Evolutionary stability of a manufacturer-buyer VMI-conduced supply chain

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Abstract

The strategy of integration known as VMI (*Vendor-Managed Inventory*) allows the coordination of inventory policies between producers and buyers in supply chains. Based on a new proposed model for the implementation of VMI in a chain of two links composed of a producer and a buyer, this paper studies the evolution of individual strategies of the producer and the buyer by a formalism derived from the theory of evolutionary games. The conditions that determine the stability of evolutionarily stable strategies are derived and analyzed. Work results specify analytical conditions that favor the implementation of VMI on traditional chains without VMI.

Keywords: Supply chain coordination, game theory, dynamic systems stability.

Introduction

A full integration of the supply chain has become one of the industry's greatest dreams, thanks to the success achieved by different businesses (Darwish and Odah 2010). Initiatives like "efficient customer response" in the grocery industry and "quick response" in the garment industry (Waller et al. 1999) are good examples of this concept. Vendor-Managed Inventory (VMI) is a supply chain initiative where the vendor decides on the appropriate inventory levels of each of the products and the appropriate inventory policies to maintain those levels. In recent years, there has been a growing interest in implementing VMI initiatives (Emigh 1999), due to important recognition from different industrial leaders (Southard and Swenseth 2008). This interest stems from the fact that there are benefits for the whole chain in terms of cost reduction, improved service levels and supplier performance (Choi et al. 2004). VMI is a coordination mechanism that improves multi-firm supply chain efficiency (Waller et al. 1999) between a supplier and its customers (Silver et al. 1998) VMI can decrease inventory levels, increase fill rates in the supply chain (Yao et al. 2007) and reduce lead times and inventory stock-outs (Daugherty et al. 1999).

The models presented in this paper analyze a two-level supply chain in which external demand for a single item takes place at the purchaser. The paper proposes models with and without VMI, and establishes a cooperative VMI system by sharing demand and inventory information between agents (Lee et al. 2009, Cachon and Zipkin 1999). Through a formalism based on evolutionary game theory, this paper characterizes the stability of individual strategies of the producer and the buyer and deduces the analytical conditions that favor the implementation of VMI for the two agents. The analysis allowed us to identify and characterize the evolutionary stable strategy (ESS) of the supply chain in implementing VMI. This article is divided as follows: Section 2 is a review of the literature. Section 3 describes the proposed model and develops the main results. Section 4 presents the evolutionary stability analysis of the supply chain and shows the results of four experiments using the model. Section 5 presents the main conclusions and future research avenues.

Literature review

The first VMI models appeared in the late 1980s, when Wal-Mart, K-Mart and Procter & Gamble implemented major projects relating to supply-chain integration (Waller et al. 1999, Blatherwick 1998). However, not until recently was this subject discussed in academic literature (Southard and Swenseth 2008). (Blatherwick 1998) analyzed some of VMI's benefits and disadvantages to the agents involved in the agreements. (Holmström 1998) studied and characterized the adaptation of SAP R/3 in a partnership relationship within the context of VMI. (Emigh 1999) presented VMI cases in different industrial sectors. A number of papers have addressed statistical characterization of VMI models, starting with (Daugherty et al. 1999) who presented the statistical results of a survey about the implementation of automatic replenishment programs. Several studies make use of discrete event simulation techniques. (Waller et al. 1999) compared order frequency in different scenarios, facing inventory reduction through experimentation with a VMI strategy. Additionally, (Disney and Towill 2002) designed a VMI system with different cost levels and proposed a simulation method to determine the optimal parameters in the chain. (Cachon and Zipkin 1999) analyzed a two-level chain with stationary stochastic demand. The same approach was developed by (Lee et al. 2000), who modeled a chain consisting of a manufacturer and a retailer in the presence of stochastic demand and information sharing between agents. (Yao et al. 2007) presented an analytical model applied to supply chains of two agents with and without VMI. (Yao and Dresner 2008) proposed a model consisting of a manufacturer and a retailer with stochastic demand. A recent VMI approach has incorporated game theory approaches. In this category, the work of (Yu et al. 2009a) used evolutionary game theory to analyze a strategy of evolutionary stability in supply chains with VMI. A contemporaneous work by (Yu and Huang 2009) formulated a model of a manufacturer and multiple retailers based on an analysis of a response function and a generic demand function. (Yu et al. 2009b) analyzed the interaction between a manufacturer and its retailers to optimize its marketing strategy.

