Performance gap between decentralized system and coordinated system with respect to number of retailers.
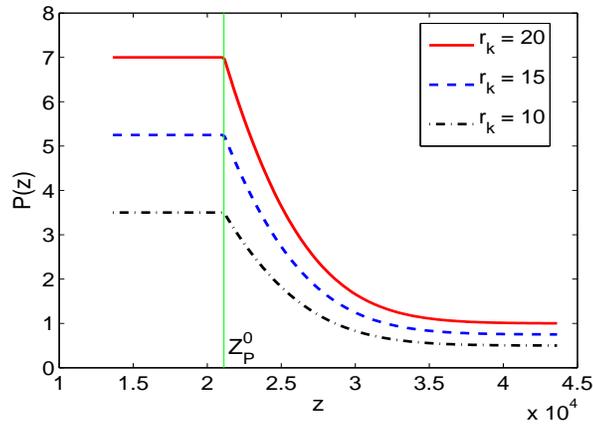


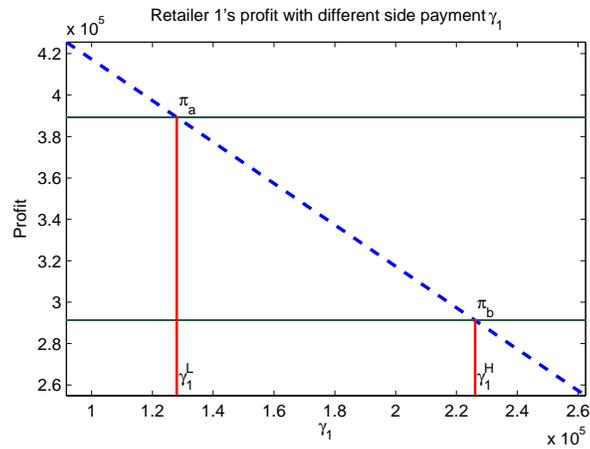Figure 2: Adjustment contract price with respect to system inventory level.



Figure 3: Coordination with 5 retailers: The coordinating range of $\gamma_1$ for retailer 1.

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