The model of coordination between producer and buyer driven by VMI

The supply chain we study consists of a manufacturer and a buyer implementing VMI for a single product. This problem has been studied by (Dong and Chu 2009), (Choi et al. 2004), and (Yao et al. 2007). We propose a coordination scheme where the key difference with existent approaches involves a synchronization mechanism between the buyer and manufacturer replenishment cycles.

The notation used in our model is described as follows:

Parameters: $C, c, c', H, h, P, r, d, \delta, g, g'$

Variables: $T, t, Q, q, T_s, k, L, U, \tau_s, I_s$

Where:

C = Setup (ordering) costs for the manufacturer (in $/setup)

c = Setup (ordering) costs for the buyer without VMI (in $/setup)

c'= Setup (ordering) costs for the buyer with VMI (in $/setup)

H = Holding cost of manufacturer inventory (in $/unit/year)

h = Holding cost of buyer inventory (in $/unit/year)

P = Manufacturer production rate (in units/year)

r = Demand rate (in units/year)

$d = H/h$ = Manufacturer and buyer inventory holding cost ratio

$\delta = r/P$ = Demand and production rate

$g = C/c$ = Manufacturer and buyer setup (ordering) cost ratio without VMI

$g' = C/c'$ = Manufacturer and buyer setup (ordering) cost ratio with VMI

T = Manufacturer replenishment time (in years)

t = Buyer replenishment time (in years)

Q = Manufacturer lot size or total quantity manufactured over replenishment time T (in units)

q = Buyer lot size or total quantity demanded over replenishment time t (in units)

$T_s = q/P$ = Manufacturing time of buyer lot size q (in years)

k = Number of buyer shipments placed during the manufacturer replenishment time (integer)

L = Number of buyer shipments placed during the manufacturer uptime (integer)

U = Manufacturer uptime (in years)

$\tau_s = U - Lt$ = Fractional manufacturer up time (in years)

I_s = Manufacturer average inventory (in units)

The system is studied before and after VMI implementation and is presented in Figure 1. We adopted the convention used by (Yao et al. 2007) that uppercase parameters are for the manufacturer and lowercase parameters are for the buyer. Production rate is constant and denoted with P and $P \geq r$. The buyer replenishment time is represented by t. The manufacturer replenishment time T is kt (with k integer) and contains L buyer replenishment cycles (with L integer). The time required to produce a lot size required for the buyer (q) is denoted by T_s. The lot size of the manufacturer is $Q = kq$. In our model we explicitly consider the uptime $U = Lt + \tau_s$. This uptime is not taken into consideration in other manufacturer-buyer VMI approaches (Dong and Chu 2009, Choi et al. 2004, Yao et al. 2007, Yang et al. 2003).

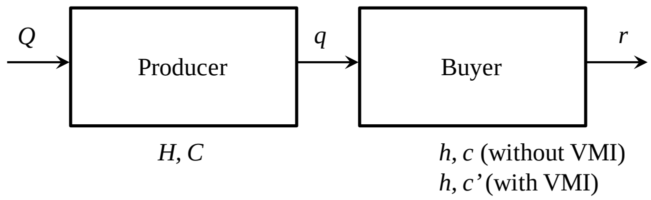

Figure 1 – The studied logistic system

The explicit replenishment coordination mechanism between the manufacturer and the buyer is represented in Figure 2. We have deduced the mathematical conditions (Equations 4-10) needed to achieve the manufacturer-buyer synchronization.

Without VMI, because the buyer average inventory level is driven by a simple EOQ model, his or her average inventory level is *q/2*. As a consequence, the buyer's average total annual holding and setup cost is given by:

$$f(q) = c\frac{r}{q} + h\frac{q}{2} \tag{1}$$

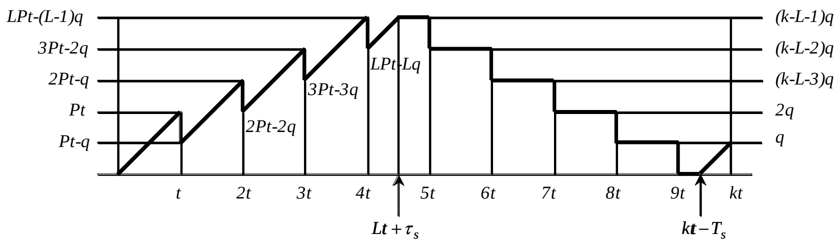

Figure 2 – Manufacturer's inventory level and the proposed manufacturer-buyer coordination mechanism under VMI

Similarly, without VMI the manufacturer is guided by a finite production rate model. The change in manufacturer inventory level over time is shown in Figure 3. Average inventory level can be deduced according to the economic production quantity (EPQ) model (Silver et al. 1998). In consequence, the manufacturer's average total annual holding and setup cost is:

$$F(Q) = C\frac{r}{Q} + H\frac{Q}{2}\left(1 - \frac{r}{P}\right) \tag{2}$$

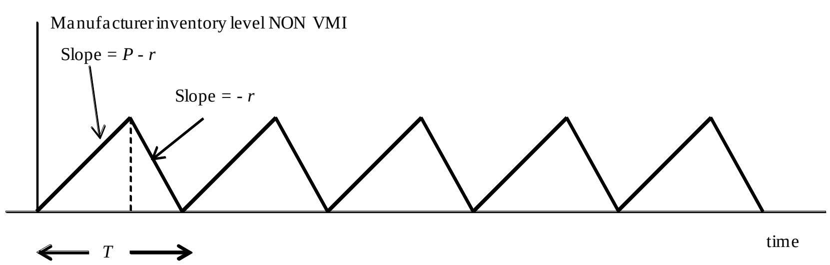

Figure 3 – Manufacturer's inventory level without VMI

It follows that the optimal total costs of the system without VMI are:

$$TC^{*}_{NON\ \ VMI} = \sqrt{2r}\left(\sqrt{CH\left(1-\frac{r}{P}\right)} + \sqrt{ch}\right) \tag{3}$$

Considering our manufacturer-buyer coordinated and synchronized VMI system shown in Figure 2, if I_S is the manufacturer's average inventory, the total cost of the manufacturer-buyer coordinated VMI system is given by:

$$TC_{VMI} = c'\frac{r}{q} + h\frac{q}{2} + C\frac{r}{Q} + HI_S \tag{4}$$

In our VMI approach, we can calculate the manufacturer's average inventory (denoted by I_s), given by Equation 5.

$$I_S = \frac{q}{2}\left[k\left(1-\frac{r}{P}\right) + 2\frac{r}{P} - 1\right] \tag{5}$$

Optimizing the expression in Equation 4 and relaxing the integrality condition on *k*, the expressions for the optimal total cost for the buyer and the producer with and without VMI are given in equations 6, 7, 8 and 9.

$$TC_{manufacturer,VMI} = \sqrt{2CHr\left(1-\frac{r}{P}\right)} + \frac{H}{2}\left(2\frac{r}{P}-1\right)\sqrt{\frac{2c'r}{\left(h+H\left(2\frac{r}{P}-1\right)\right)}} \tag{6}$$

$$TC_{buyer,VMI} = \frac{1}{2}\sqrt{\frac{2c'r}{\left(h+H\left(2\frac{r}{P}-1\right)\right)}}\left[2h + H\left(2\frac{r}{P}-1\right)\right] \tag{7}$$

$$TC_{manufacturer,non\ VMI} = \sqrt{2CHr\left(1-\frac{r}{P}\right)} \tag{8}$$

$$TC_{buyer,non\ VMI} = \sqrt{2crh} \tag{9}$$

Two cost matrices associated with the implementation of VMI for the producer and the buyer (Yu et al. 2009a) are proposed in Table 1. Each of the actors in the chain can take these individual decisions: implement or not VMI. In the proposed cost matrices, *m* and *n* are the investment amounts made by the manufacturer and the buyer, respectively, when they adopt the VMI strategy. Parameters p_1, p_2 are the goodwill losses of the manufacturer and the buyer respectively when they violate the VMI contract.

Table 1 – The cost matrices of the manufacturer and buyer strategies

	Buyer VMI	Buyer non VMI
Manufacturer VMI	$\begin{bmatrix} TC_{manufacturer\ VMI} \\ TC_{buyer\ VMI} \end{bmatrix}$	$\begin{bmatrix} TC_{manufacturer\ non\ VMI} + m \\ TC_{buyer\ non\ VMI} + p_2 \end{bmatrix}$

Manufacturer non VMI	$\begin{bmatrix} TC_{manufacturer\ non\ VMI} + p_1 \\ TC_{buyer\ non\ VMI} + n \end{bmatrix}$	$\begin{bmatrix} TC_{manufacturer\ non\ VMI} \\ TC_{buyer\ non\ VMI} \end{bmatrix}$

Analysis of evolutionary stability of the supply chain driven by VMI

Evolutionary game theory can be used to study the evolutionary stability of strategies followed by buyer and producer in the game depicted in Table 1. Evolutionary game theory combines static feature of an evolutionary stable strategy (ESS) (Yu et al. 2009a) with dynamic nature of Replicator dynamics by (Taylor and Jonker 1978). The concept of ESS was initially proposed by (Maynard Smith 1973, 1974, 1982). (Taylor and Junker 1978) proposed a dynamic equation, called "Replicator dynamics" which reflects the dynamics and interaction among the agents involved in the game (Yu et al. 2009a). For the game we are analyzing, the cost matrices of Table 1 should be multiplied by -1 to obtain fitness matrices ***A*** and ***B*** for producer and buyer, respectively. Assuming that α is the probability that the producer selects the VMI strategy and β is the probability that the buyer selects the VMI strategy, Replicator dynamics equations are:

$$\frac{d\alpha}{dt} = \alpha\left[U_{manufacturer,VMI}(t) - \tilde{U}_{manufacturer}(t)\right] \quad (11)$$

$$\frac{d\beta}{dt} = \beta\left[U_{buyer,VMI}(t) - \tilde{U}_{buyer}(t)\right] \quad (12)$$

Where $U_{manufacturer,VMI}(t)$ and $U_{buyer,VMI}(t)$ correspond to the expected payoff of the producer and the buyer at time *t* when they implements VMI; $\tilde{U}_{manufacturer}(t)$ is the producer average payoff at time *t* and $\tilde{U}_{buyer}(t)$ is the buyer average payoff at time *t*.

Evolutionary stability of the producer-buyer supply chain VMI-conduced

The stable states of the Replicator dynamic equation are the Nash equilibrium (NE). They are often referred to as the evolutionary equilibriums (EE) (Ellison and Fudenberg 2000) When $\frac{d\alpha}{dt} = 0$ and $\frac{d\beta}{dt} = 0$, the EE are: $E_1 = (0,0), E_2 = (0,1), E_3 = (1,0), E_4 = (1,1),$

and $E_5 = \left(\frac{n}{(n + p_2 + TC_{buyer,non\ VMI} - TC_{buyer,VMI})}, \frac{m}{(m + p_1 + TC_{manufacturer,non\ VMI} - TC_{manufacturer,VMI})} \right)$

The stability of EE (Friedman 1991) can be analyzed by the Jacobi matrix ***J***, which is given by:

$$J = \begin{bmatrix} \frac{\partial}{\partial\alpha}\left(\frac{d\alpha}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\alpha}{dt}\right) \\ \frac{\partial}{\partial\alpha}\left(\frac{d\beta}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\beta}{dt}\right) \end{bmatrix}$$

Stability of the EE depends on sign of tr(***J***) y det(***J***). The results of this analysis are represented in Table 2.

Table 2 – The local stability of EE

EE	tr(***J***)	det(***J***)	*Type of stability*

E_1	$-m-n < 0$	$mn > 0$	*ESS*
E_2	$n+p_1+TC_{man,non\ vmi}-TC_{man,vmi}$	$n(p_1+TC_{man,non\ vmi}-TC_{man,vmi})$	*Unstable or Saddle*
E_3	$m+p_2+TC_{buyer,non\ vmi}-TC_{buyer,vmi}$	$m(p_2+TC_{buyer,non\ vmi}-TC_{buyer,vmi})$	*Unstable or Saddle*
E_4	$-p_1-TC_{man,non\ vmi}+TC_{man,vmi}$ $-p_2-TC_{buyer,non\ vmi}+TC_{buyer,vmi}$	$(p_1+TC_{man,non\ vmi}-TC_{man,vmi})$ $(p_2+TC_{buyer,non\ vmi}-TC_{buyer,vmi})$	*ESS or Saddle or Unstable*
E_5	0	<0	*Saddle*

The stability analysis shows that when penalties p_1 and p_2 become large, the equilibrium E_4 -associated with producer and buyer both selecting as their preferred strategy to implement VMI- becomes an ESS. In this case, the equilibrium E_5 is located inside the square of probabilities and merges with E_1. Under these circumstances for the supply chain is more favorable than both agents support the implementation of VMI.

Experiments to study the effect of the parameters on the evolutionary stability of the supply chain

The simulation software "Dynamo" (Sandholm and Dokumaci 2007) was used to study the evolutionary dynamics of producer and buyer strategies in the supply chain. Varying the system parameters, we simulated 4 different logistics systems. With the help of the software, phase diagrams of each experiment were obtained. The selected parameter sets are shown in Table 3.

Table 3 – The selected parameter sets of each experiment

Parameters	Experiment 1	Experiment 2	Experiment 3	Experiment 4
C	300	300	300	300
c	100	100	100	100
c'	80	80	80	99
H	19	19	19	19
h	20	20	20	20
P	2000	2000	4000	4000
r	200	1800	320	3990
m	300	300	300	300
p_1	100	100	100	10
p_2	200	200	200	200
n	500	500	500	500

Phase diagrams obtained in each experiment are shown in Figure 4. The payoff matrices and characterization of the stability of each EE in each experiment are presented in Table 4.

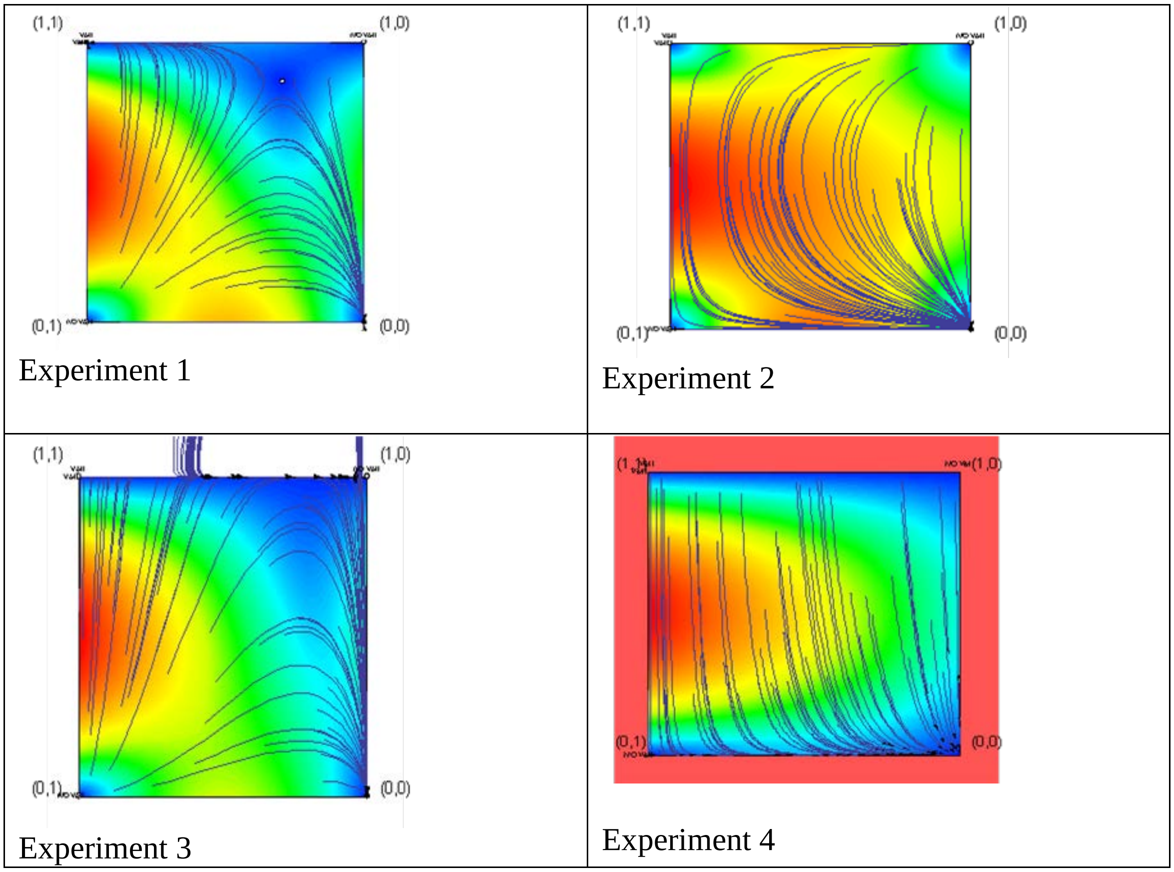

Figure 4 – Phase Diagrams of the experiments

The logistic system presented in experiment 1, shows saddle equilibrium in E_5. In this case, the supply chain can evolve into stable equilibriums characterized in that either both agents implement or do not implement VMI. The convergence in to one of these two ESS states E_1 or E_4 depends on the initial conditions. Experiments 2, 3 and 4 show that demand and capacity are critical parameters because they affect stability type of each EE. These experiments also illustrate that the equilibrium E_4 -associated with producer and buyer both selecting as their preferred strategy to implement VMI- can easily evolve to any type of equilibrium depending of parameter settings and that equilibrium E_5 does not always exist inside the feasible square of probabilities $[0,1]^2$. The analytical condition that ensures that E_4 is a ESS corresponds to tr($\boldsymbol{J}$)<0 and det($\boldsymbol{J}$)>0. In consequence, agents must carefully select the supply chain parameters to ensure that the optimal strategy for both is to implement VMI. For example, from local stability conditions, we can deduce the next results:

1. If $\frac{r}{P} < \frac{1}{2}$ and $\frac{c'}{c} < \frac{\left[1+\frac{H}{h}\left(2\frac{r}{P}-1\right)\right]}{\left[1+\frac{H}{2h}\left(2\frac{r}{P}-1\right)\right]^2}$, the equilibrium E_4 is ESS for any value of the parameters p_1 and p_2.

2. If $p_1 > \frac{H}{2}\left(2\frac{r}{P}-1\right)\sqrt{\frac{2c'r}{\left[h+H\left(2\frac{r}{P}-1\right)\right]}}$ and $p_2 > \sqrt{2rch}\left\{\sqrt{\frac{c'}{c}}\frac{\left[1+\frac{H}{2h}\left(2\frac{r}{P}-1\right)\right]}{\sqrt{1+\frac{H}{h}\left(2\frac{r}{P}-1\right)}}-1\right\}$, the equilibrium E_4 is ESS.

Table 4 – The analyses results of each experiment

Experiment 1

The payoff matrix:

	Buyer VMI	Buyer non VMI
Manufacturer VMI	(-811.94 , -1012.46)	(-1732.48, -1094.43)
Manufacturer non VMI	(-1532.48, -1394.43)	(-1432.48, -894.43)

EE	Stability
E_1	ESS
E_2	Unstable
E_3	Unstable
E_4	ESS
E_5 = (.86, .29)	Saddle

Experiment 2

The payoff matrix:

	Buyer VMI	Buyer non VMI
Manufacturer VMI	(-2119.93 , -2496.51)	(-1732.48, -2888.28)
Manufacturer non VMI	(-1532.48, -3183.28)	(-1432.48, -2683.28)

EE	Stability
E_1	ESS
E_2	Saddle
E_3	Unstable
E_4	Saddle
E_5 does not exist	-

Experiment 3

The payoff matrix:

	Buyer VMI	Buyer non VMI
Manufacturer VMI	(-933.63 , -1353.16)	(-2131.98, -1331.37)
Manufacturer non VMI	(-1931.98, -1631.37)	(-1831.98, -1131.37)

EE	Stability
E_1	ESS
E_2	Unstable
E_3	Saddle
E_4	Saddle
E_5 does not exist	-

Experiment 4

The payoff matrix:

	Buyer VMI	Buyer non VMI
Manufacturer VMI	(-1684.63 , -4197)	(-637.22, -4195)
Manufacturer non VMI	(-347.22, -4195)	(-337.21, -3995)

EE	Stability
E_1	ESS
E_2	Saddle
E_3	Saddle
E_4	Unstable
E_5 does not exist	-

Conclusions and future work

This paper presents an analysis of a supply chain between a producer and a buyer driven by VMI as coordination strategy. The analysis is carried out using an approach based on evolutionary game theory and the study of evolutionary stability of the producer and buyer strategies according to the replicator dynamics. The analysis allowed us to identify and characterize the ESS in implementing VMI. The dynamics equations show that both agents of the supply chain can adopt VMI as the preferred coordination strategy under certain parameter settings. A preliminary analysis shows that increasing the penalties for non-adoption of VMI strategies, favoring the implementation of the VMI strategy for both agents. The proposed dynamic model and stability analysis presented serve to quantify the value of these penalties and predict long-term behavior of individual agents based on system parameters and initial conditions. A deeper sensitivity analysis will study the effect of all other parameters on the stability of VMI-conduced collaborative strategies.

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